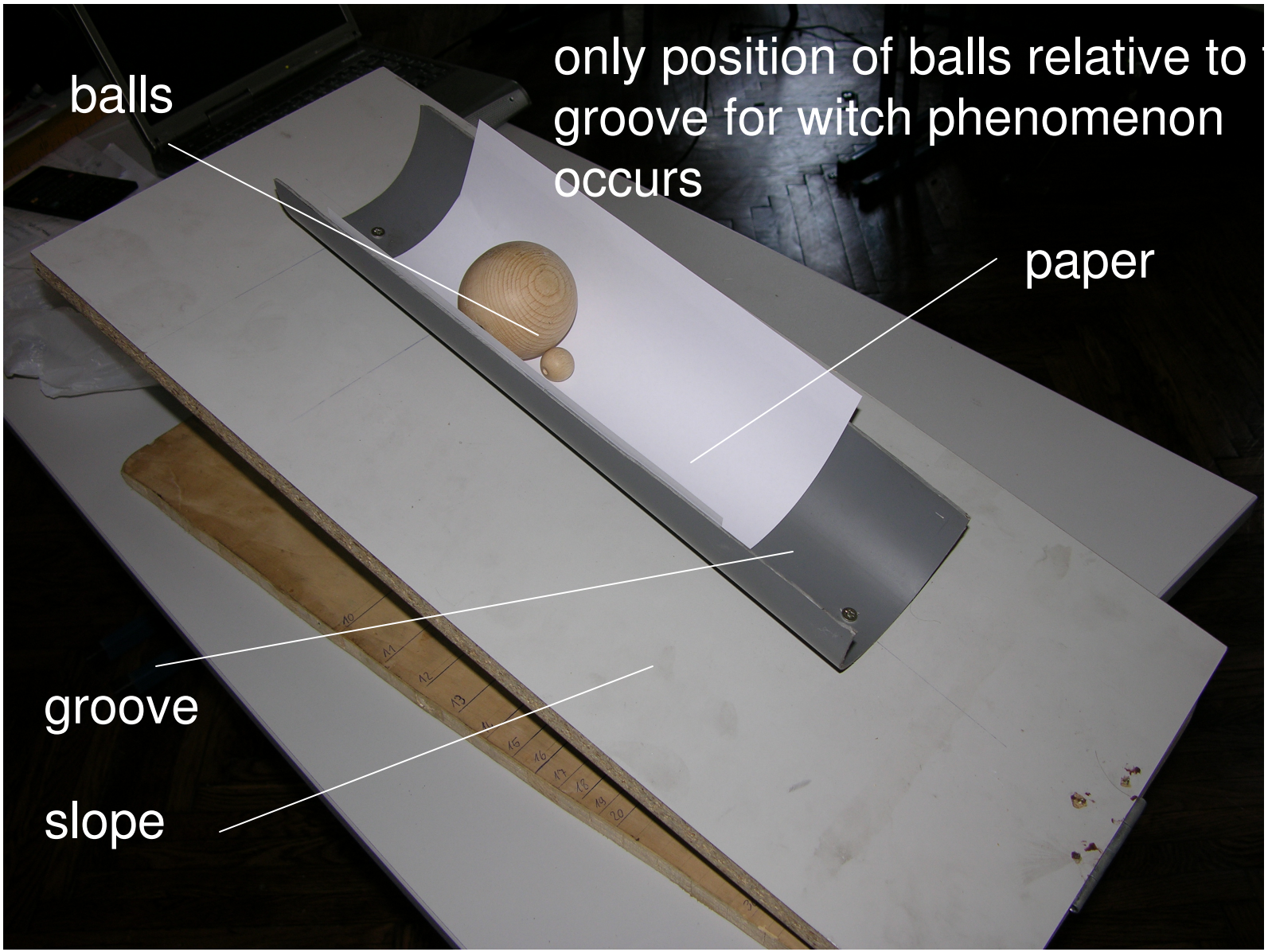
2.The two ball problem

Two balls placed in contact on a tilted groove sometimes do not roll down. Explain the phenomenon and find the conditions, under which it occurs.

2.1 Experiment

- Effect:

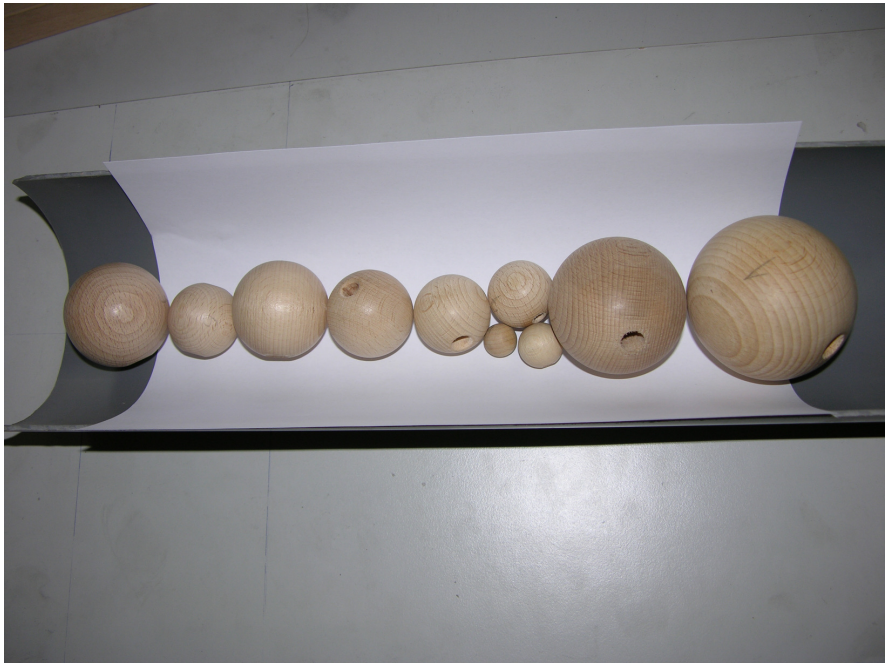


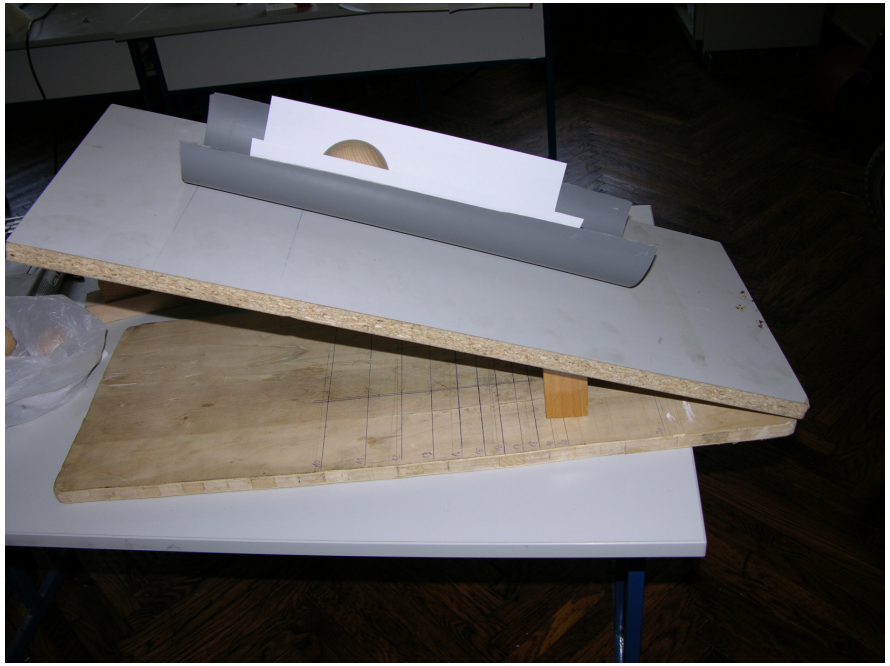
2.2 Experiment

- we worked with wooden balls of radius ranging from 1.5 cm to 6.9 cm
- balls were made of beech-wood and the groove was covered with paper
- we measured maximum tilt angle dependence on radius ratio φ = upper ball radius / lower ball radius
- phenomenon occurred for values of φ bigger than 2.667 (maximum φ we could get was 4.6)

2.2 Experiment

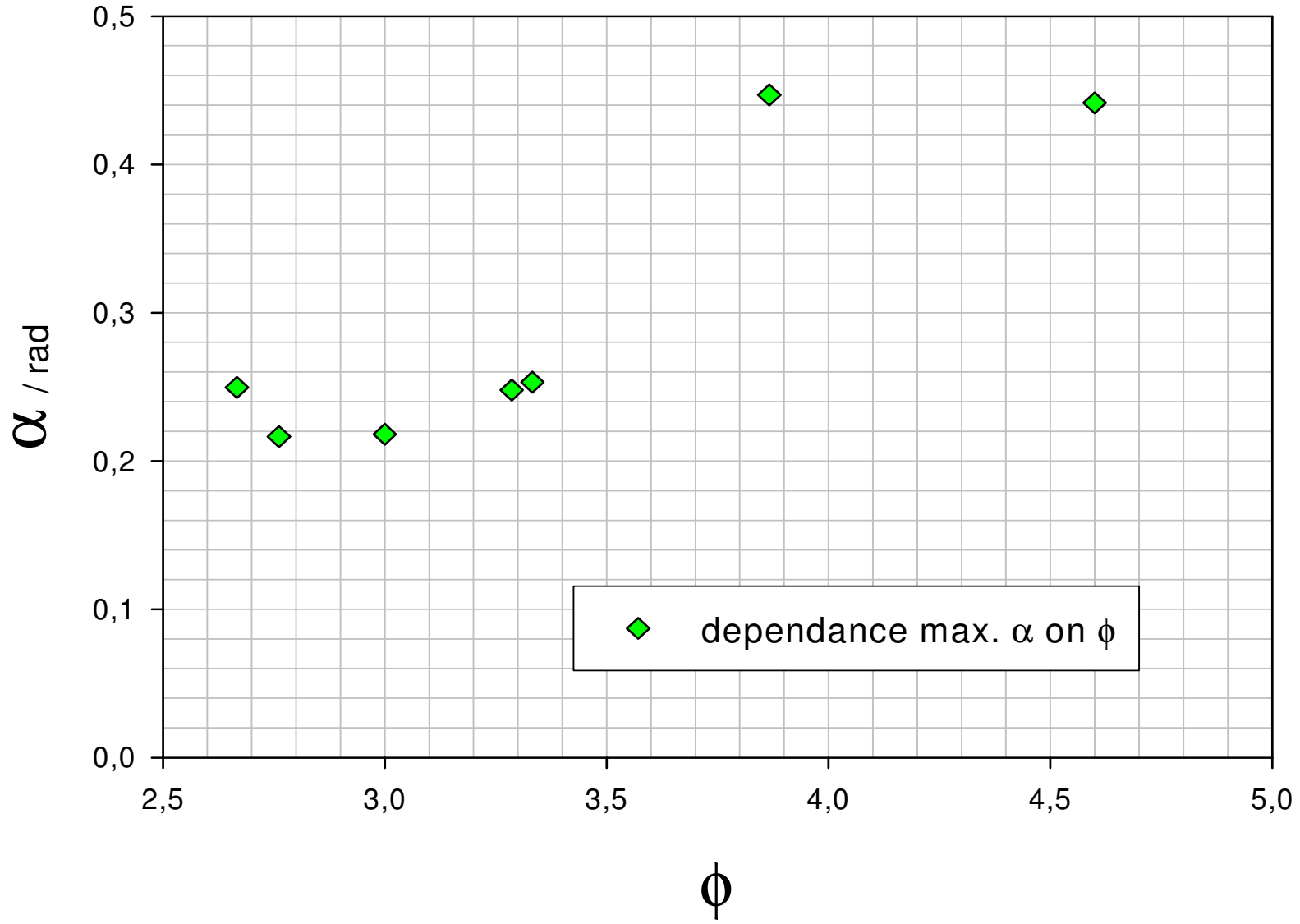
- balls and tilted groove:





2.4 Experiment

- dependence max. α on φ:



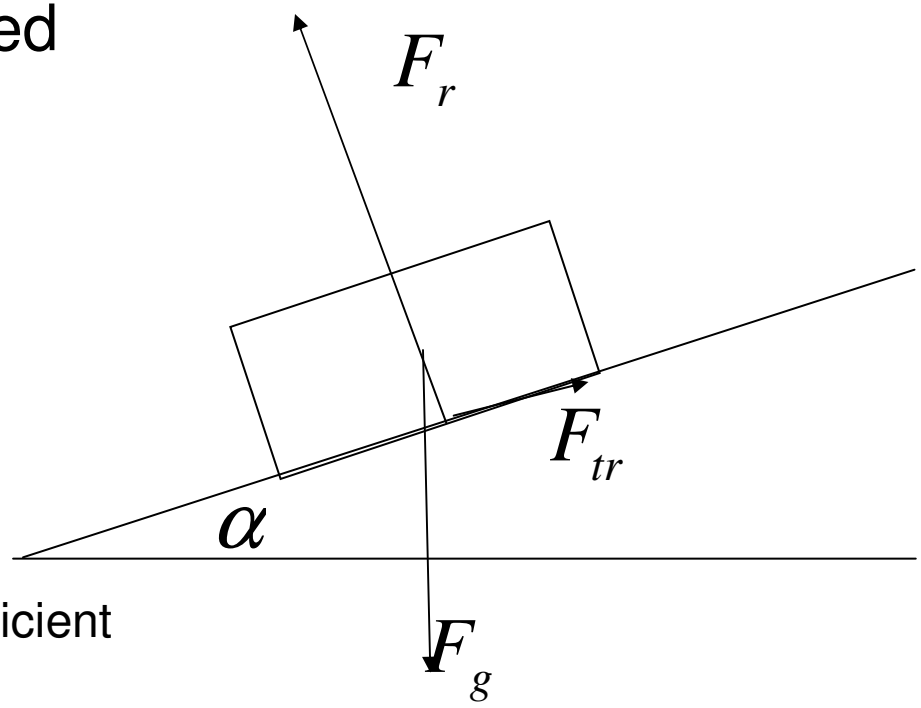
2.5 Experiment

- we also had to measure statically friction coefficient between beech and beech (between balls) and between beech and paper (ball and groove)
- following method was used to measure statically friction coefficient:
  - object is put on a slope and it is tilted until object slips, then max. tilt angle α is measured

$$mg \sin \alpha = \mu mg \cos \alpha$$

tj.

$$\mu = \tan \alpha$$



F_r – surface reaction

F_g – weight

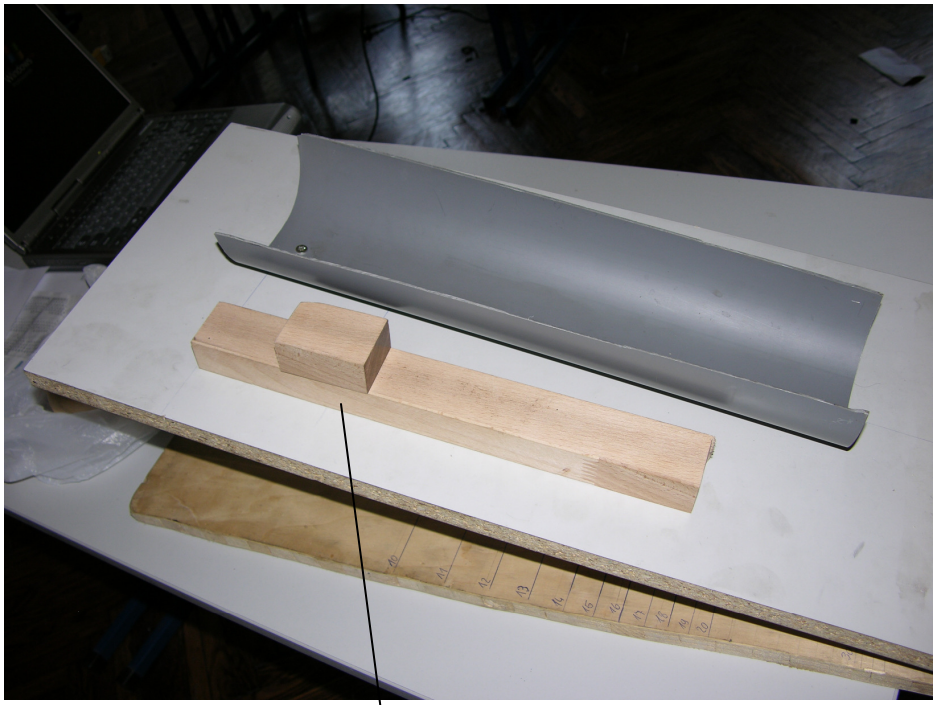
F_{tr} – friction

α – max. tilt angle

μ – statical friction coefficient

2.6 Experiment

Measuring statically friction coefficient :



beech - beech

$$\mu = 0.7$$



beech - paper

$$\mu = 0.5$$

2.1 Theory

There are 3 ways balls can be put in groove relative to it:

- ball centers are in plane of groove cross-section

- ball centers are in plane perpendicular to the plane of groove cross-section

- any other position

2.2 Theory

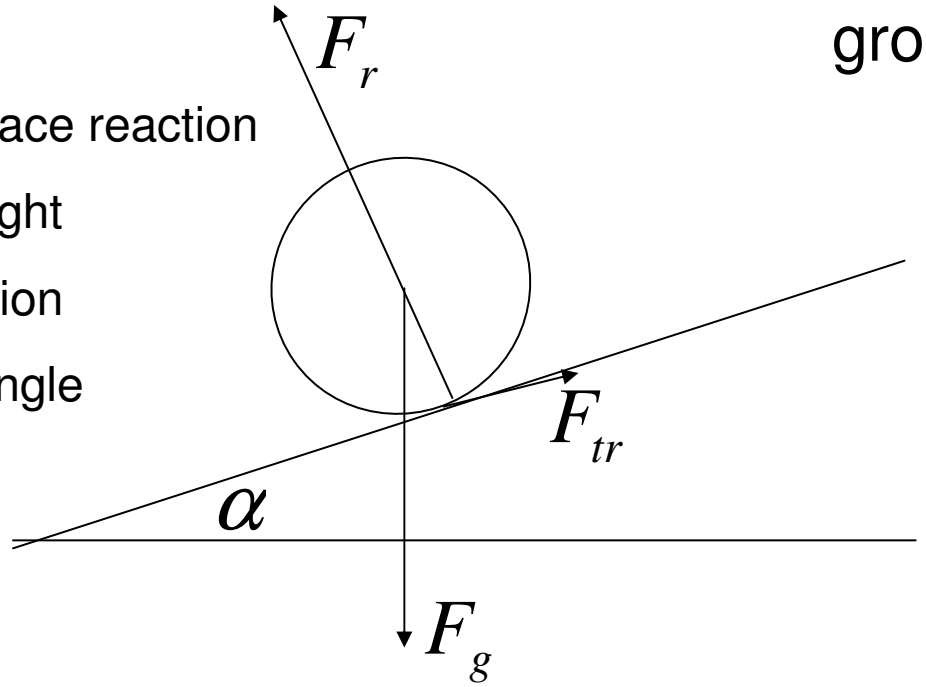
- ball centers are in plane of groove cross-section

F_r – surface reaction

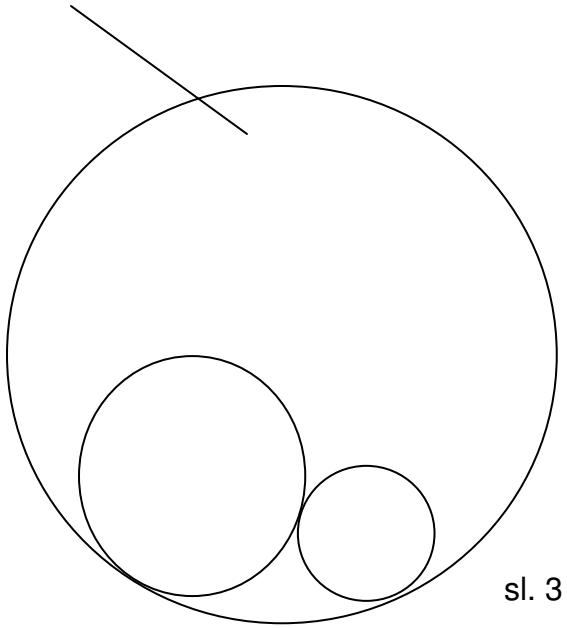
F_g – weight

F_{tr} – friction

α – tilt angle



groove cross-section



sl. 3

-equilibrium is possible only for α = 0, otherwise no force can create moment to annul moment of force of friction

2.3 Theory

- ball centers are in plane perpendicular to the plane of groove intersection

  - only forces on lower ball are shown in this diagram

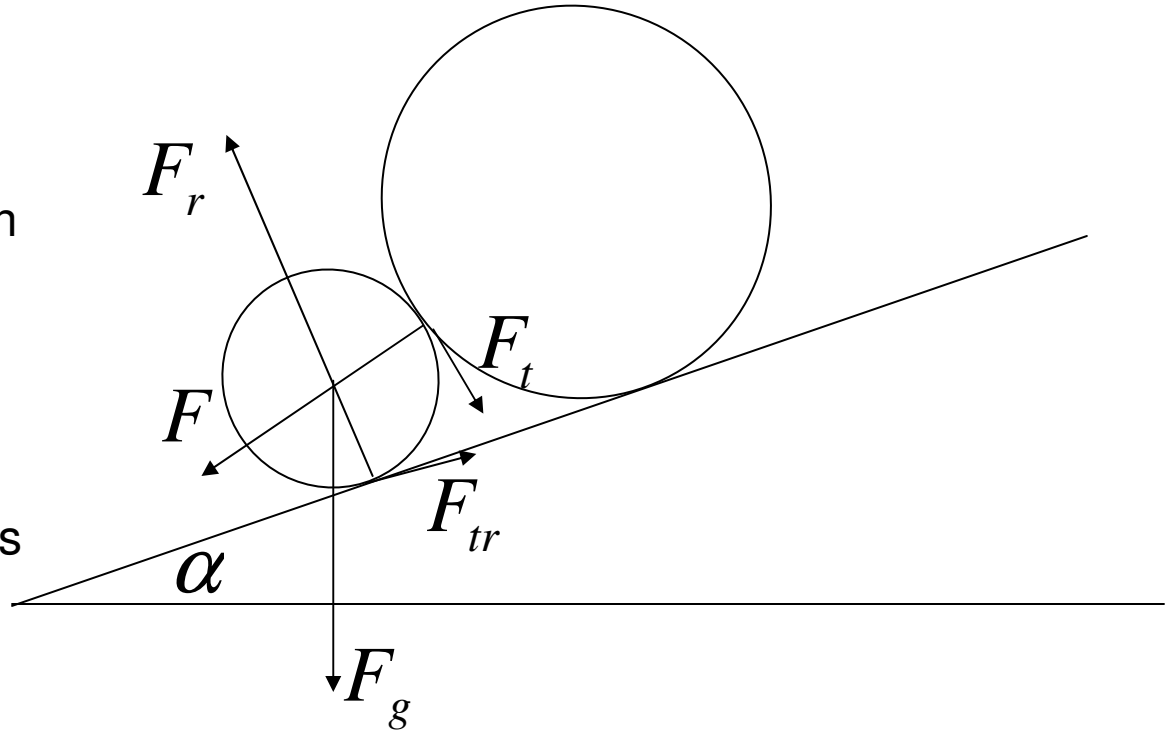
F_r – surface reaction

F – pressure force between balls

F_g – weight

F_{tr} F_t – friction forces

α – tilt angle



- Equilibrium maybe possible, we will examine this case deeply later

2.4 Theory

- any other position

- there exists friction force component in plane of intersection and no force can create moment to annul its moment

2.5 Theory

- hence only possible position is when ball centers are in plane perpendicular to the plane of groove intersection

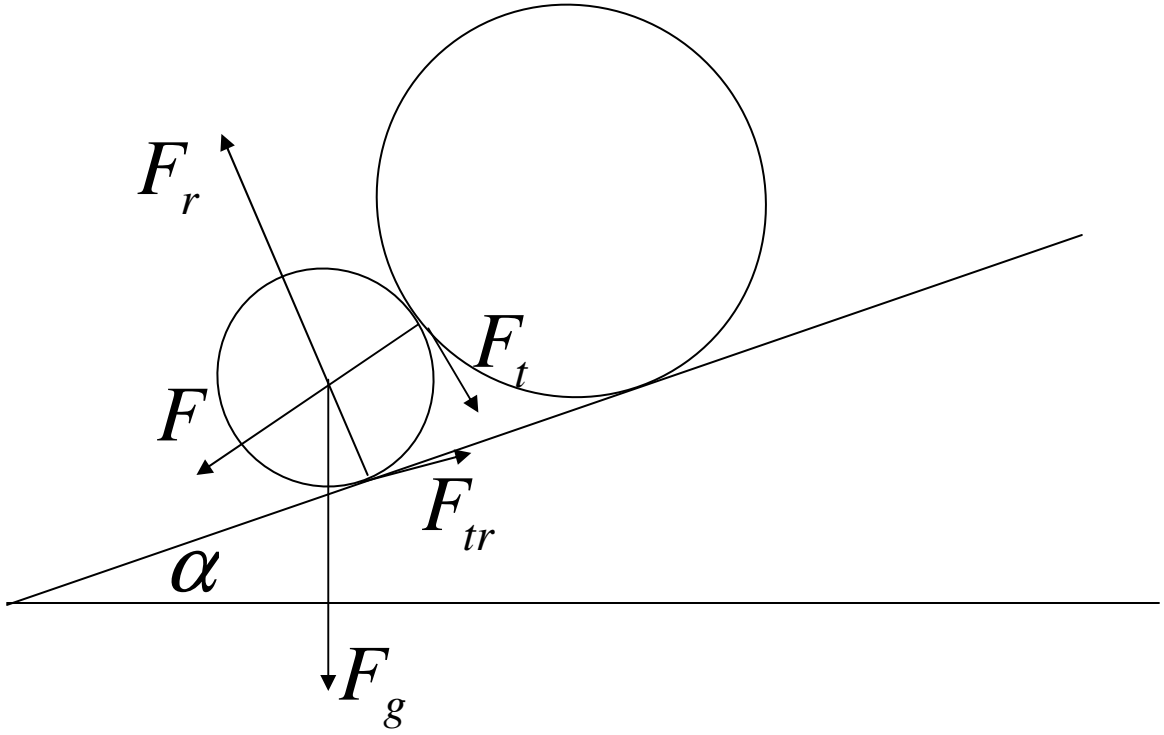
F_r – surface reaction

F – pressure force between balls

F_g – weight

F_{tr} F_t – friction forces

α – tilt angle

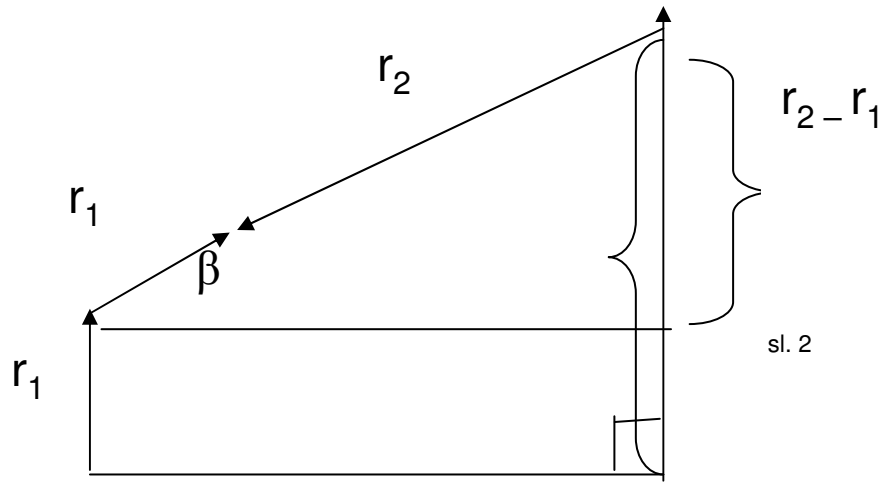


Namely, moments of forces F_{tr} and F_t can, under some conditions, annul.

2.6 Theory

We will introduce following labels:

- index 1 denotes lower and index 2 upper ball
- r – ball radius
- angle $\beta = \arcsin \frac{r_2 - r_1}{r_2 + r_1}$
- angle α – tilt angle

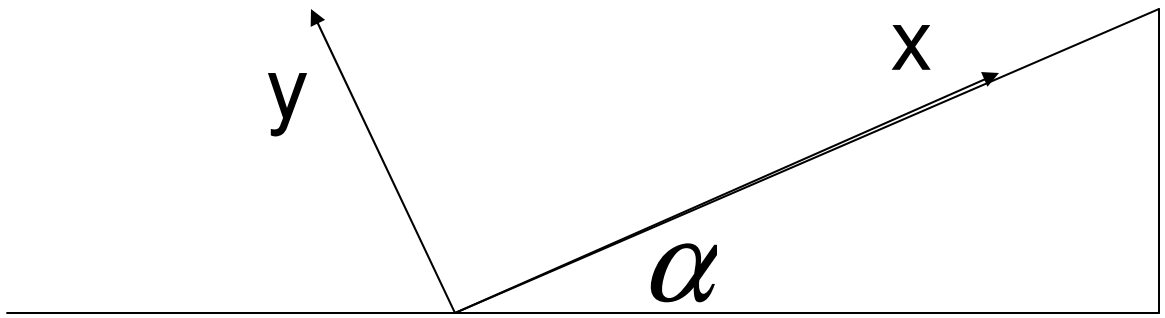


2.7 Theory

- equilibrium conditions are:

$$\sum^{\mu} F = 0 \qquad \sum^{\mu} M = 0$$

- we will use following coordinate system:



2.8 Theory

$\sum \vec{F} = 0$ - for ball 1:

$$F\cos\beta - F_{tr1} + F_{g1}\sin\alpha - F_t\sin\beta = 0$$
$$-F_{r1} + F\sin\beta + F_{g1}\cos\alpha + F_t\cos\beta = 0$$

- for ball 2:

$$F_t\sin\beta - F\cos\beta + F_{g2}\sin\alpha - F_{tr2} = 0$$
$$-F_{r2} - F\sin\beta + F_{g2}\cos\alpha - F_t\cos\beta = 0$$

F – pressure force between balls

F_t – friction between balls

α – tilt angle

$\sin\beta = (r_2 - r_1)/(r_2 + r_1)$

F_{r1} – surface reaction

F_{g1} – weight

F_{tr1} – friction

} for lower ball

F_{r2} – surface reaction

F_{g2} – weight

F_{tr2} – friction

} for upper ball

2.9 Theory

$$\sum \vec{M} = 0$$

- for ball 1:

$$F_t - F_{tr\,2} = 0$$

- for ball 2:

$$F_t - F_{tr1} = 0$$

F_t – friction between balls

F_{tr1} – friction for ball 1

F_{tr2} – friction for ball 2

2.10 Theory

- solving that system for forces:

$$F = \frac{1}{2} g \sec\beta \sin\alpha(-m_1 + m_2 + (m_1 + m_2)\sin\beta)$$

$$F_{r2} = \frac{1}{2} g(2m_2 \cos\alpha + \frac{\sin\alpha}{\cos\beta}(-m_1 - m_2 - (m_2 - m_1)\sin\beta))$$

$$F_{r1} = \frac{1}{2} g(2m_1 \cos\alpha + \frac{\sin\alpha}{\cos\beta}(m_1 + m_2 + (m_2 - m_1)\sin\beta))$$

$$F_t = F_{tr1} = F_{tr2} = \frac{1}{2} g(m_1 + m_2)\sin\alpha$$

F – pressure force between balls

F_t – friction between balls

α – tilt angle

$\sin\beta = (r_2 - r_1)/(r_2 + r_1)$

F_{r1} – surface reaction

F_{g1} – weight

F_{tr1} – friction

} for lower ball

F_{r2} – surface reaction

F_{g2} – weight

F_{tr2} – friction

} for upper ball

2.11 Theory

We used ball of the same material.

We can introduce following label:

$$\varphi = \frac{r_2}{r_1} \quad \Rightarrow \quad \frac{m_2}{m_1} = \varphi^3$$

To study statically friction coefficients we should find ratios:

$$\frac{F_t}{F}, \frac{F_{tr1}}{F_{r1}}, \frac{F_{tr2}}{F_{r2}}$$

2.12 Theory

$$\frac{F_t}{F} = \frac{\sqrt{\varphi}\left(\varphi^3 + 1\right)}{\varphi^4 - 1} \leq \mu_{12}$$

As: $\mu_{12} = 0.70$

Follows (numerically): $\varphi \geq 2.44$

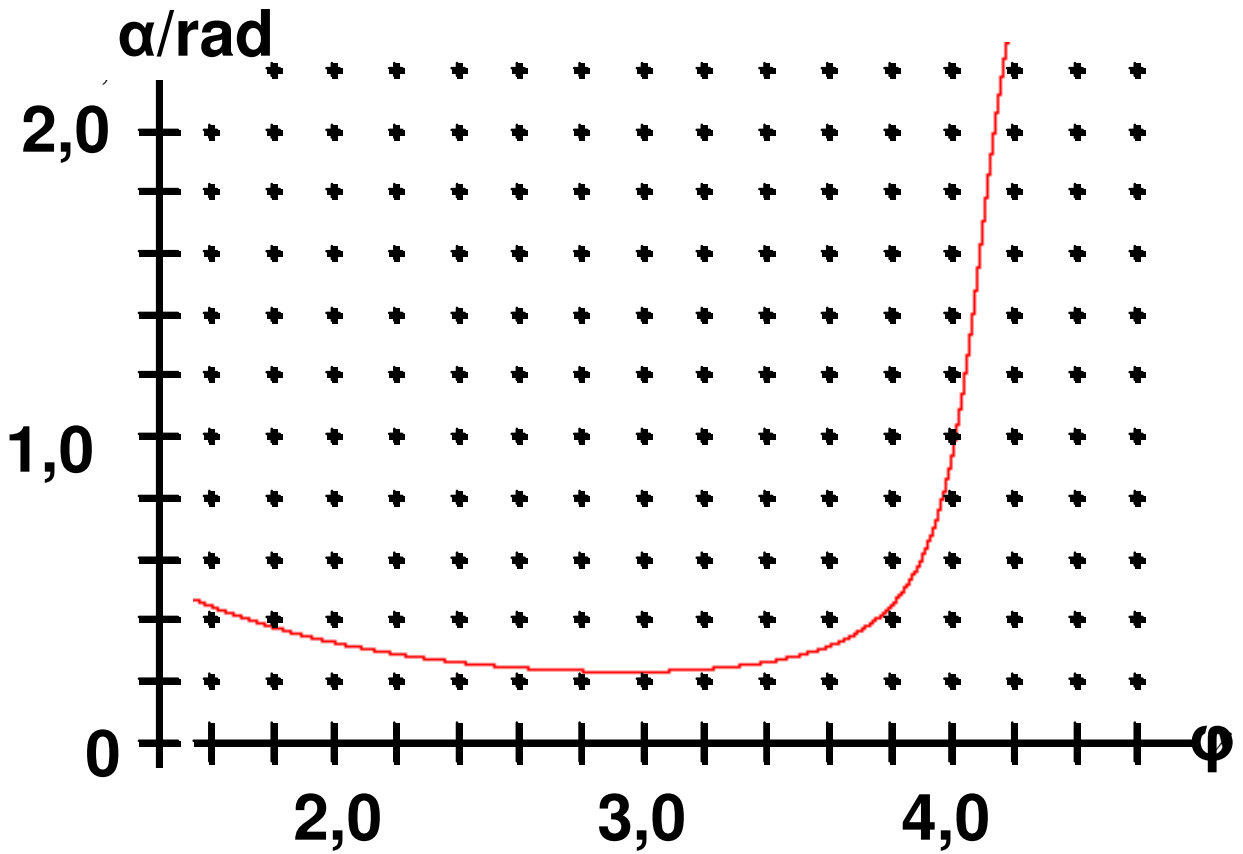
2.13 Theory

$$\frac{F_{tr1}}{F_{r1}} = \frac{\sqrt{\varphi}\left(\varphi^3+1\right)}{2\sqrt{\varphi}\,\frac{1}{\tan\alpha}+\varphi^4+1} = \mu_1$$

for max. α:

$$\mu_1 = 0.50$$

Theoretical dependence max. α on φ concerning lower ball.



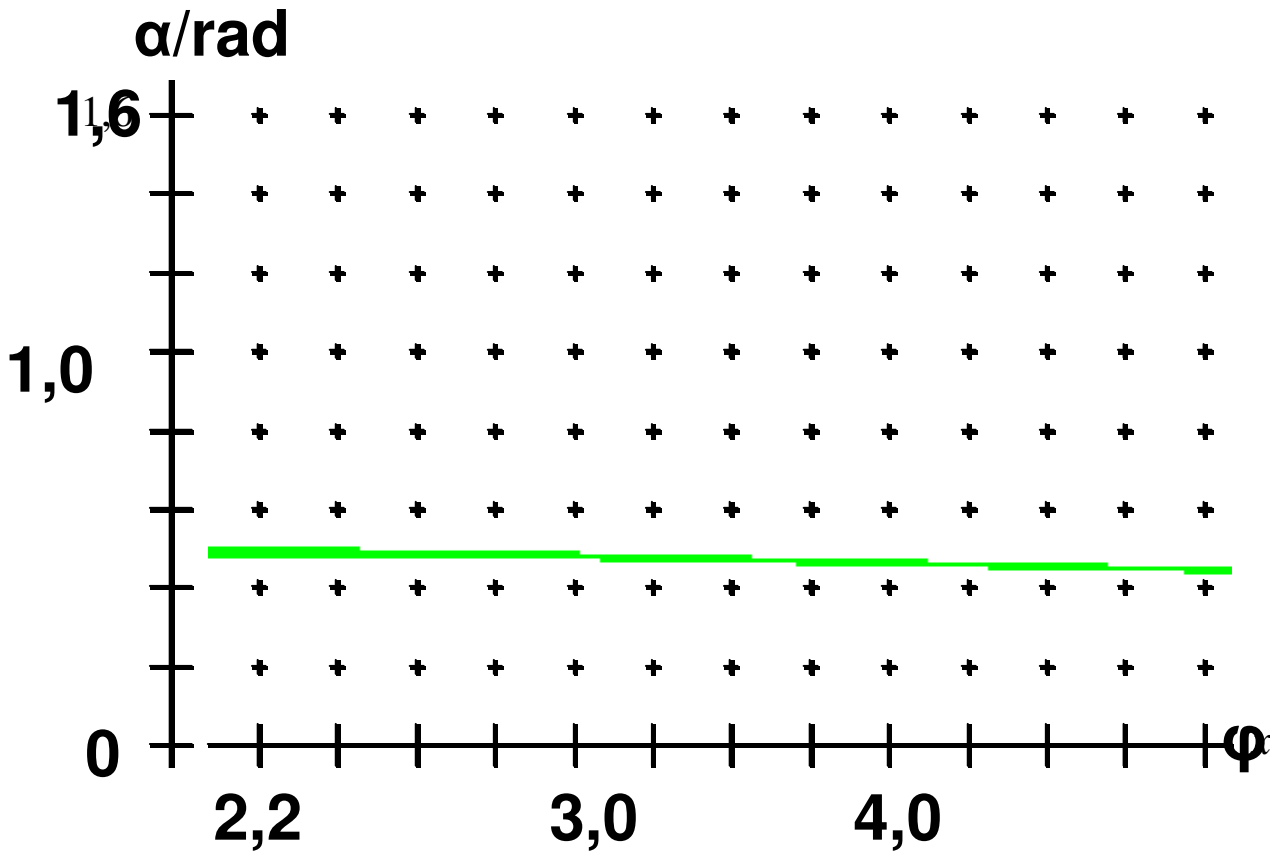
2.14 Theory

$$\frac{F_{tr2}}{F_{r2}} = \frac{\sqrt{\varphi}\left(\varphi^3 + 1\right)}{2\sqrt{\varphi}\,\frac{\varphi^3}{\tan\alpha} - \varphi^4 - 1} = \mu_2$$

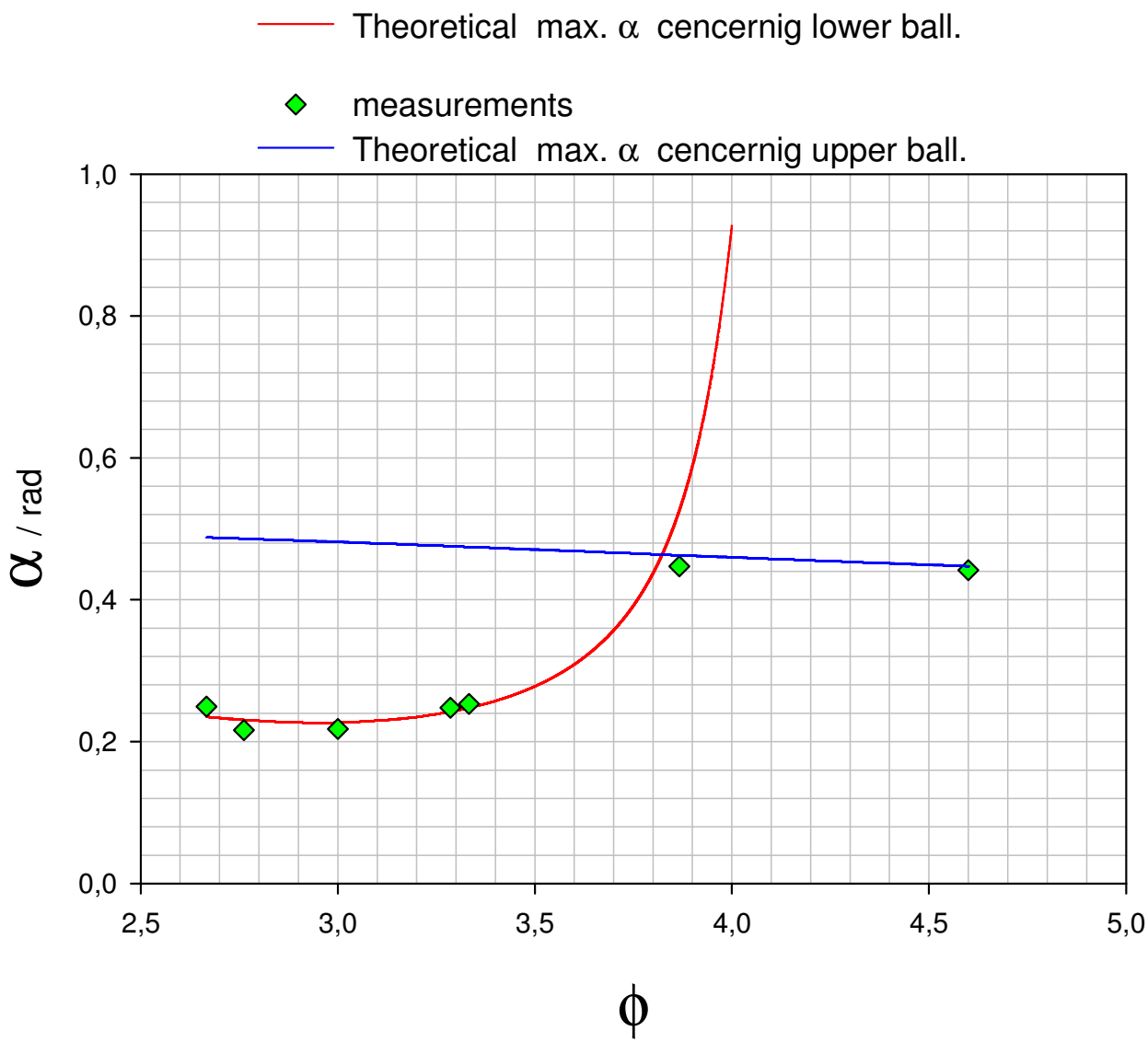
for max. α

$$\mu_2 = 0.50$$

Theoretical dependence max. α on φ concerning upper ball.



Theory and experiment comparison



Zakljucak

- we got the phenomenon
- we constructed mathematical model which is describing parameters very good
- we used balls of same material to reduce number of variables which simplifies model
- remark: phenomenon occurred for under some other conditions for small tilt angles α, but as our balls are not perfectly spherical model can’t describe these situations