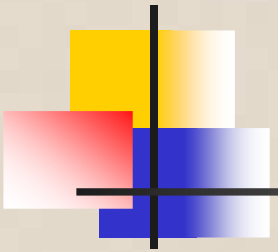


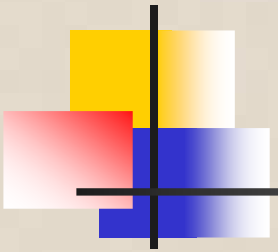
Problem 17.

Magnetohydrodynamics



Problem 17.

A shallow vessel contains a liquid. When an electric and magnetic field are applied, the liquid can start moving. Investigate this phenomenon and suggest a practical application.



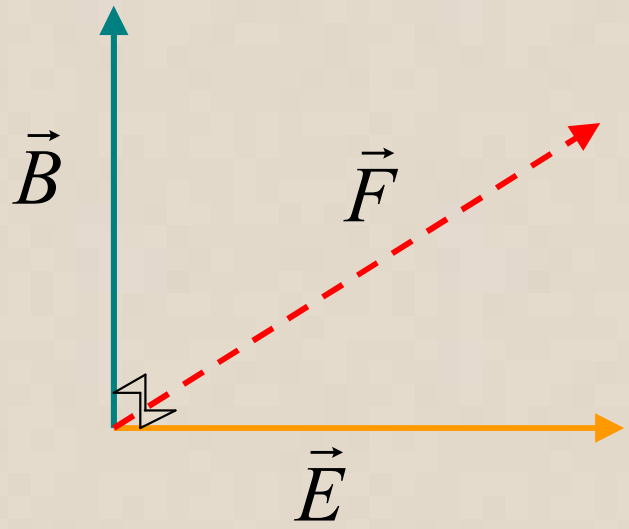
Overview

- Basic explanation
- Experimental approach
- Modeling
- Practical applications

Basic explanation

- **Liquid – must be conducting**
- **Lorentz force on conducting particles – driving pressure**

$\Rightarrow$ **Orientation:**



B – magnetic field

E – electric field

F – liquid driving force

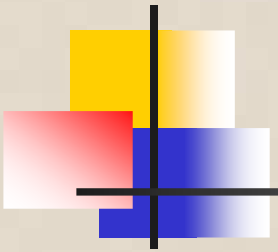
Basic explanation cont.

- Two possible liquid classes:
  - Electrolytes
  - Liquid metals
- In both – positive and negative particles
- Force on particles in fields region:

$$\mathbf{F}_E = q\mathbf{E}$$

$$\mathbf{F}_L = q\mathbf{v} \times \mathbf{B}$$

B – magnetic field
E – electric field
F_L – Lorentz force
v – particle velocity
q – particle charge



Basic explanation cont.

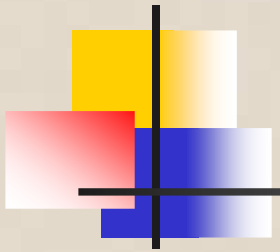
⇒ **Lorentz force:**

$$F_L \sim q^2$$

q – particle charge

⇒ **both charges move in the same direction**

- **In real liquid flow – two force contributions:**
  - **Electromagnetic Lorentz force**
  - **Viscous drag**

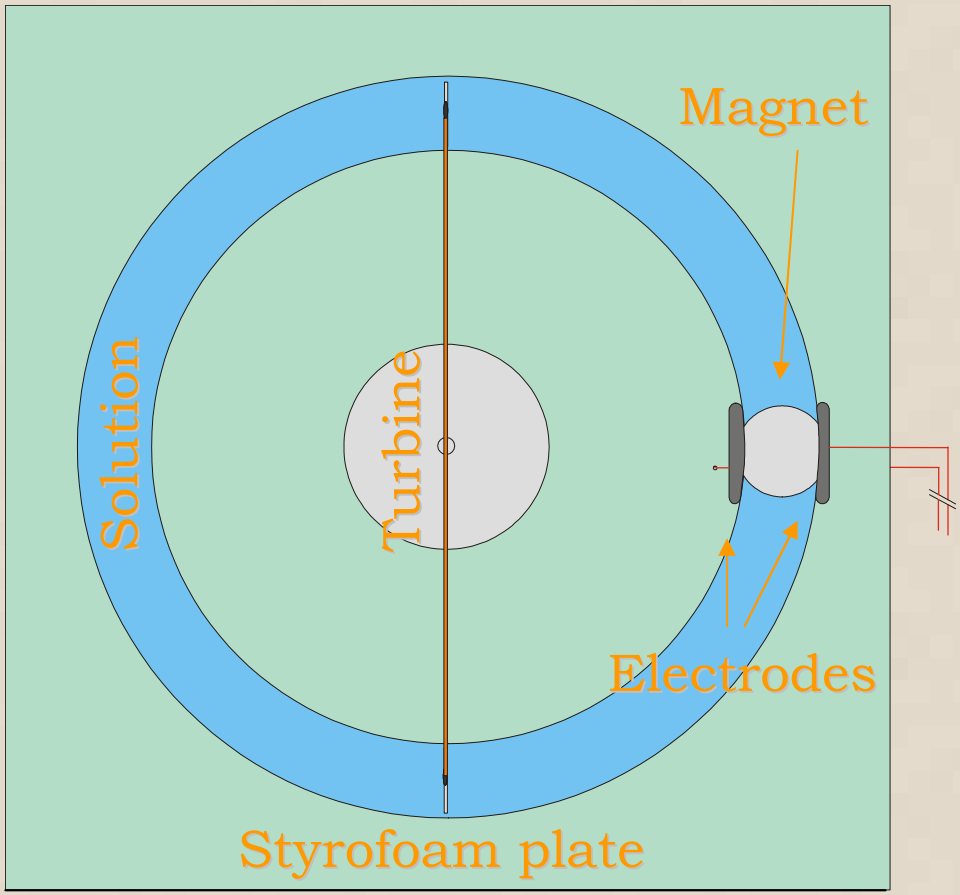


Experimental approach

- Vessel – shallow circular channel
- Liquid – Na_2CO_3 solution
- Electric field – graphite plates in channel
- Magnetic field – permanent FeNdB magnet placed under channel
- Terminal velocity measurement – low friction turbine
- Magnet field measurement – Hall probe on translator

Apparatus

- **Apparatus general:**



Channel width	2.0 cm
Circle mean radius	10.0 cm
Electrode width	3.2 cm
Electrode height	4.4 cm

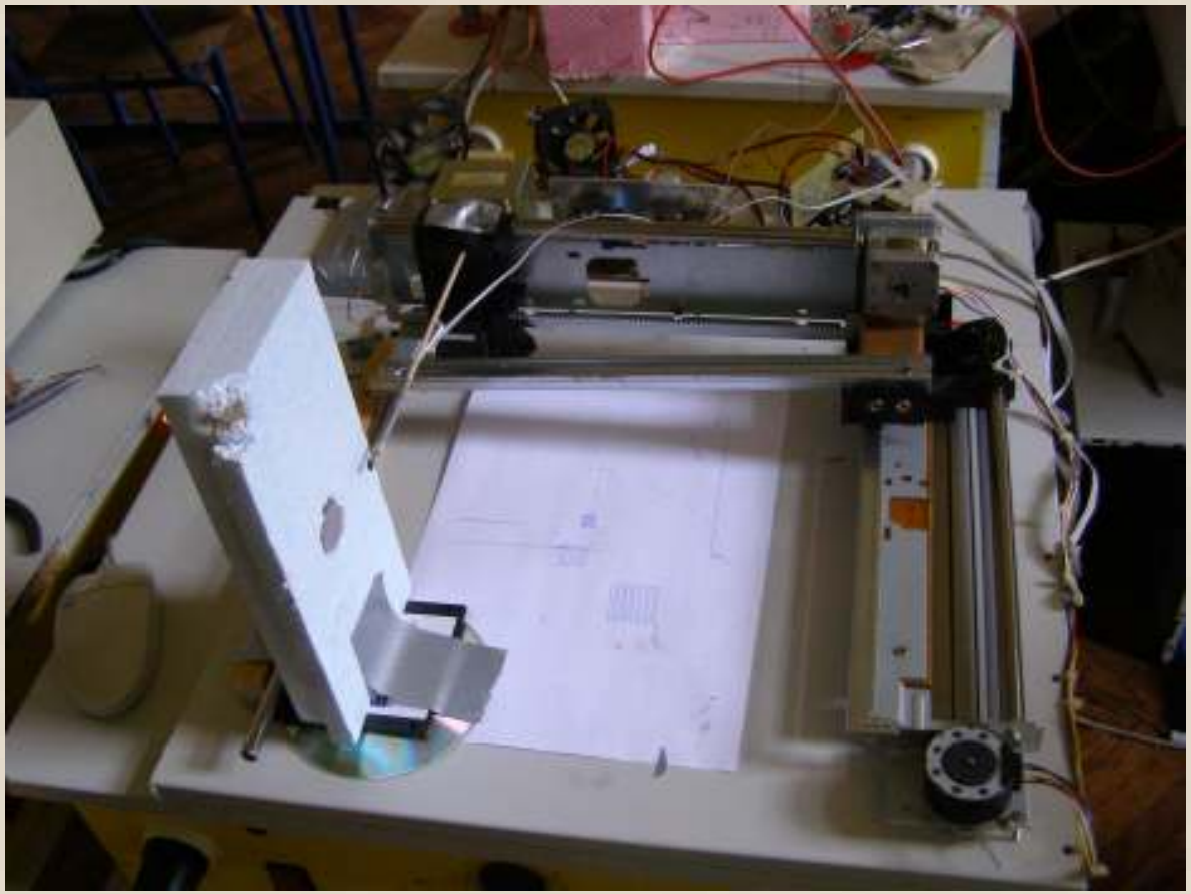
- **Magnet surface induction 1.5 T**
- **Turbine sensitivity 10^{-4} m/s**

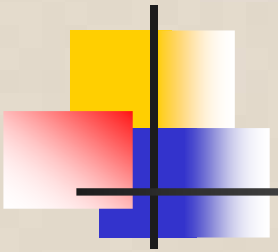
Apparatus cont.



Apparatus cont.

- For magnetic field scanning – Hall probe on translator:



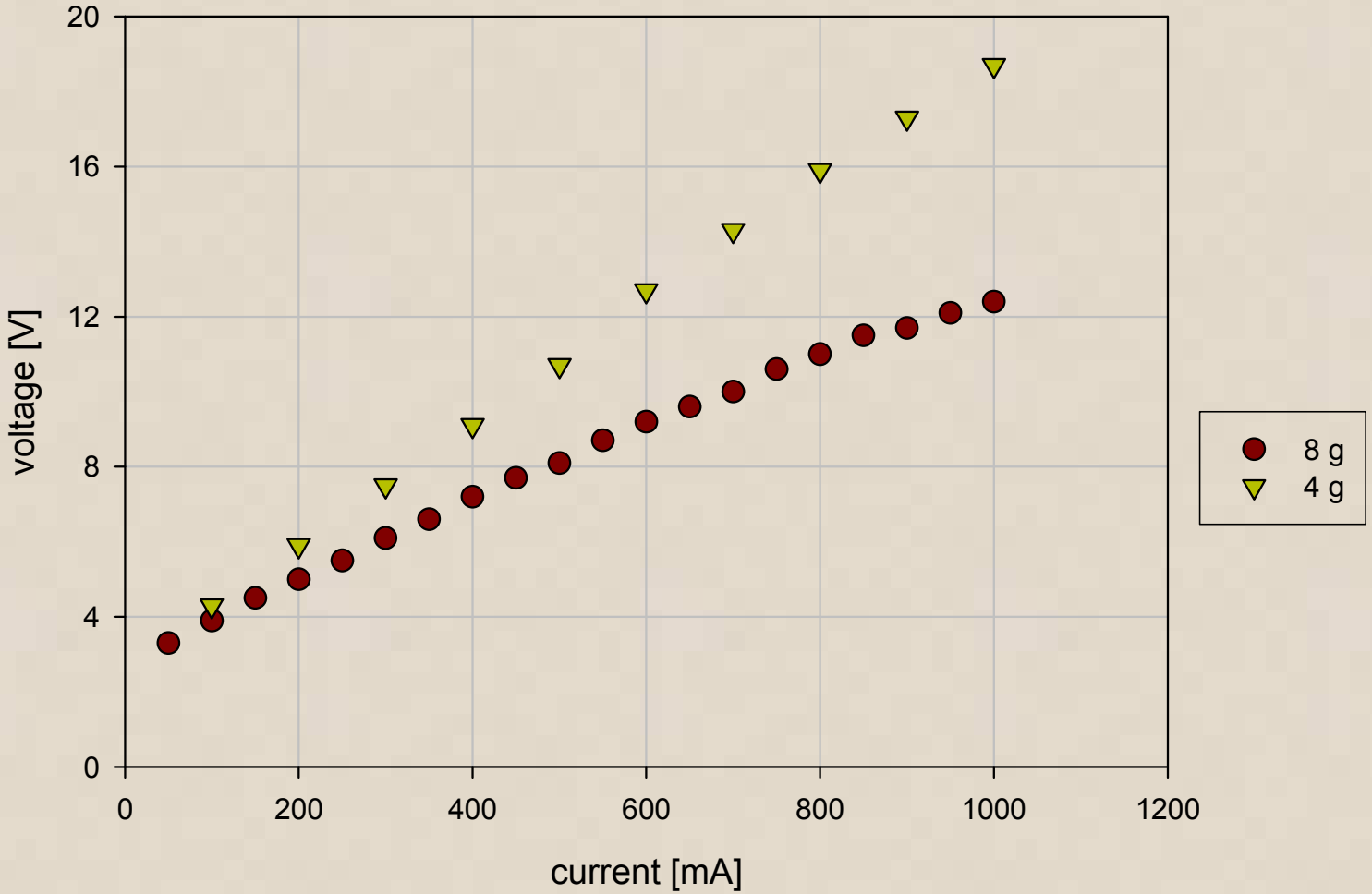


Measurements

- **Measurements:**
  - **U – I characteristic of solution**
  - **Terminal velocity dependence on electric field (current) – two electrolyte concentrations**
  - **Terminal velocity dependence on liquid mass**
  - **Magnet field distribution**

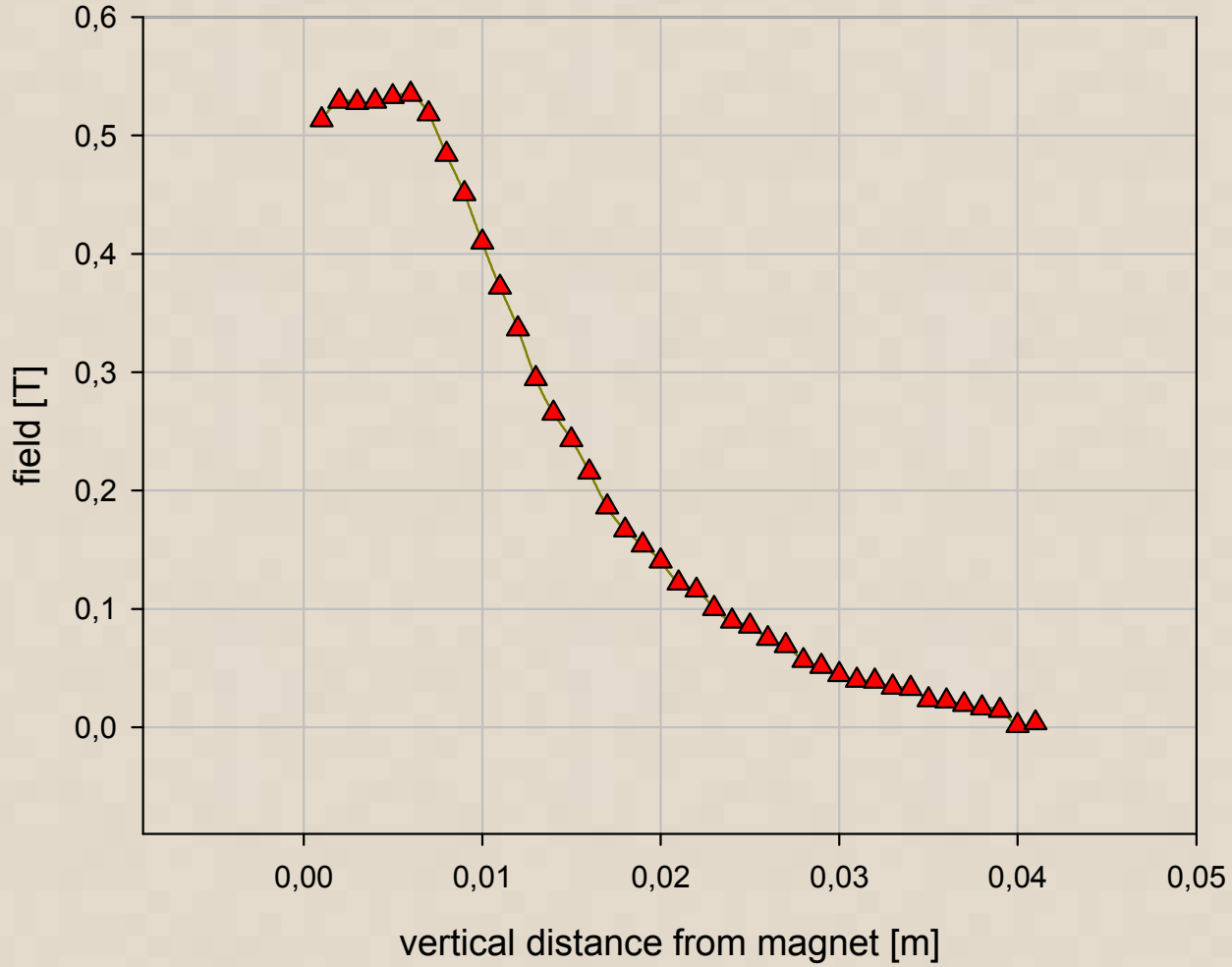
Measurements cont.

- **U – I characteristics:**



Measurements cont.

- **Magnetic field *z* – component distribution:**



Modeling

- **Lorentz force – on current:**

$$\mathbf{F}_L = I\mathbf{L} \times \mathbf{B}$$

B – magnetic field
L – electrode distance
F_L – Lorentz force
I – current in solution

- **In our system – Lorentz force balanced by drag in terminal state:**

$$\mathbf{F}_L = \mathbf{F}_D$$

F_D – drag force

- **Drag force estimate – flow range dependent**

Modeling cont.

- **Flow regime determination – Reynolds number:**

$$\mathrm{Re} = \frac{L\rho v}{\eta}$$

L – electrode distance
ρ – solution density
v – mean flow velocity
η – solution viscosity

- **In our geometry Re ~ 100 to 1000**

⇒ transitional flow, power law drag

$$F_d = \Gamma v^{\alpha}$$

Γ – constant
α - exponent
v – mean flow velocity

Modeling cont.

⇒ **Mean flow velocity in channel:**

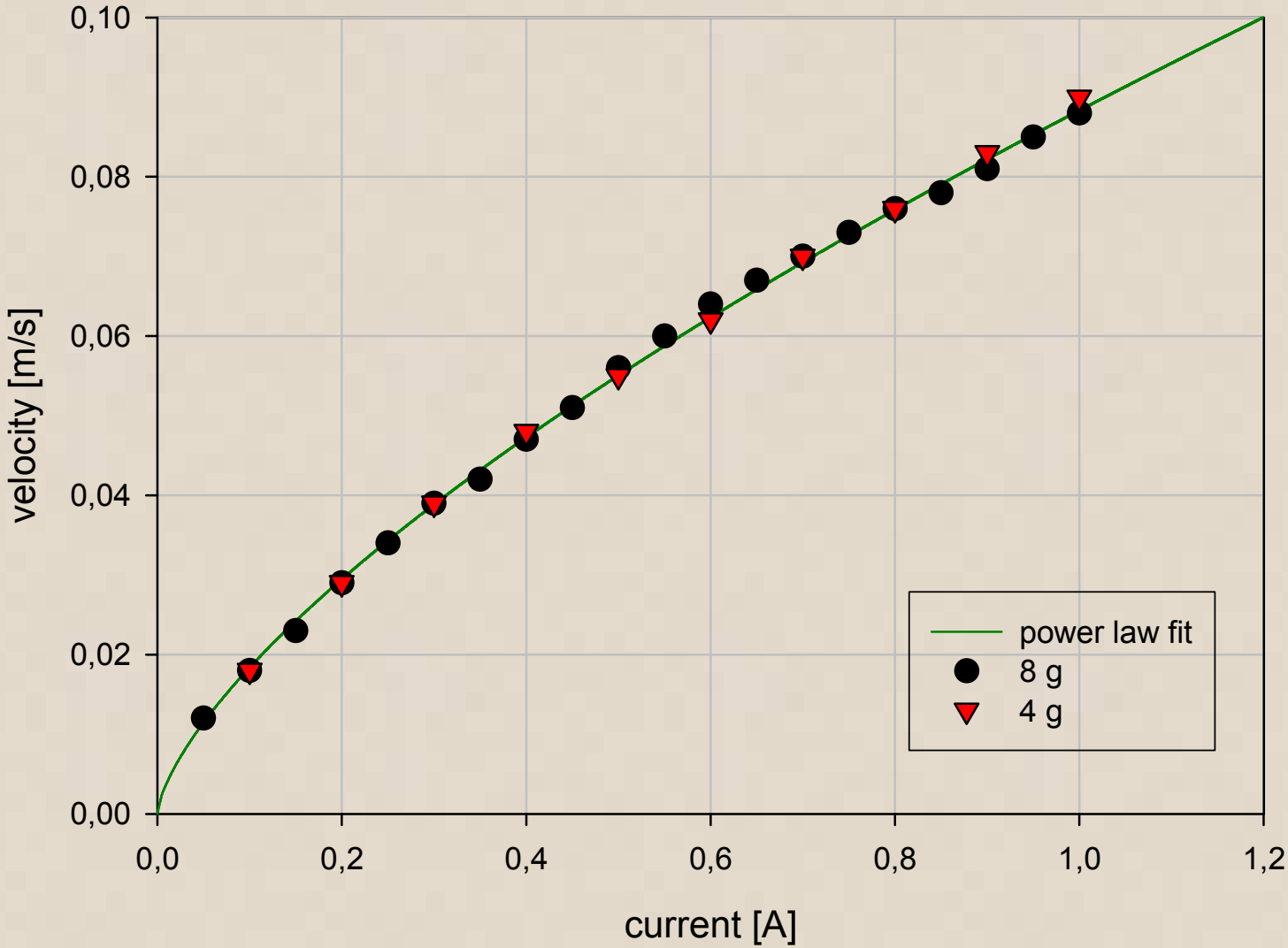
$$\bar{v} = \left(\frac{IL}{\Gamma}\right)^{\frac{1}{\alpha}} \frac{1}{H} \int_0^H B_z^{\frac{1}{\alpha}} dz$$

$$\bar{v} = I^{\frac{1}{\alpha}} \Lambda$$

L – electrode distance
I – current through solution
H – solution level in channel
B_z – z - component of magnetic field
z – vertical coordinate
Λ - constant

- **Velocity depends only on *current* – salt concentration change – no influence**
- **Experiments confirm power law drag:**

Modeling cont.



Liquid mass:
325 g

$\alpha = 1.5$

$\Gamma = 0.19$

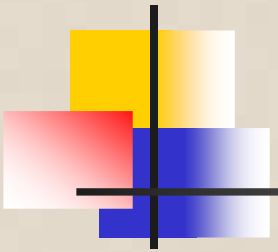
Modeling cont.



Salt mass: 8 g

Current: 0.5 A

$\Gamma = 0.17$



Modeling cont.

⇒ Agreement in damping constant to 10%

⇒ Theory confirmed

⇒ Acceptable model

⇒ Discrepancy – horizontal fields gradients