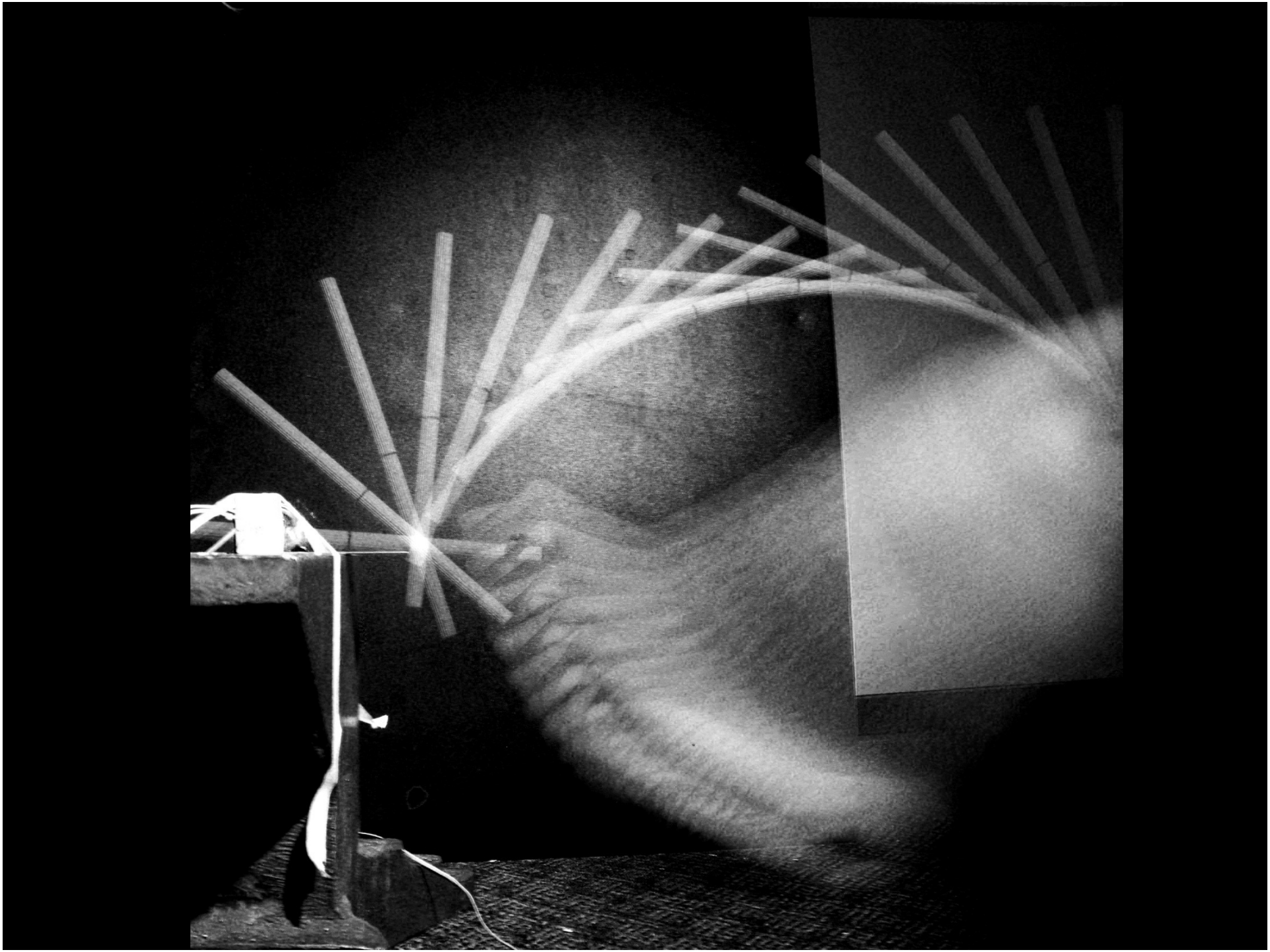
Place a small wooden stick over the edge of a desk. Hit the end of the stick overhanging the table so that it flies away. How is the flight distance related to the relevant parameters? What is the condition for a maximum horizontal distance?

1. Tipcat

Contents

1. Theory
   1. Collision
   2. Gliding
   3. Flight
2. Apparatus
3. Results
   1. Path of the stick
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4. Interpretation of the results
5. Conclusion



Theory

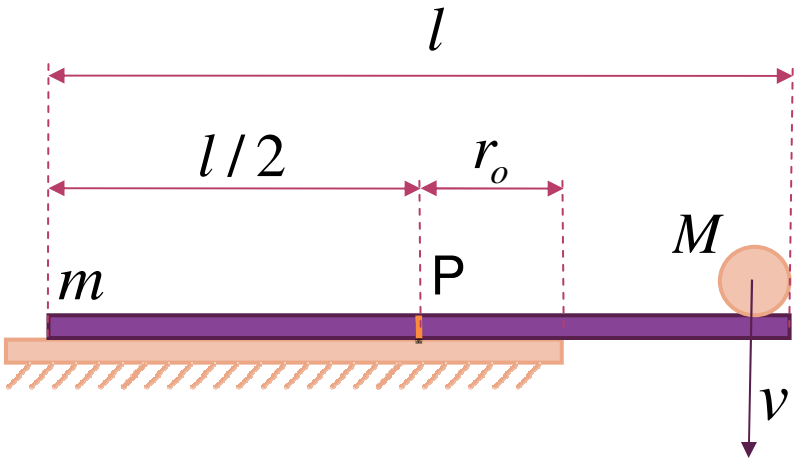
- Motion in three stages

1. Collision with the falling object
   - Interaction force between the bodies
   - (Friction force)
2. Sliding along the edge of the table
   - Reaction of the table
   - Gravitational force
3. Flying through the air
   - Gravitational force
   - (Air resistance)

The Collision

- $M \geq\geq m$
- Angular momentum conservation
- Energy conservation
  - λ– free parameter obtained due to coefficient in energy conservation

$$\omega = \frac{2\lambda v_0}{\frac{l}{2} - r_0}$$



l – length of the stick

r_o – distance of the center of mass (P) from the edge of the table

m – mass of the stick

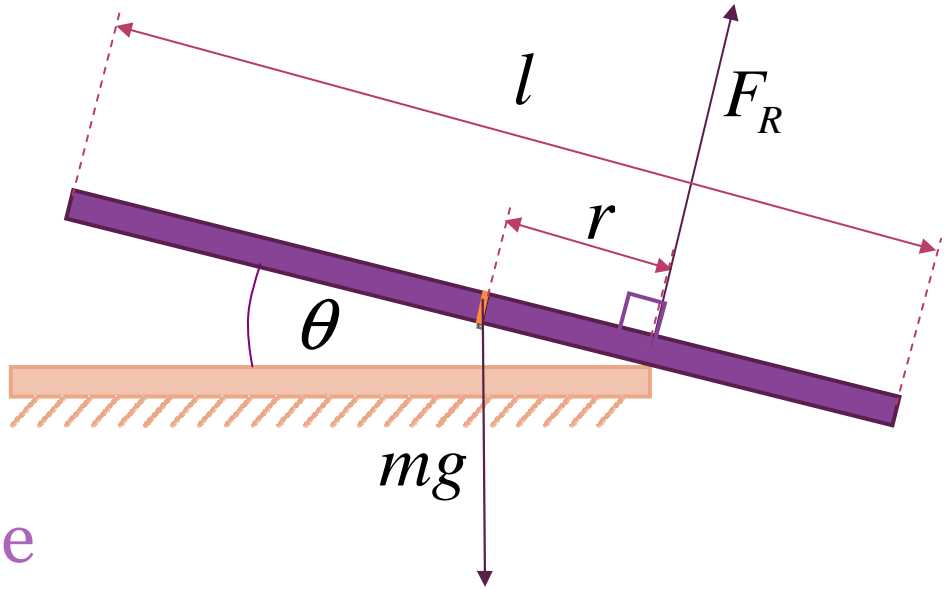
M – mass of the falling body

v_o – velocity of the falling body before the collision

ω – angular velocity of the stick after the collision

Sliding Along the Edge of the Table

- Position of the stick defined by
  - θ – angle between the stick and the table
  - r – distance of the center of mass (P) from the edge



l – length of the stick

m – mass of the stick

F_R – reaction force between the stick and the table

Equations of motion

- r-direction

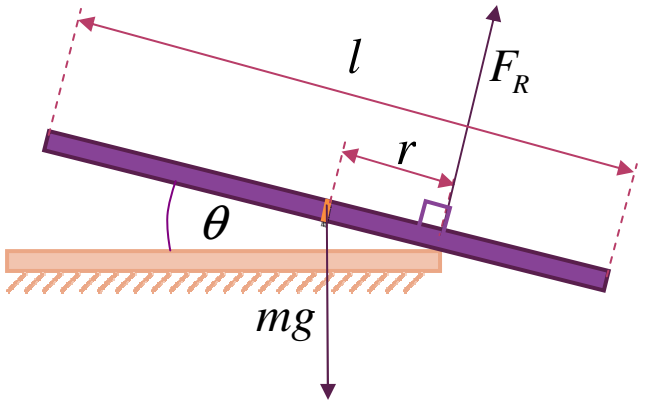
$$-mg\sin\theta = m(\ddot{r} - r\dot{\theta}^2)$$

- direction perpendicular to r

$$F_R - mg\cos\theta = 0$$

- rotation around the edge of the table

$$-mgr\cos\theta = \dot{L} = \dot{I}\dot{\theta} + I\ddot{\theta}$$



- L – angular momentum of the stick
- I – moment of inertia of the stick

- Can be analitically solved by estimating $\omega^2 l \geq\geq g$
- Numerically integrated for better accuracy

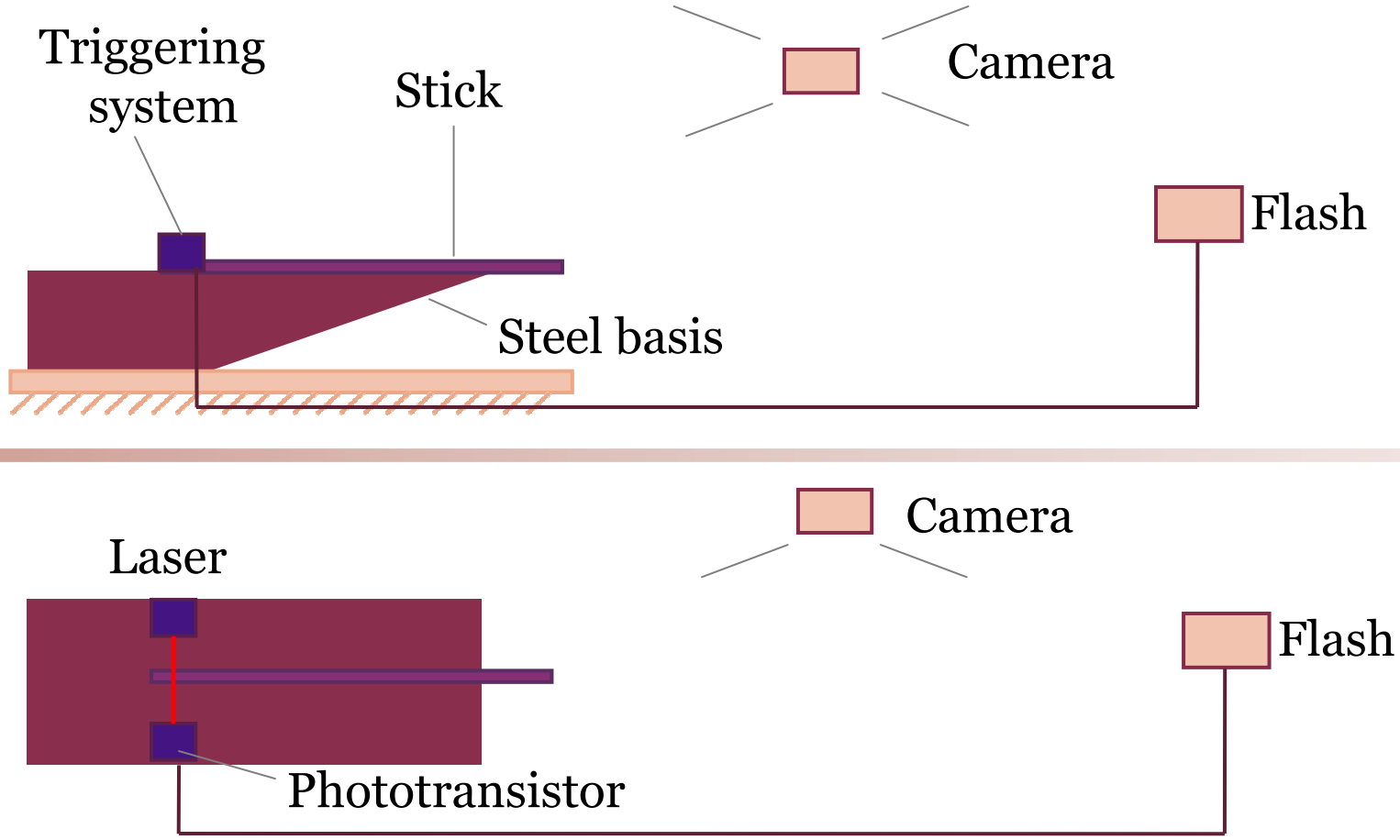
The Flight

- Initial conditions obtained from the previous stage (sliding)
  - Initial position
  - Initial speed of the center of mass
  - Angular speed around the center of mass
- Only force acting on the body is the gravitational force
  - Center of mass moves along a parabolic path
  - Angular speed around the center of mass constant
- Path of the flying object obtained

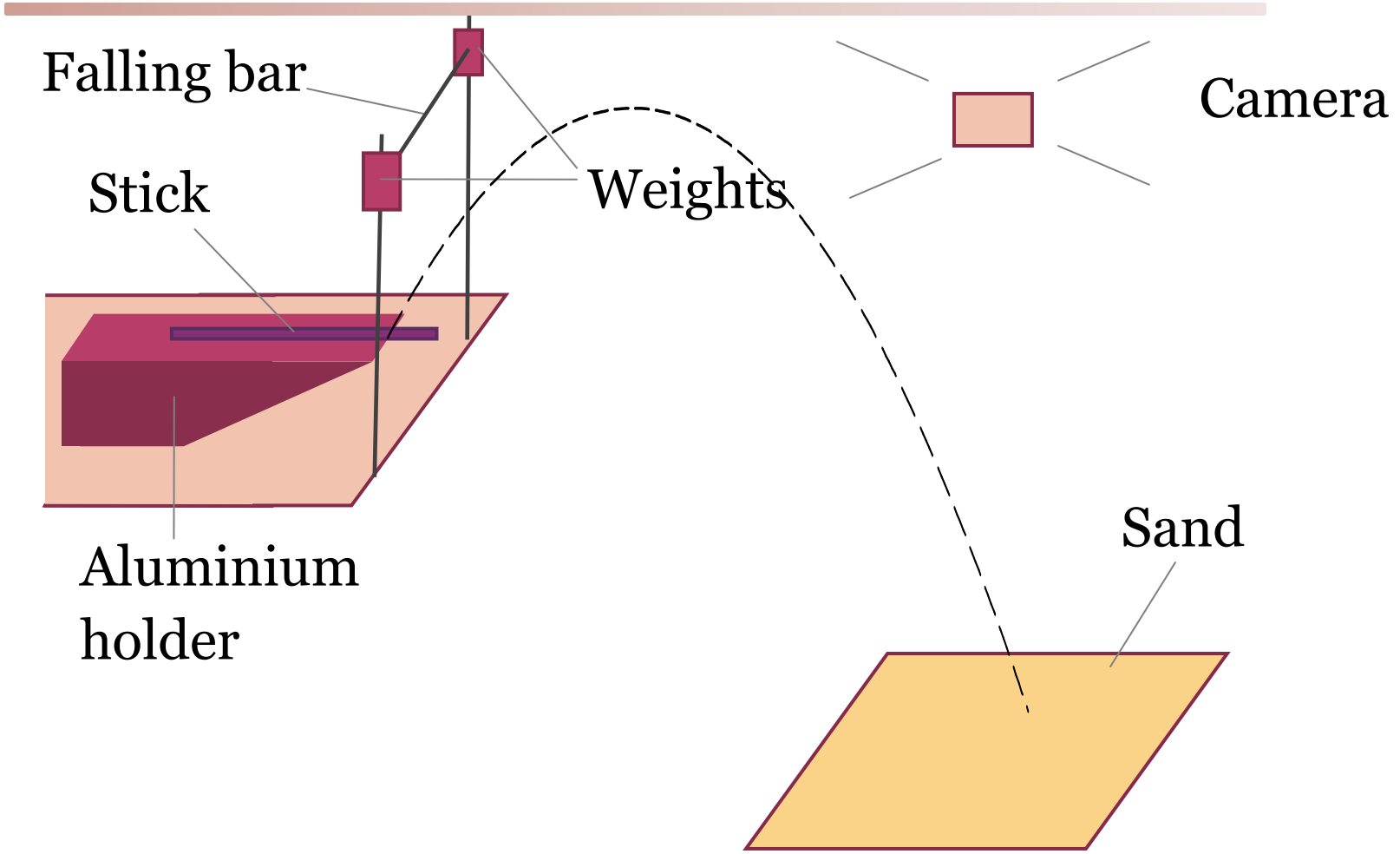
The Experiment

- Two aparatus were made
- Apparatus 1
  - Recording the sliding motion
- Apparatus 2
  - Recording the flight
  - Measuring the flight distance depending on parameters of the system
- Parameters observed
  - Initial position of the stick
  - Height of the falling object

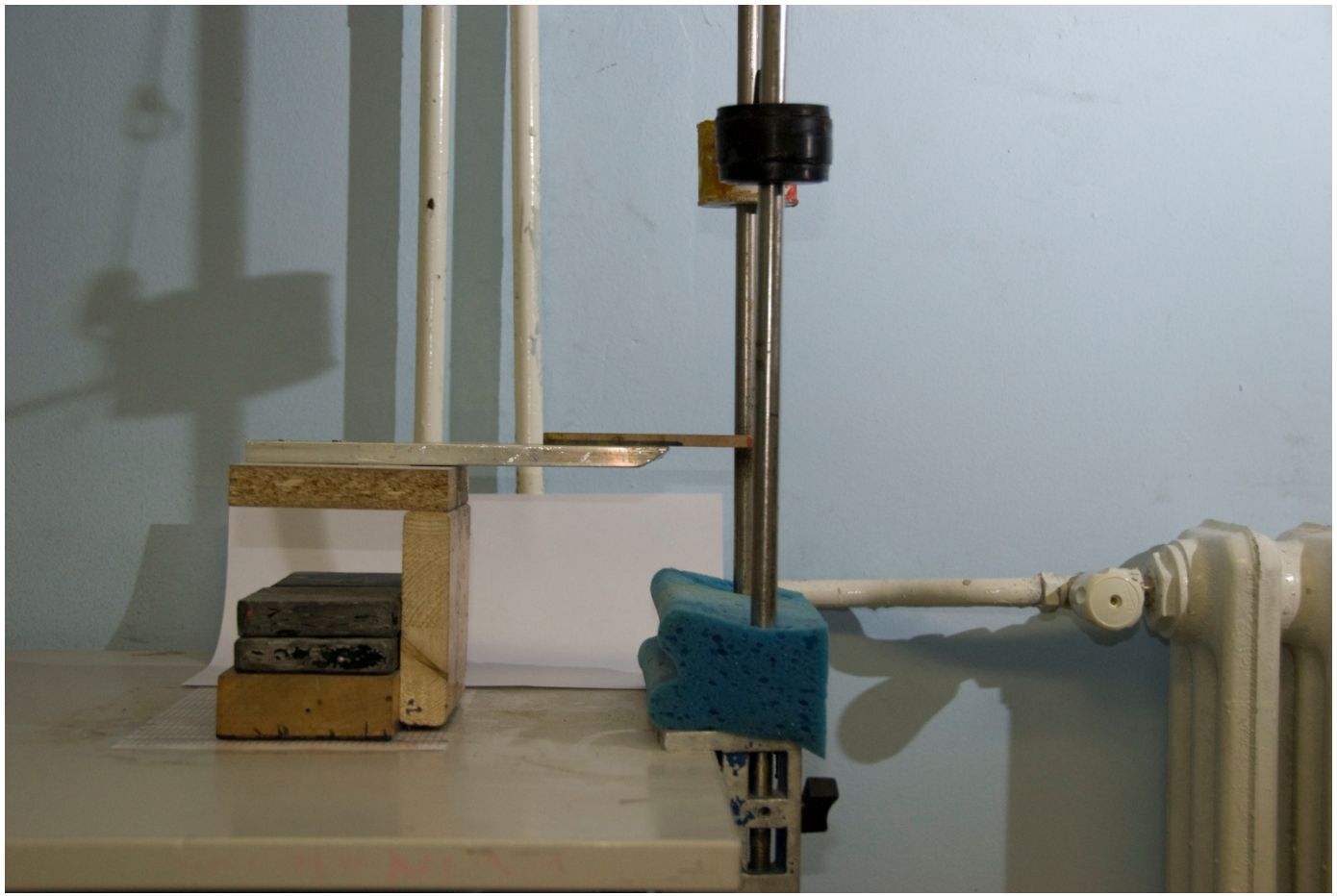
Apparatus 1


Triggering system
Stick
Camera
Flash
Steel basis
Camera
Laser
Flash
Phototransistor

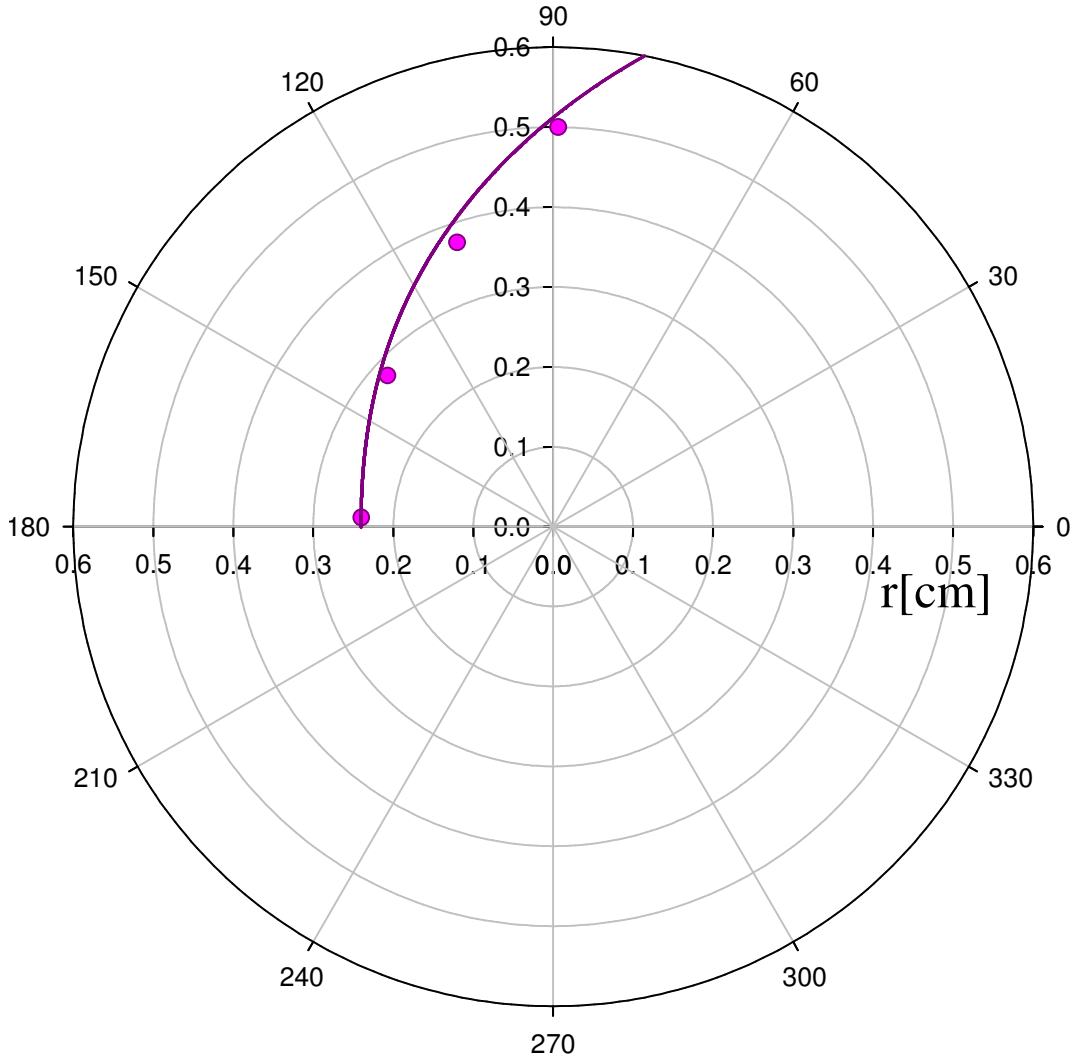
Apparatus 2



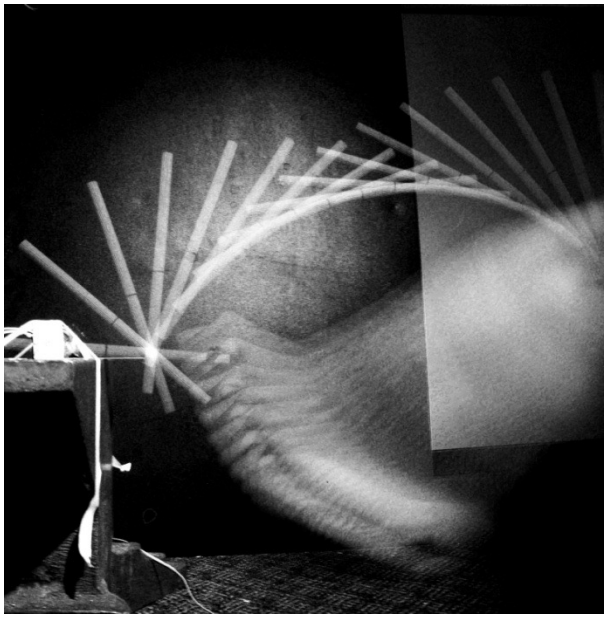
Apparatus 2



Results

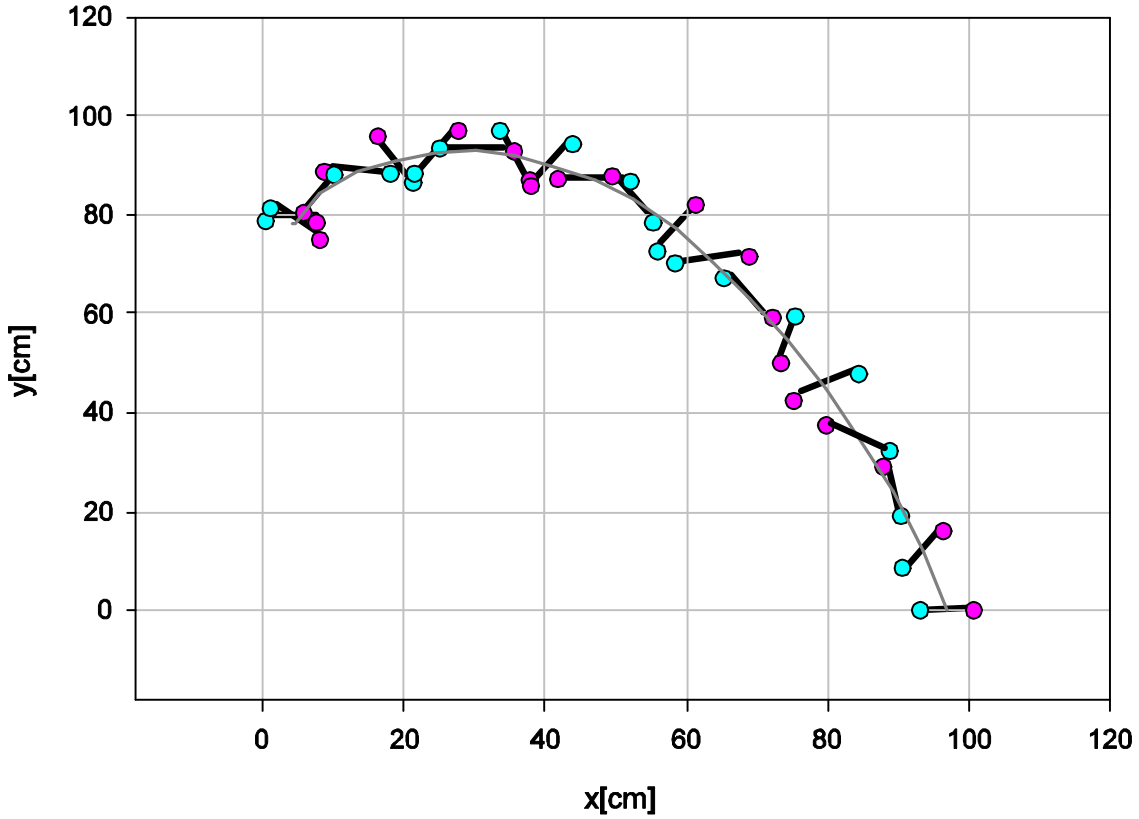


l=30 cm
r_o=5 cm
v_o=2 m/s



Results cont.

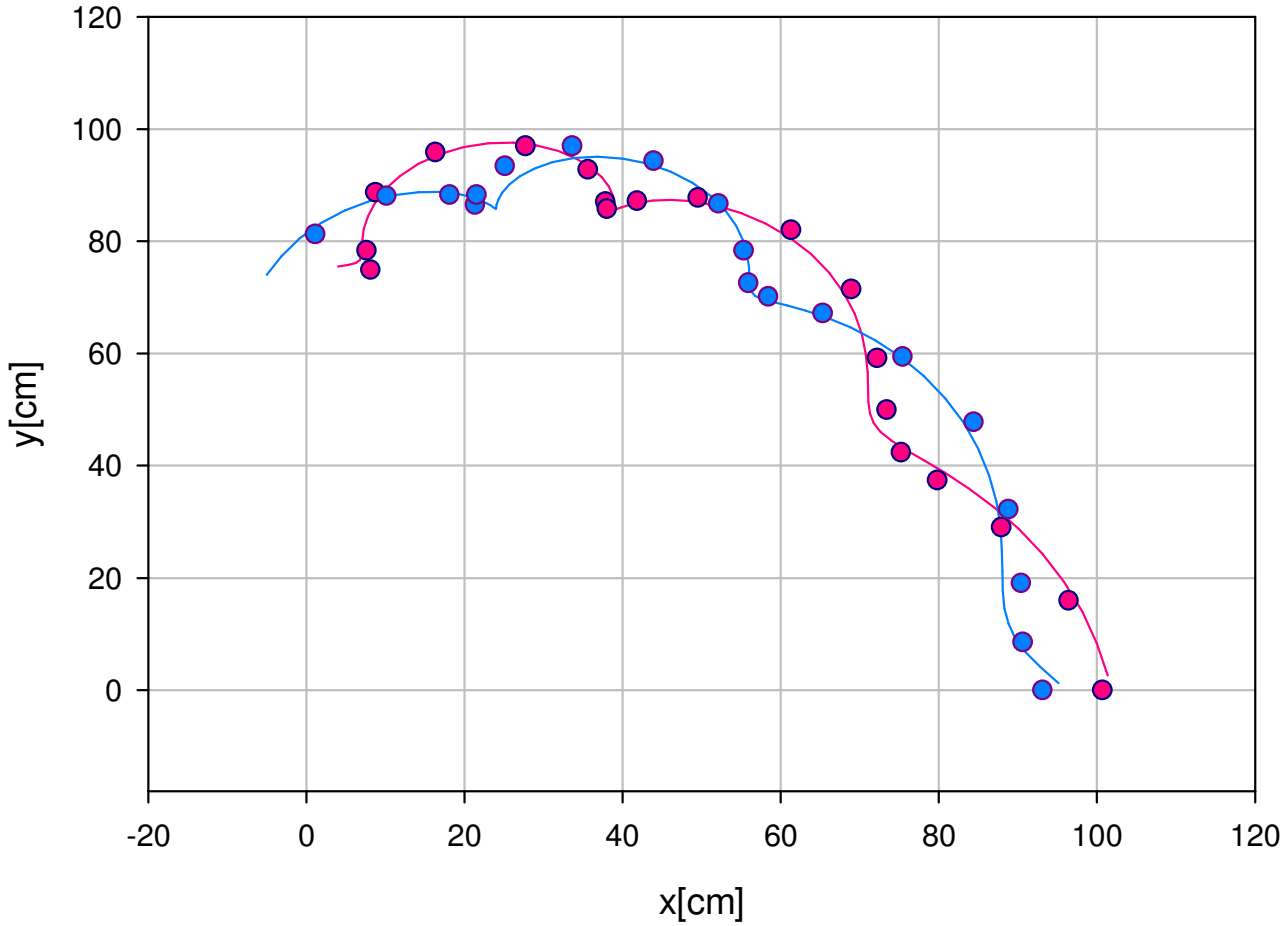
- 30 fps



l=12 cm
r_o=3 cm
v_o=1.5 m/s

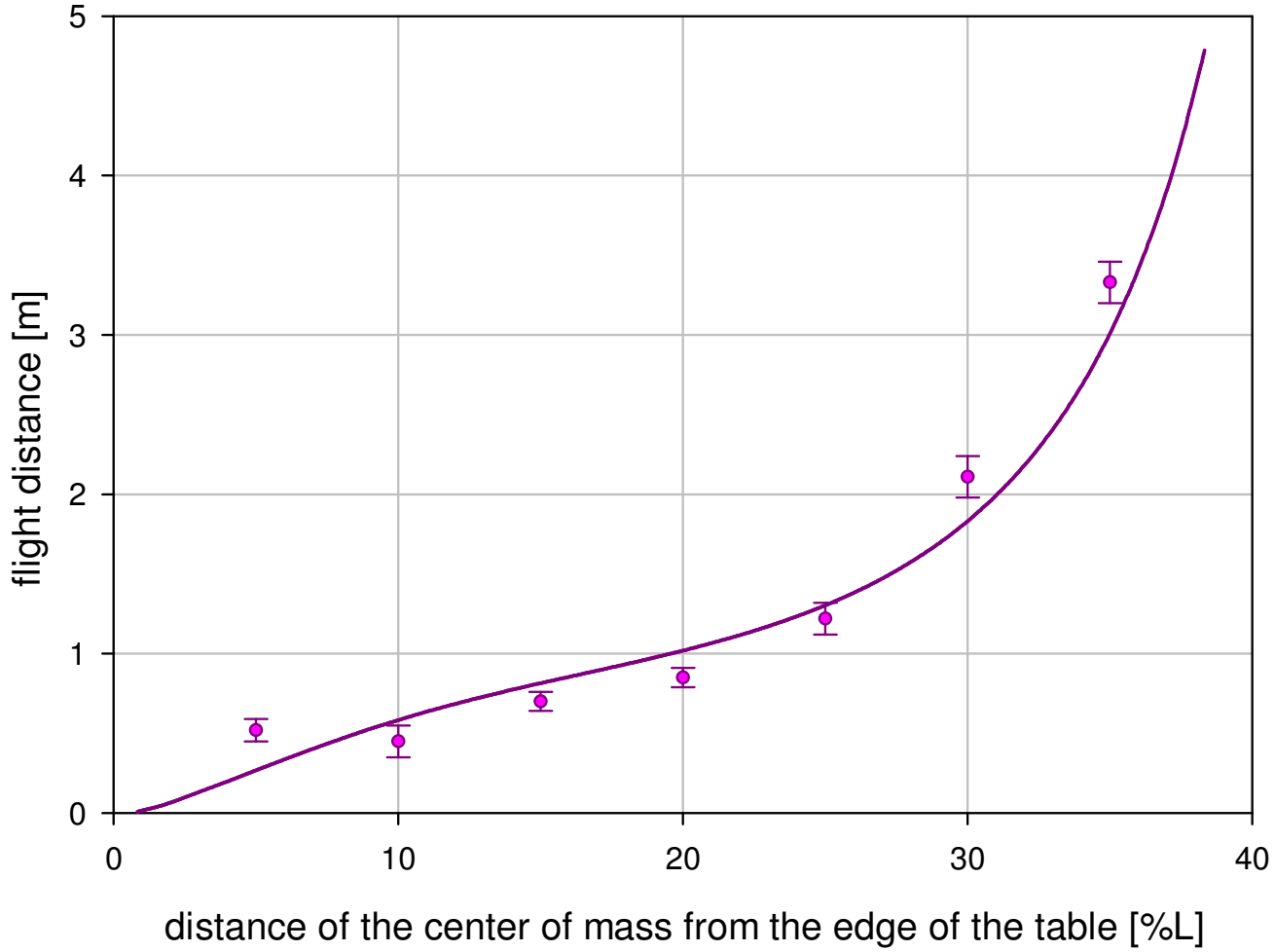
Results cont.

- 30 fps



l=12 cm
r_o=3 cm
v_o=1.53 m/s

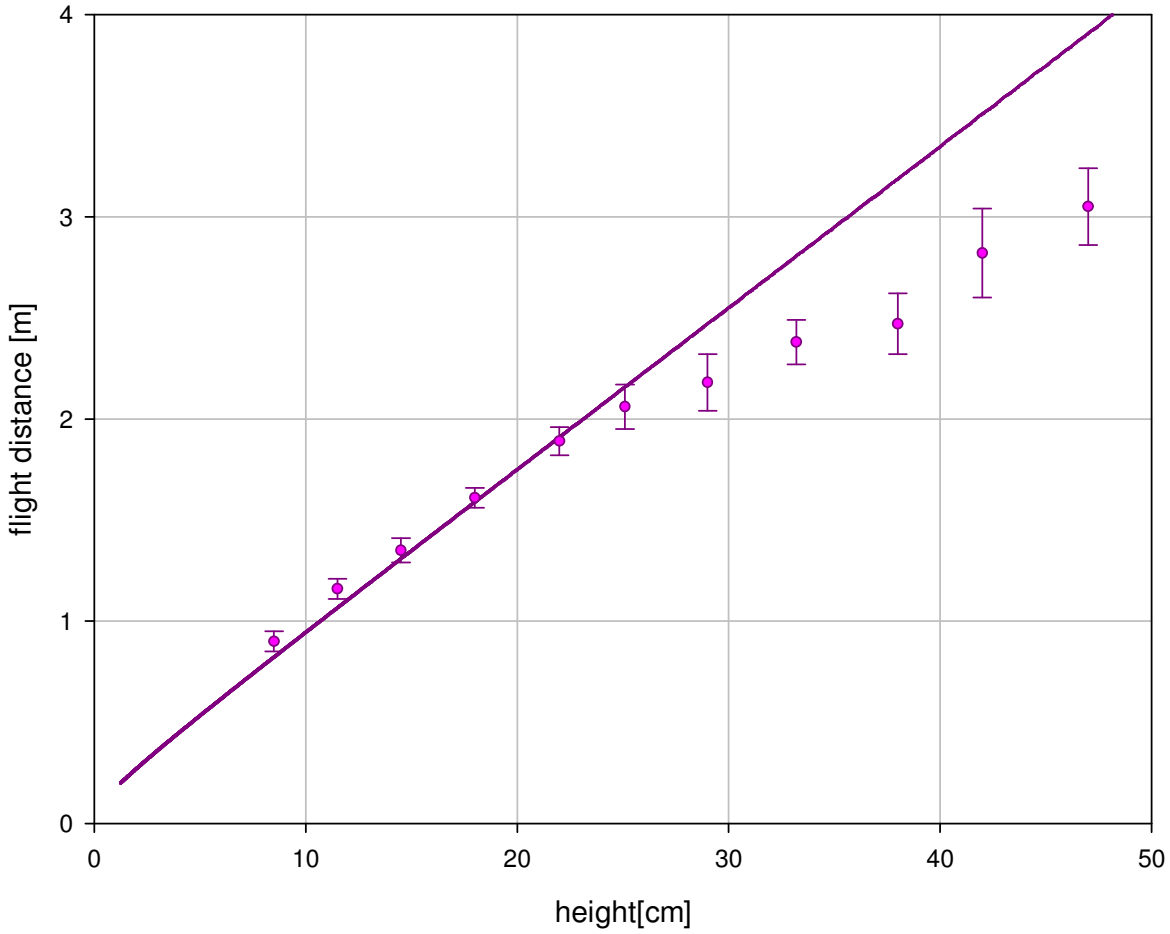
Results cont.



l=12 cm

v_o=1.72 m/s

Results cont.



l=12 cm

r_o=3 cm

Results summary

- Path of the stick
  - Almost perfect matching with the theory
  - Slight deflection in sliding due to forces of friction
- Dependence on the position
  - Increase of the flight distance with the distance of the center of mass from the edge of the table
  - Maximum for $r_o = \frac{1}{2}l$ (practical boundary)
- Dependence on height of the falling object
  - Deflection due to air resistance and the free parameter
  - Increase of the flight distance with height

Conclusion

- Theory
  - Path depending on the parameters of the system obtained from equations of motion
- Experiment
  - Full path of the stick was recorded
  - Dependence on
    - Position of the stick
    - Height of the falling object
- Maximum obtained for
  - $r_o = \frac{1}{2}l$ (with a practical boundary)
  - Maximum height

Thank you for your attention!