Problem 1.

Froth

The problem

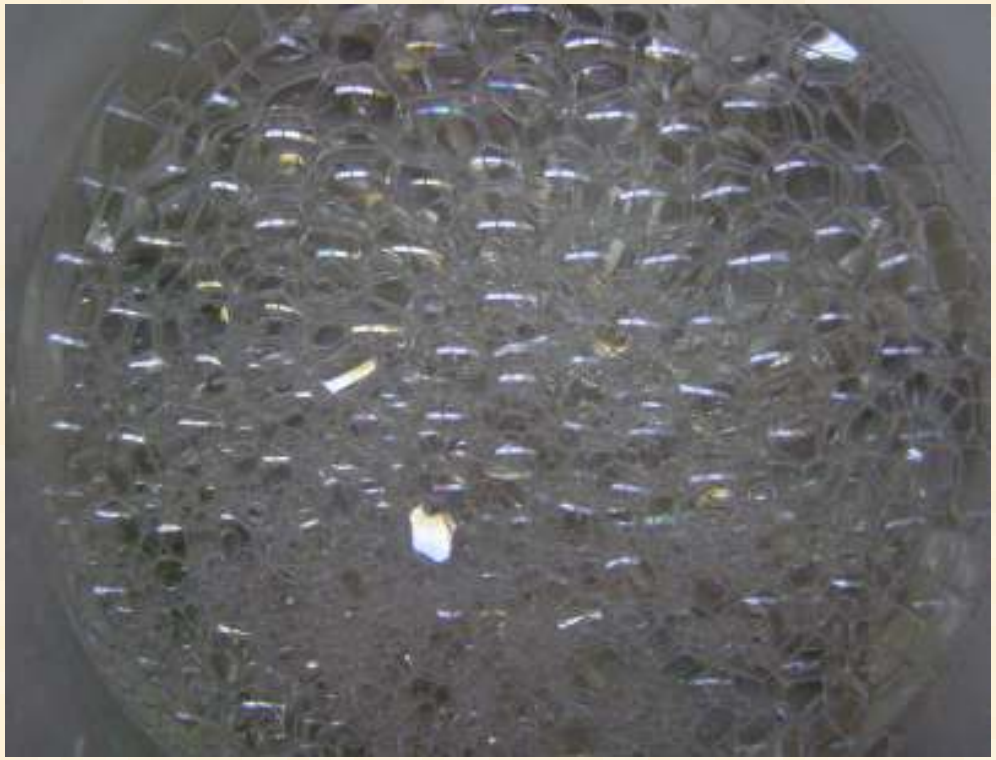
Investigate the nature of the decay in height of the 'froth' or 'foam' on a liquid. Under what conditions does the froth remain for the longest time?

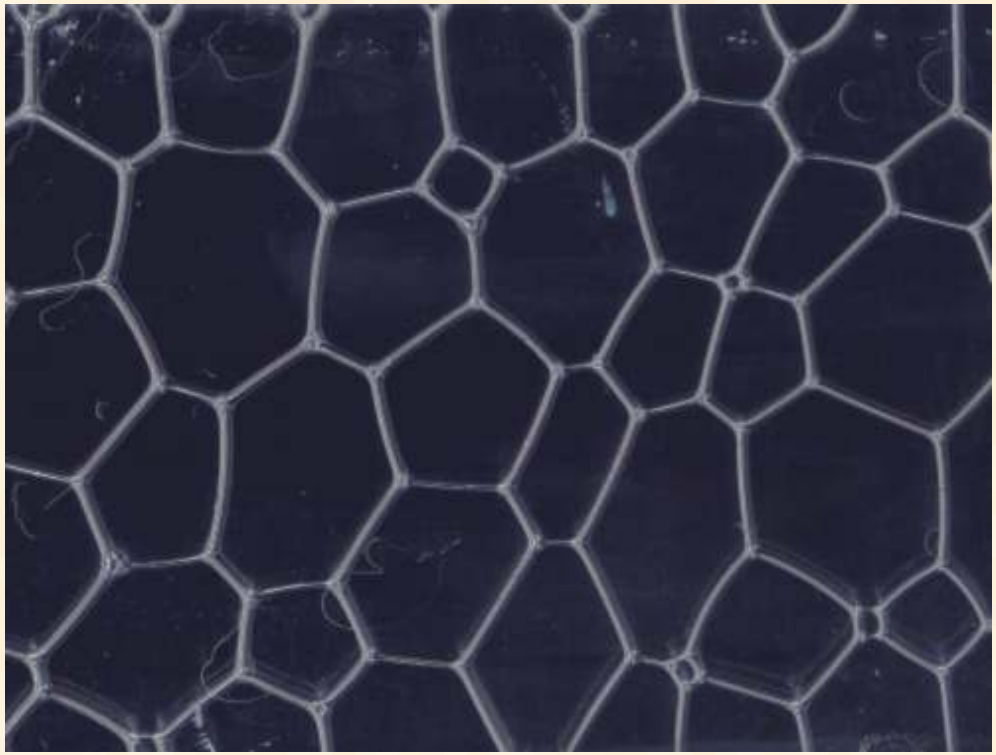
Overview

- Introduction and general distinctions
- Foam structure
- Dynamical processes:
    - Coarsening and diffusion
    - Drainage
    - Film collapse
- Sinthesys and foam lifespan predictions – De – number

Introduction

- Foam – metastable system of gas-filled cells with liquid film walls - 2D or 3D!

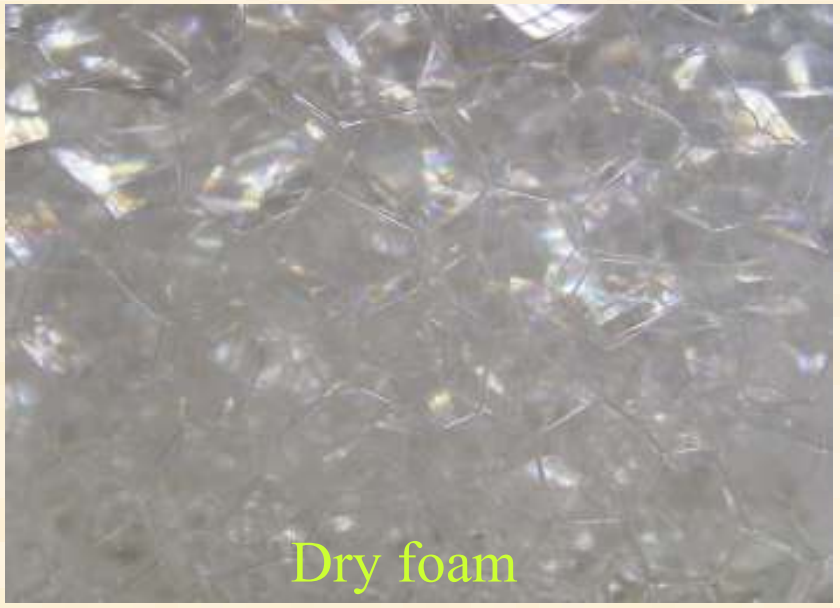




Introduction cont.

- Division with respect to liquid fraction:
  - Dry foams ($\Phi_l < 5\%$)
  - Wet foams ($5\% < \Phi_l < 36\%$)
- $\Phi_l = 36\%$ cells detach and form separate bubbles





Introduction cont.

- Our work – mainly dry foams
- Coarsening:
  - 2D foams
- Drainage:
  - 3D foams
- Film collapse:
  - 3D foams and single films
- Sinthesys – realistic foam systems

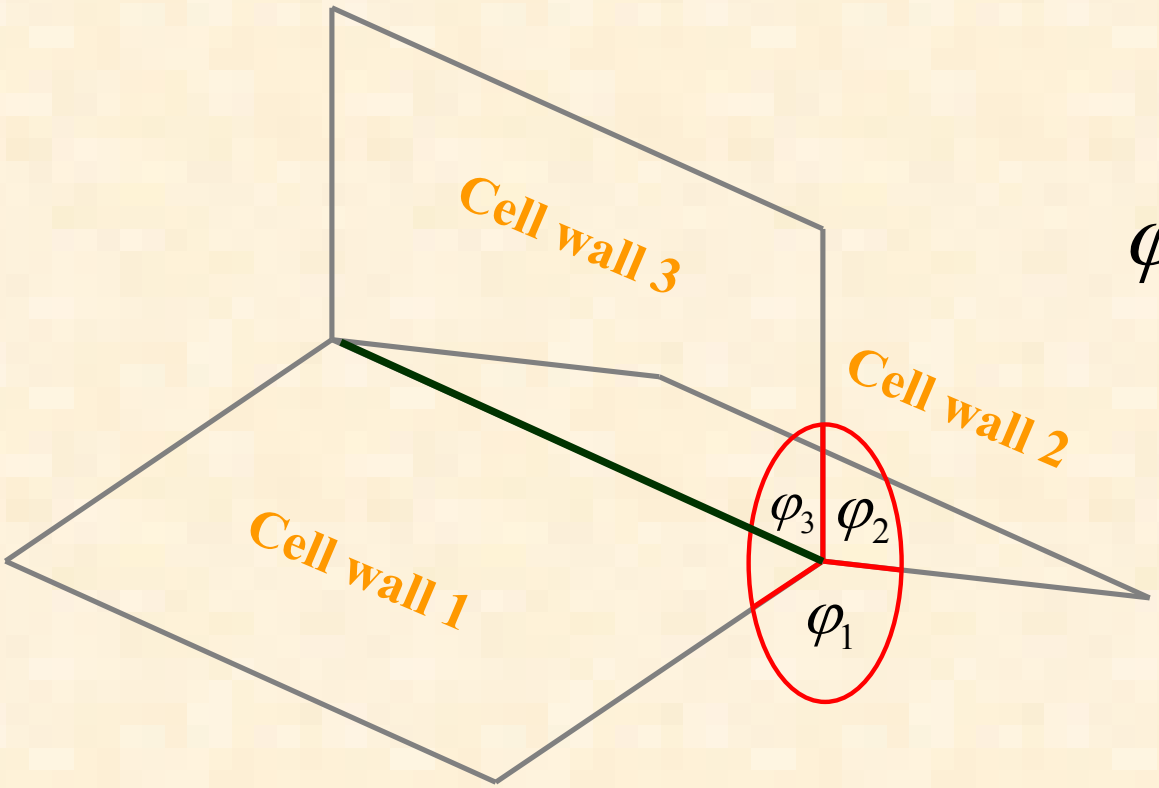
Foam structure

- "Responsible" for foams – surfactants
  - Film stability
  - Smaller surface energy

    ⇒ Metastability enhanced
- Surface tension => geometrical structural rules – Plateau rules
- Structure investigation – small focal length microscope

Plateau rules

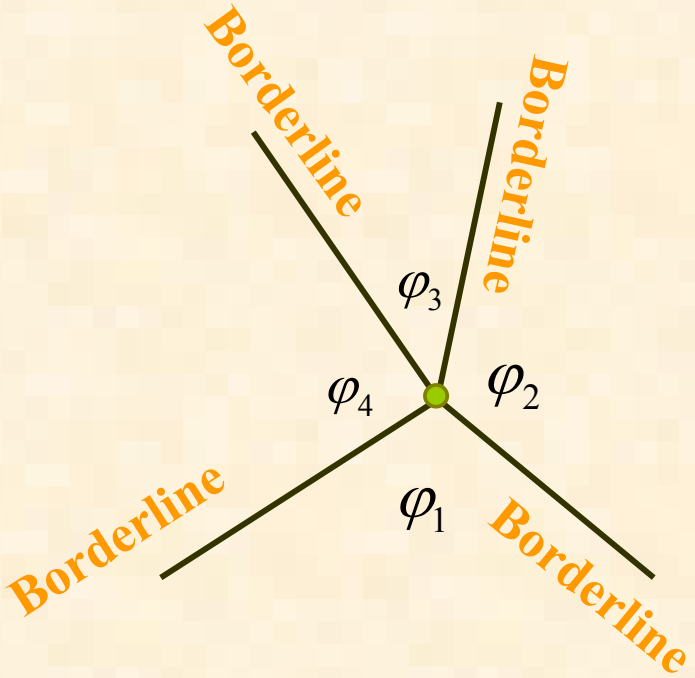
Rule 1: three cell walls meet at a borderline under equal angles

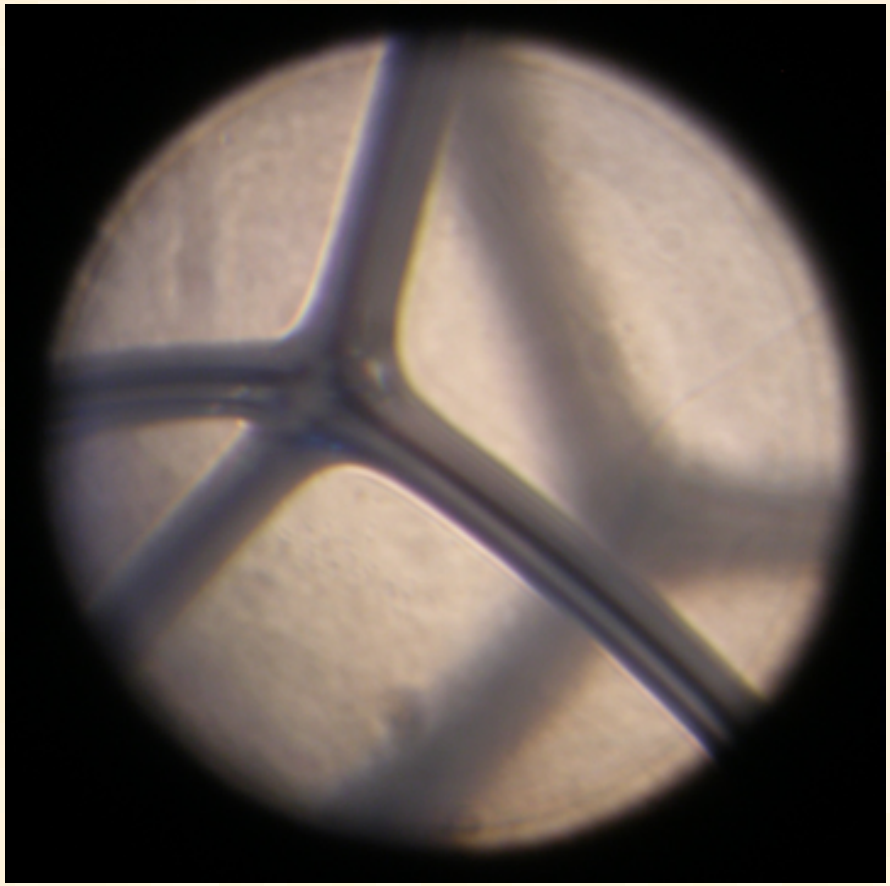


$$\varphi_i = \frac{2\pi}{3}, \quad i = 1,2,3$$

Plateau rules cont.

Rule 2: four borderlines meet at a vertex under equal (tetrahedral) angles

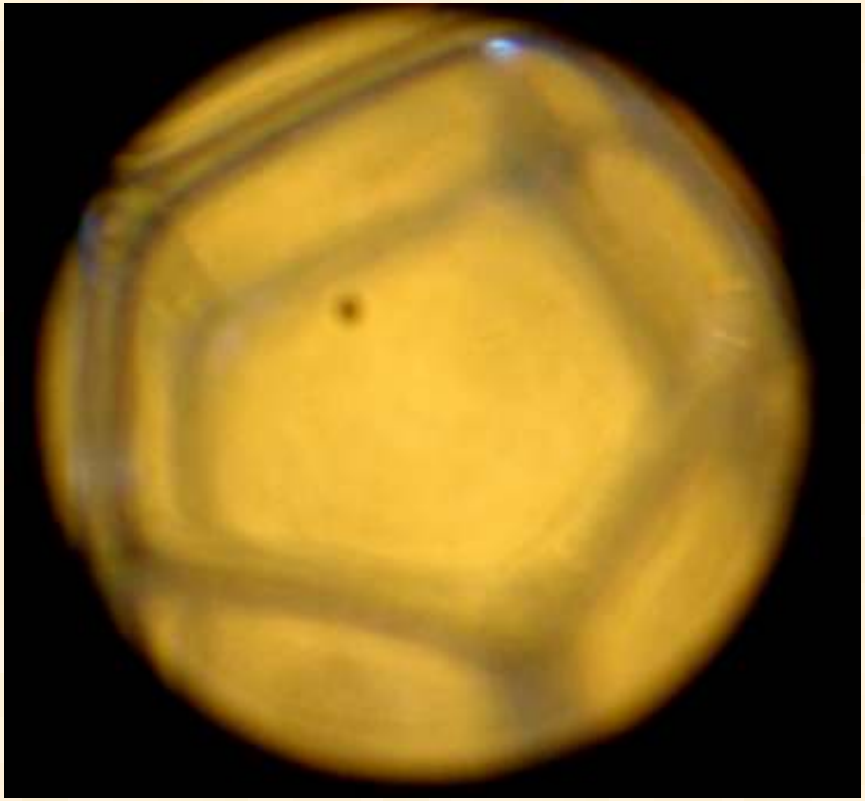


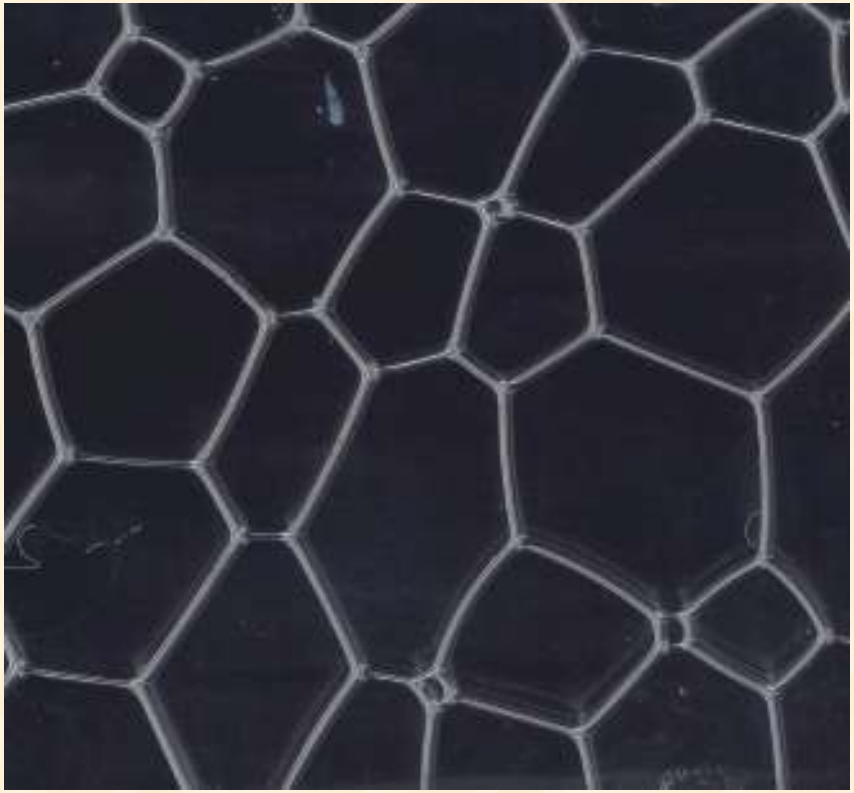


$$\varphi_i = 109.5°, \quad i = 1,2,3,4$$

Structural forms

- In 2D – polygons (3 to 11 or 12 sides at most)
- In 3D – polyhedra
- Minimal forms!





Diffusion and coarsening

- In dry foam – cell walls extremely thin (down to 5 nm)
- Gas able to diffuse through walls
- Diffusion due to pressure differences between cells
- Pressure difference on two sides of a membrane:

$$\Delta p = \frac{4\sigma}{r}$$

σ – liquid surface tension

r – radius of curvature of membrane

Diffusion and coarsening cont.

- In 2D simple diffusion calculation from:
  - Plateau rules
  - Pressure difference (single curvature)

$$\frac{dV}{dt} = \frac{4\pi}{3}\kappa h \sigma (n-6)$$

V – cell volume

t – time

κ – film permeability (constant of gas)

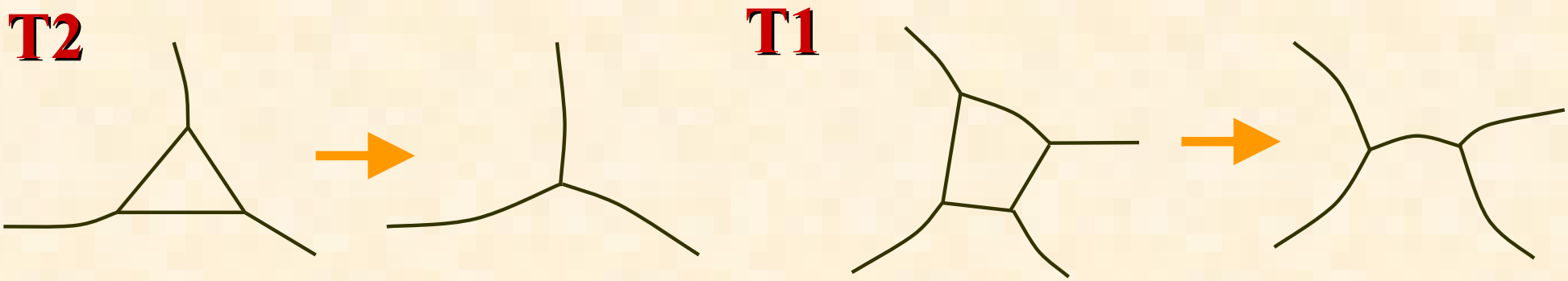
h – height of cell walls

n – number of sides of cell

- In 3D – analogous law only superstitious
- Experimental check: scanning foam in glass cell in regular time intervals

Diffusion and coarsening cont.

- Due to diffusion – cell face number changes in time
- Two transition types (equivalent in 3D):



- Cell number decrease
- Cell mean area increase - coarsening

Diffusion and coarsening cont.

- Cell & measurement:



Length [cm]	15.3
Width [cm]	6.7
Plate dist. [cm]	0.2

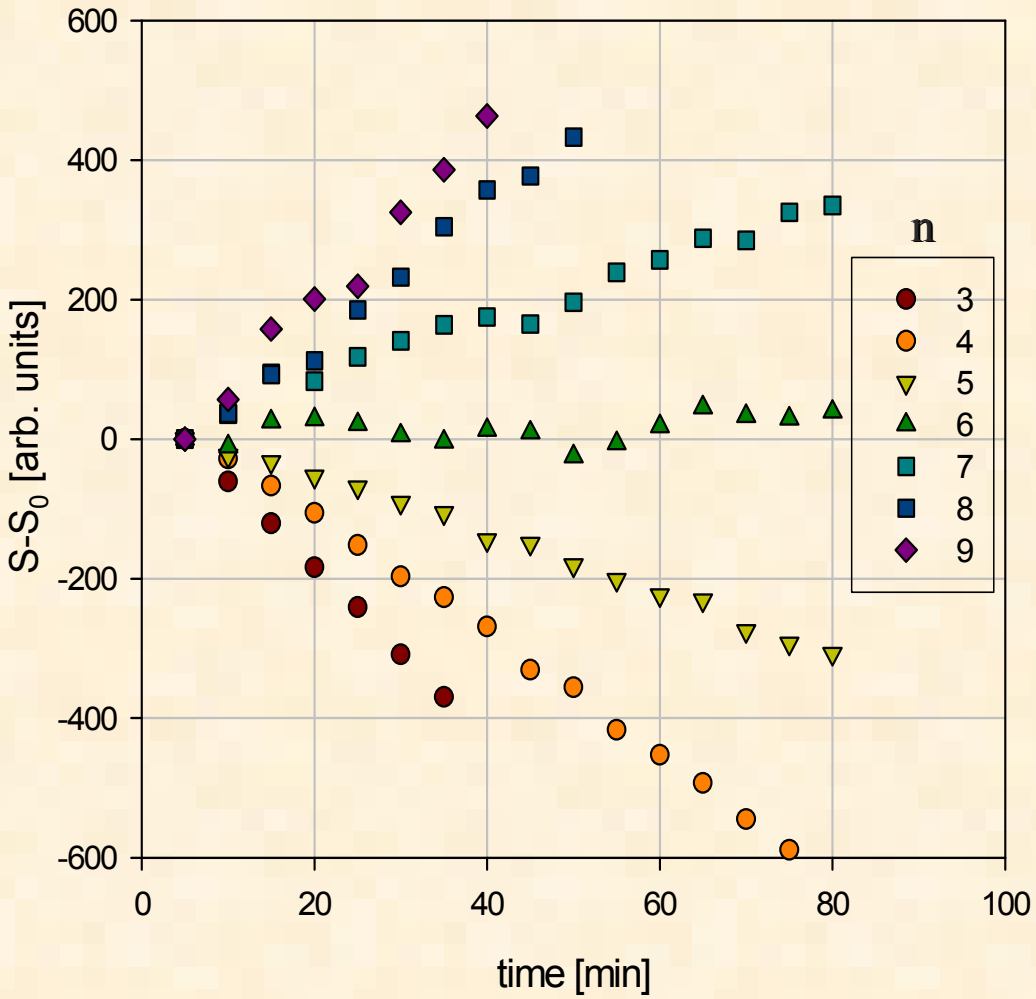
Gas used: CO_2

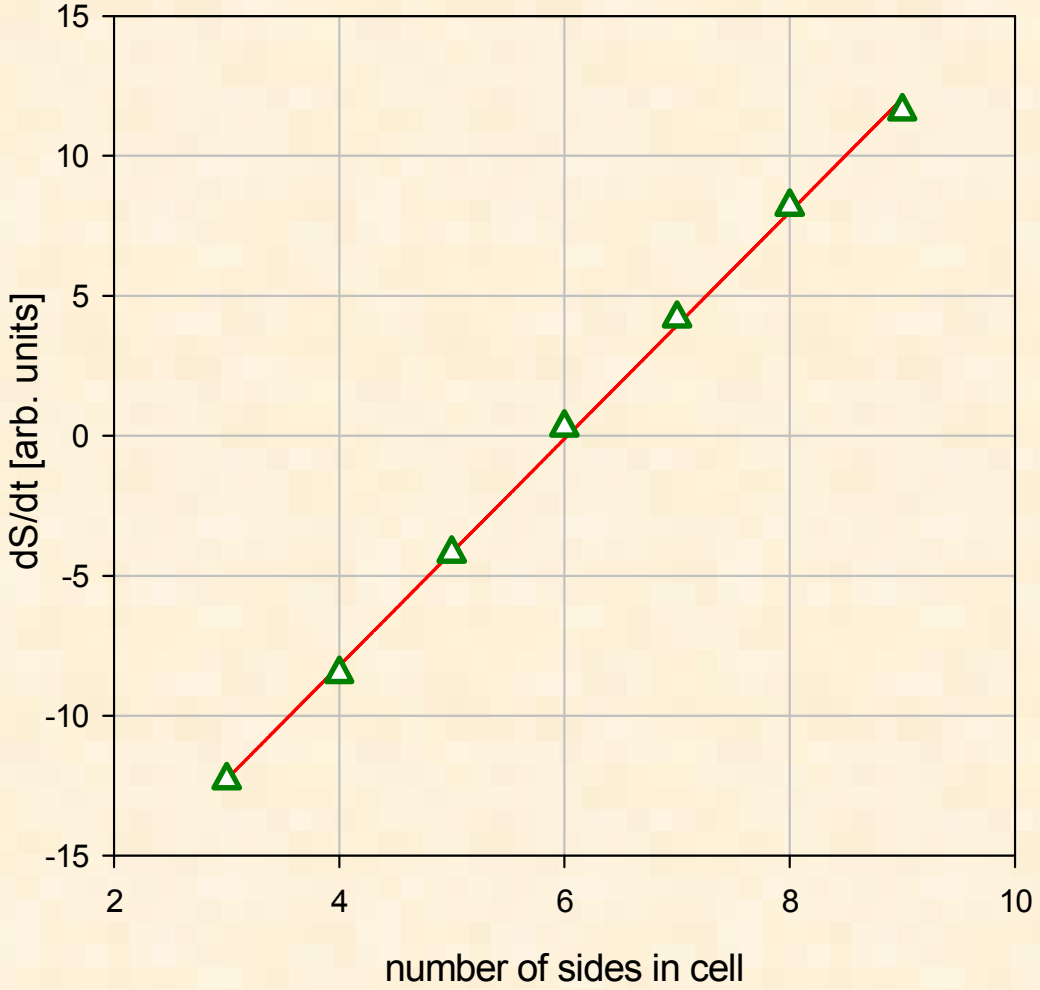
Avg. wall length: 0.5 cm

Scanning interval: 60 s

Duration: 80 min

Diffusion and coarsening cont.





Diffusion and coarsening cont.

- Air foams – coarsen much slower – CO_2 is water soluble!
- In 3D measurements very difficult

Drainage

- Drainage – liquid flow through foam borderlines and plates
- Changes film thickness
- Influence on coarsening rates and film duration
- Forces on liquid parcel in foam in gravity field:
    - Gravity and hydrostatic pressure
    - Viscous friction
    - Capillary forces (relatively small)
- Continuity

Drainage cont.

⇒ Drainage equation (capillary effects neglected):

$$\frac{\partial \Phi_w}{\partial t} + \frac{2}{3}\frac{g\bar{l}_i^{\,2}}{\Gamma \nu}\frac{\partial \Phi_w}{\partial z}\Phi_w = 0$$

Φ_w – liquid precentage

g – gravity acceleration

l_i – average borderline length

Γ – geometrical factor

ν – liquid kinematic viscosity

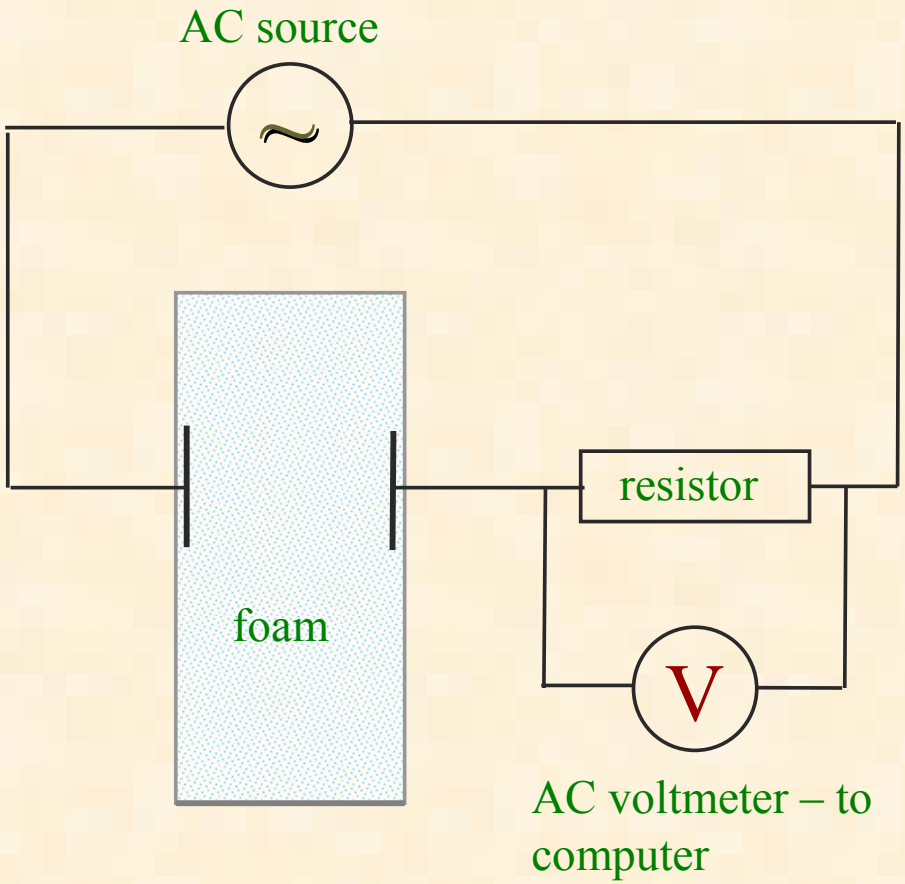
z – vertical coordinate

- Solution by separation:

$$\Phi_w = \frac{3}{2}\frac{\Gamma \nu}{g\bar{l}_i^{\,2}}\frac{z}{t}$$

Drainage cont.

- Measuring – oxygen foam AC conductivity:



$$\sigma = \frac{1}{3}\Phi_w \sigma_w$$

Source voltage 220 V (rms)

Resistor 49 Ω

Foam height 30 cm

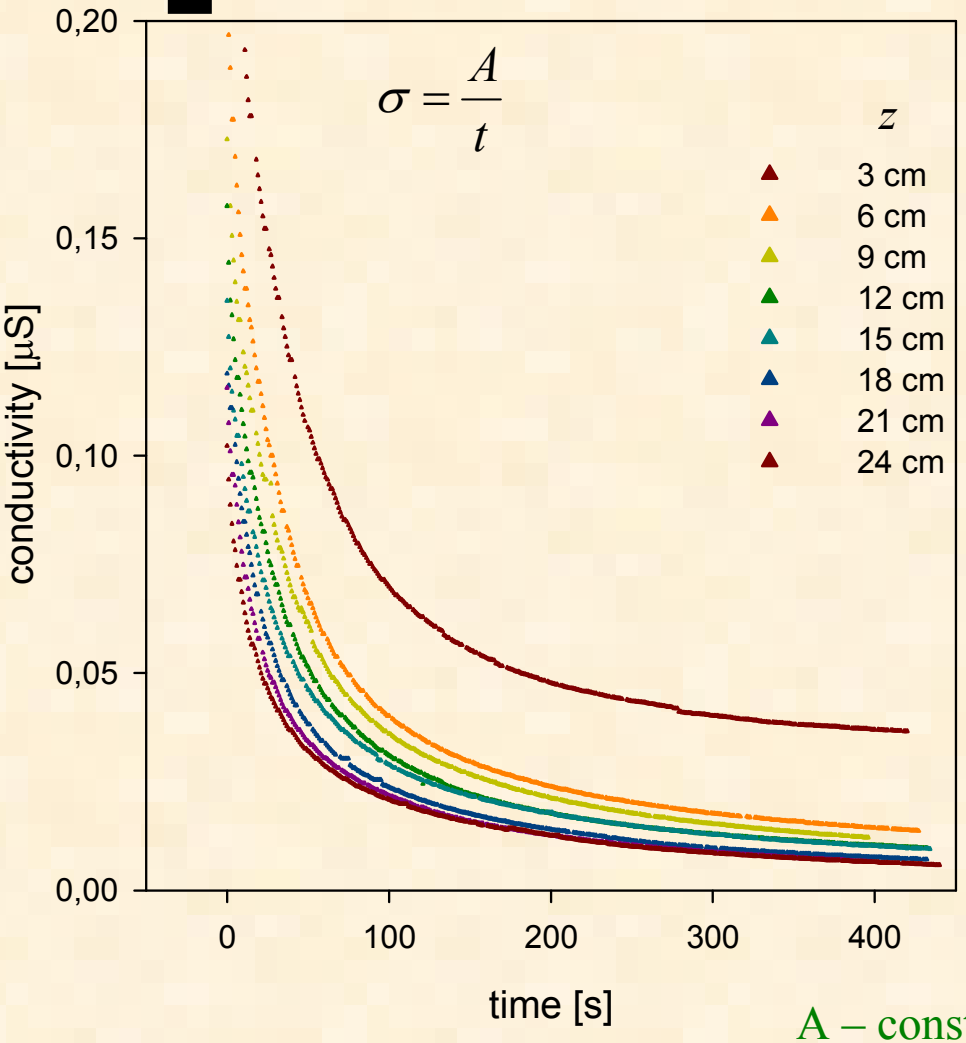
Electrodes:

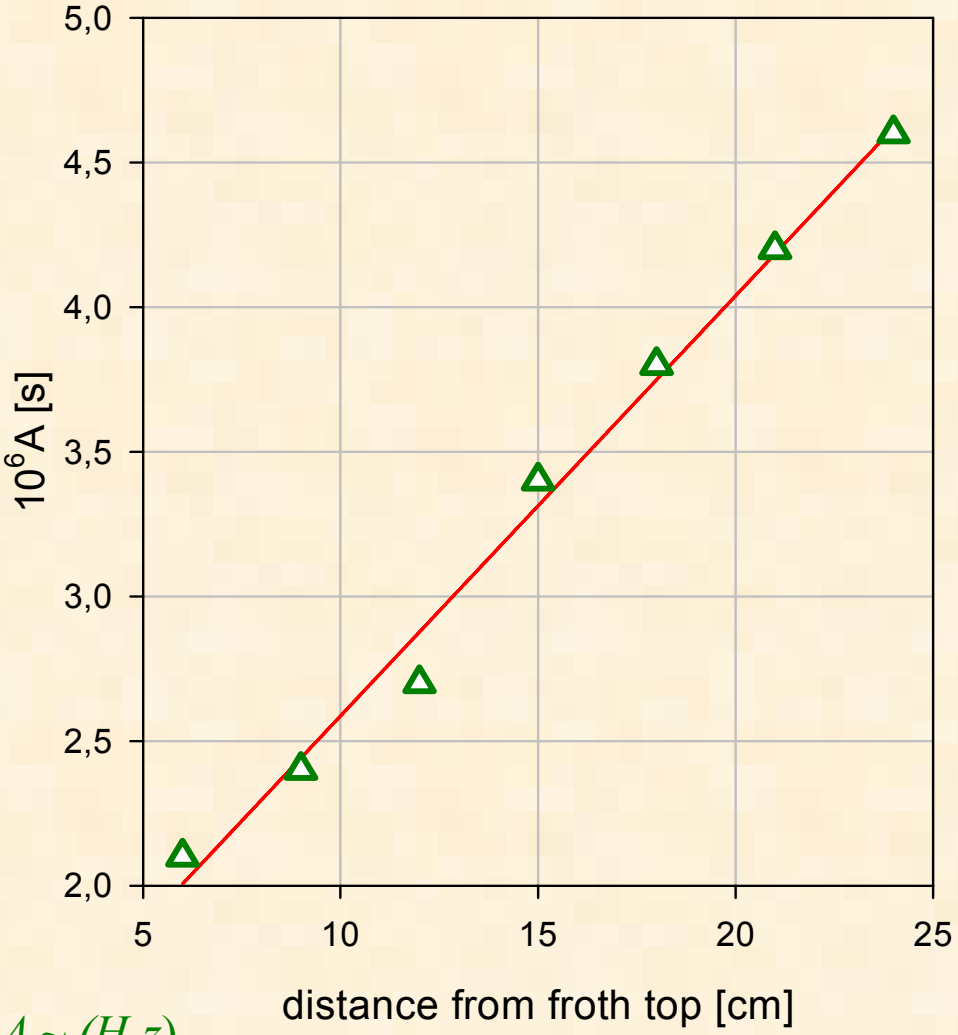
Height [cm]	2.9
Width [cm]	3.7
Distance [cm]	6.2 – 7.0

Drainage cont.



Drainage cont.





A – constant, $A \sim (H-z)$

z – distance from froth top

H – froth height

Drainage cont.

- Measured curves agree with theoretical in shape
- Discrepancies at small heights – capillary effects
- Constant calculation – numerical analysis of flow thorugh bordertubes
- From measurements:

$$\Gamma = 310\ m^{-2}$$

Film stability

- Two influences:
    - Film drainage
    - Evaporation
- Stability enhancing – surfactant molecules rearrangements
- In dry foams – evaporation dominant
- Mainly surface film collapse
- At constant pressure – linear film thickness decrease
- Influence of external parameters

Film stability cont.

- Measurements – single film
- Laser reflected intensity – measure of film thickness:

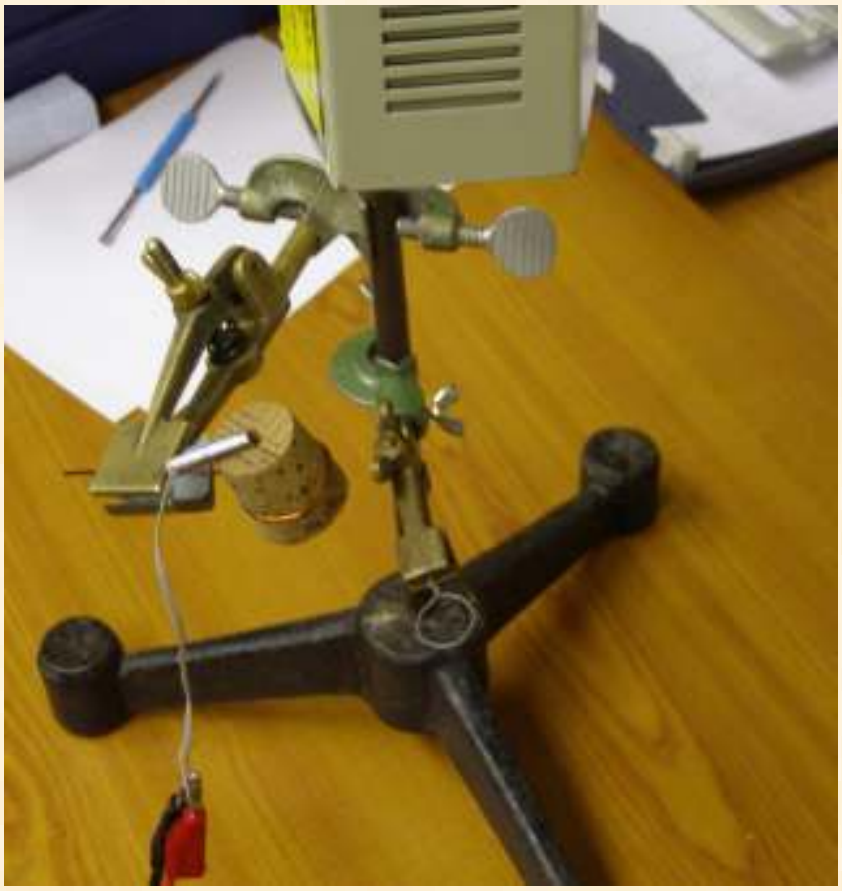
$$I_r \sim D^2 \quad \text{for} \quad D \ll \lambda$$

I_r – reflected intensity
D – film thickness
λ – light wavelength

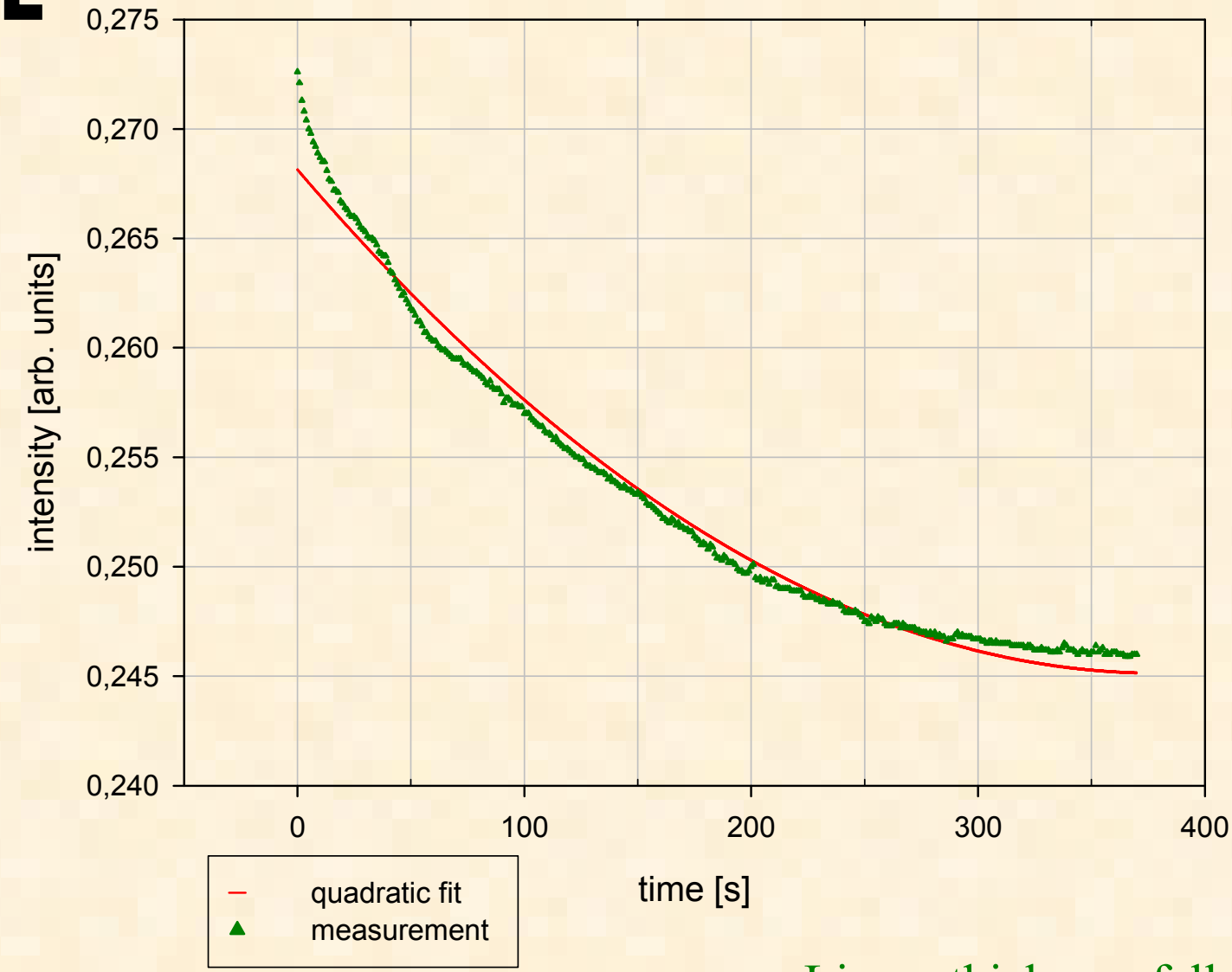
- Used in experiment:

Laser wavelength 633 nm

Photodiode sampling 1 /s (DVM)



Film stability cont.



⇒ Linear thickness fall confirmed!

Real froth fall

- All effects involved
- Two regimes:
  - Coarsening dominated
  - Film rupture dominated
- Decidive parameters:
  - Gas diffusivity
  - Film thickness
  - Liquid kinematic viscosity
  - Liquid content
  - Cell size
  - Ambient conditions (temperature)

Real froth fall cont.

- Diagnostic number – film and cell duration ratio:

$$De = \frac{\tau_f}{\tau_c}$$

De – diagnostic number

τ_f – average film duration

τ_c – average cell duration

$De >> 1 \Rightarrow$ coarsening dominates

$De << 1 \Rightarrow$ film rupture dominates

$De \sim 1 \Rightarrow$ intermediate regime

- Durations determined by relevant parameter analysis:

Real froth fall cont.

- Film duration – influenced by:
  - Drainage – viscosity, liquid content
  - Evaporation – surface tension, ambient

$$\tau_f \sim \nu \Phi_w \sigma$$

ν – viscosity
Φ_w – liquid content
σ – surface tension

- Cell duration – influenced by:
  - Diffusion – cell size, diffusivity, surface tension

$$\tau_c \sim \frac{\bar{l}_i^3}{\kappa \sigma}$$

l_i – mean border length
κ – diffusivity
σ – surface tension

Real froth fall cont.

$$\Rightarrow \quad De = \Lambda \frac{\nu \kappa \Phi_w \sigma^2}{\overline{l}_i^{-3}}$$

Λ – geometrical & ambient conditions dependent constant

- Examples – ratio of *De* numbers for beer and soap froth (same ambient conditions):



$$\frac{De_{beer}}{De_{soap}} \sim 10^6$$



Real froth fall cont.

- Longest foam duration – two conditions:
  - Both mechanisms of same magnitude
  - Both mechanisms slow

$\Rightarrow$

- $De \sim 1$
- **Small bubbles**
- **Large liquid content**
- **Large viscosity**
- **Small diffusivity (nonsoluble gas)**