Problem 9.

Sound in the glass

Problem

Fill a glass with water. Put a tea-spoon of salt into the water and stir it. Explain the change of the sound produced by the clicking of the glass with the tea-spoon during the dissolving process.

Experiment

- Measurement:

  How frequency depends on:

  - concentration of salt (density solution) with a constant level of water
  - the level of water

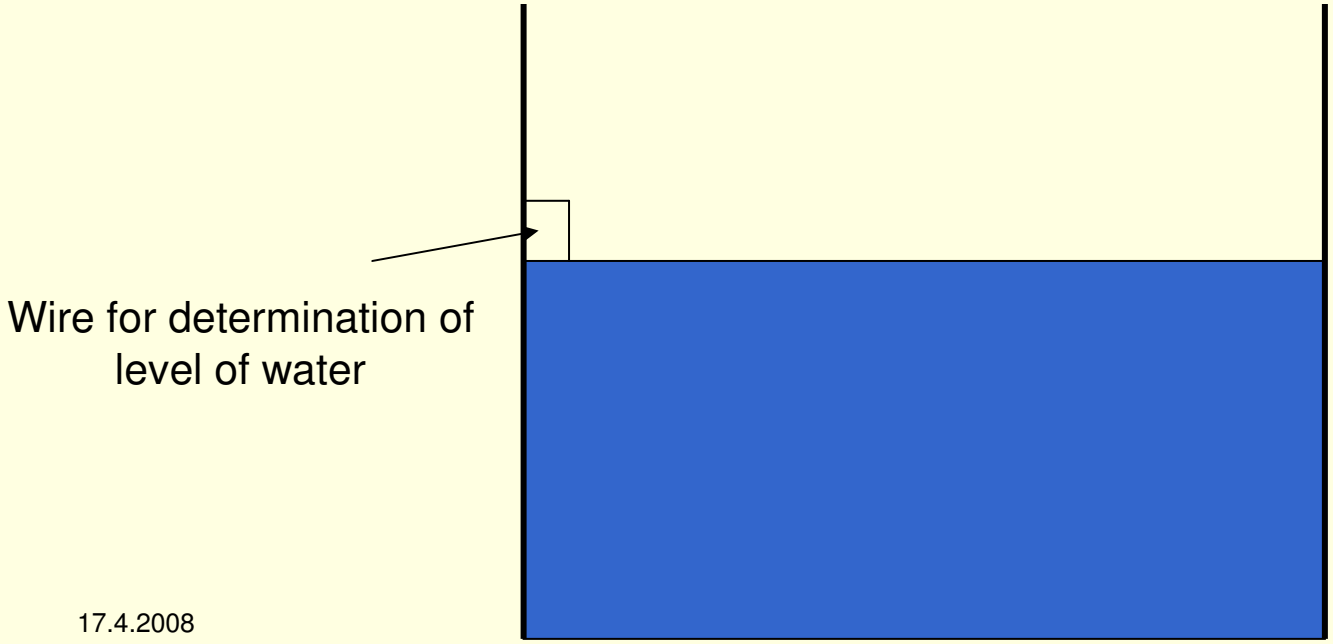
- Frequency was measured through the lock in amplifier ,which has accuracy up to the 10^{-5} Hz

Experiment cont.

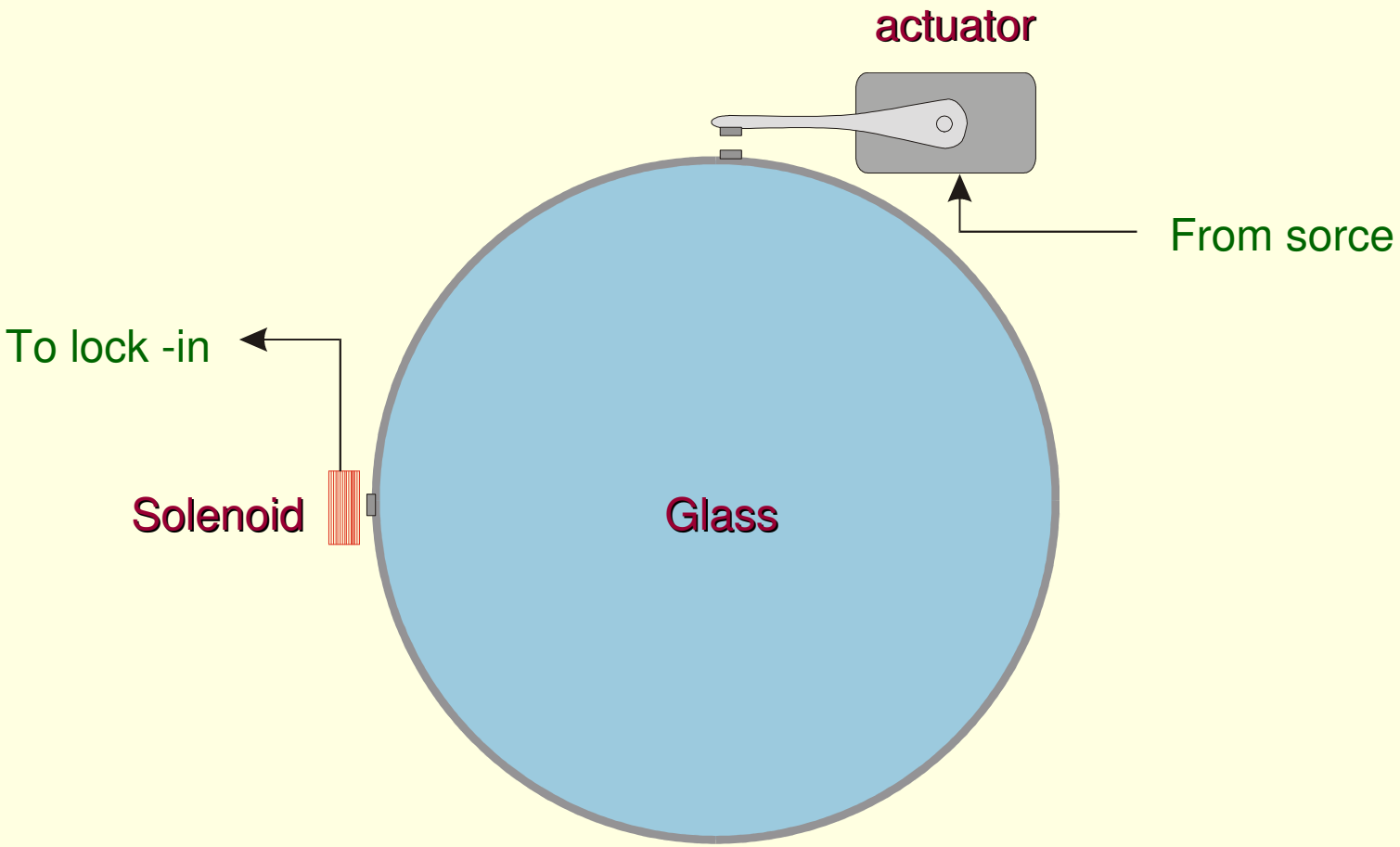
- Parts of apparatus:
    1. Glass with two attached magnets
    2. Driving actuator
    3. Sensor – solenoid
    4. Lock - in amplifier
    5. Modulated power source
    6. Water level control

Experiment cont.

- Water level control
  - It is controlled through surface tension



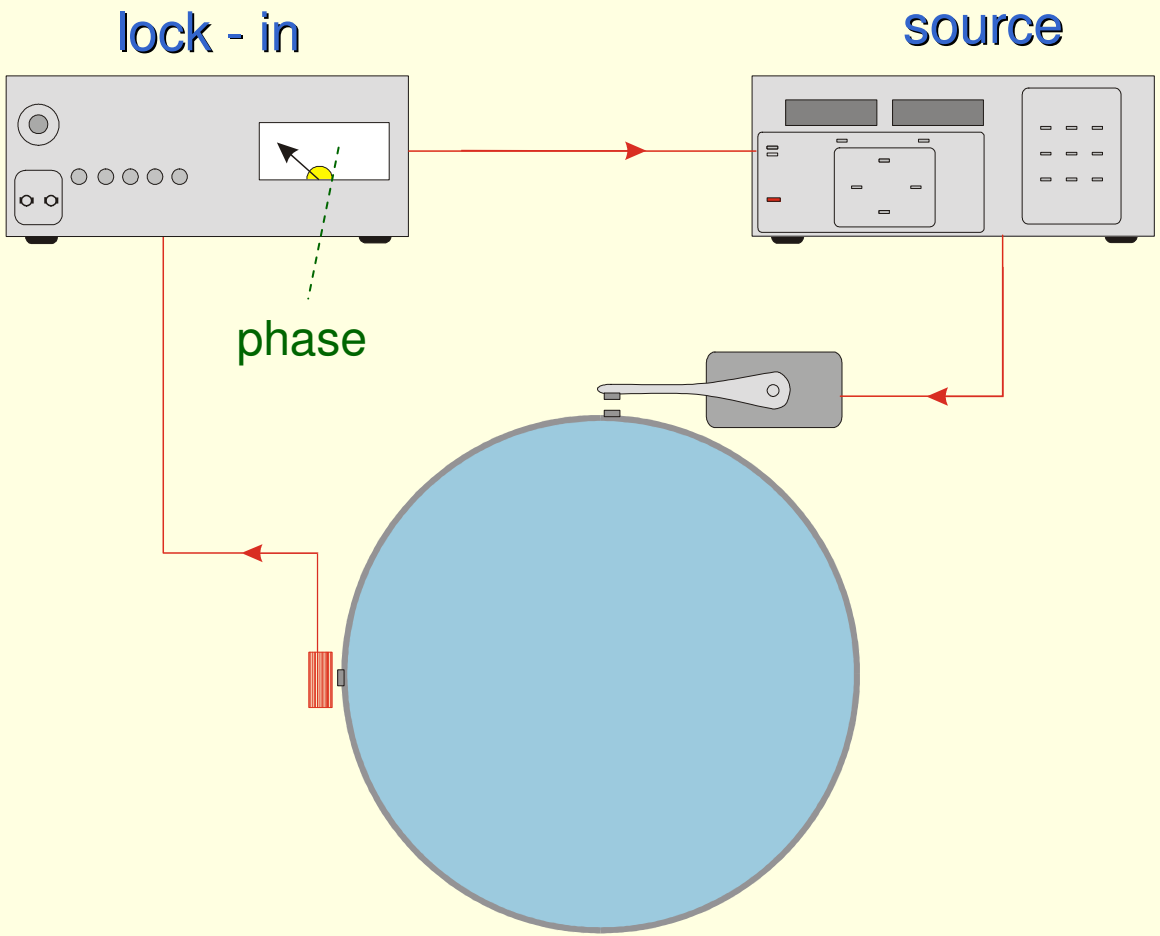
Experiment cont.



Experiment cont.



Experiment cont.



Experiment cont.





Source + lock - in

Explanation

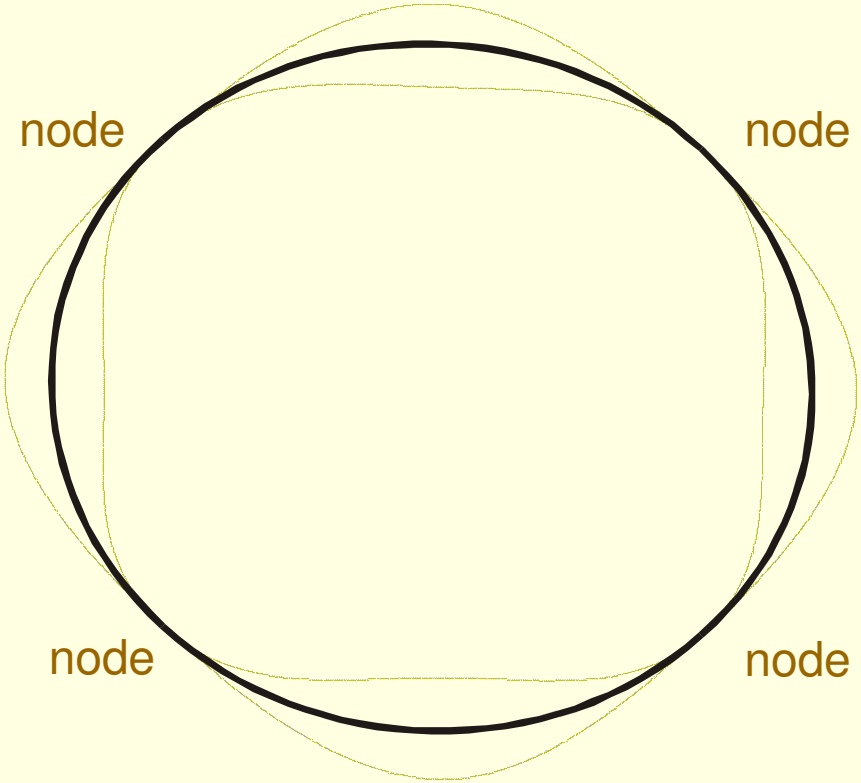
- On impact standing waves are formed
- Their frequency depends on the flexibility and mass of the glass
- When glass is filled with liquid, vibration will transfer to that liquid
- By doing so effective oscillating mass is incrised while the flexibility remains the same
- Therefore the frequency decreases

Explanation cont.

- Extraction of decresing frequency:
  - Waves on the empty glass – flexibility
  - Glass with water – influence of liquid
  - Water + salt – influence od density
- Basic principle – energy conservation

1. Waves on the empty glass

- Basic transversal mode:





1. Waves on the empty glass cont.

- Energy conservation (temporal average):

$$\overline{E_k} = \overline{E_p}$$

E_k – kinetic energy

E_p – potential energy

- We modulate the glass as a linear spring:

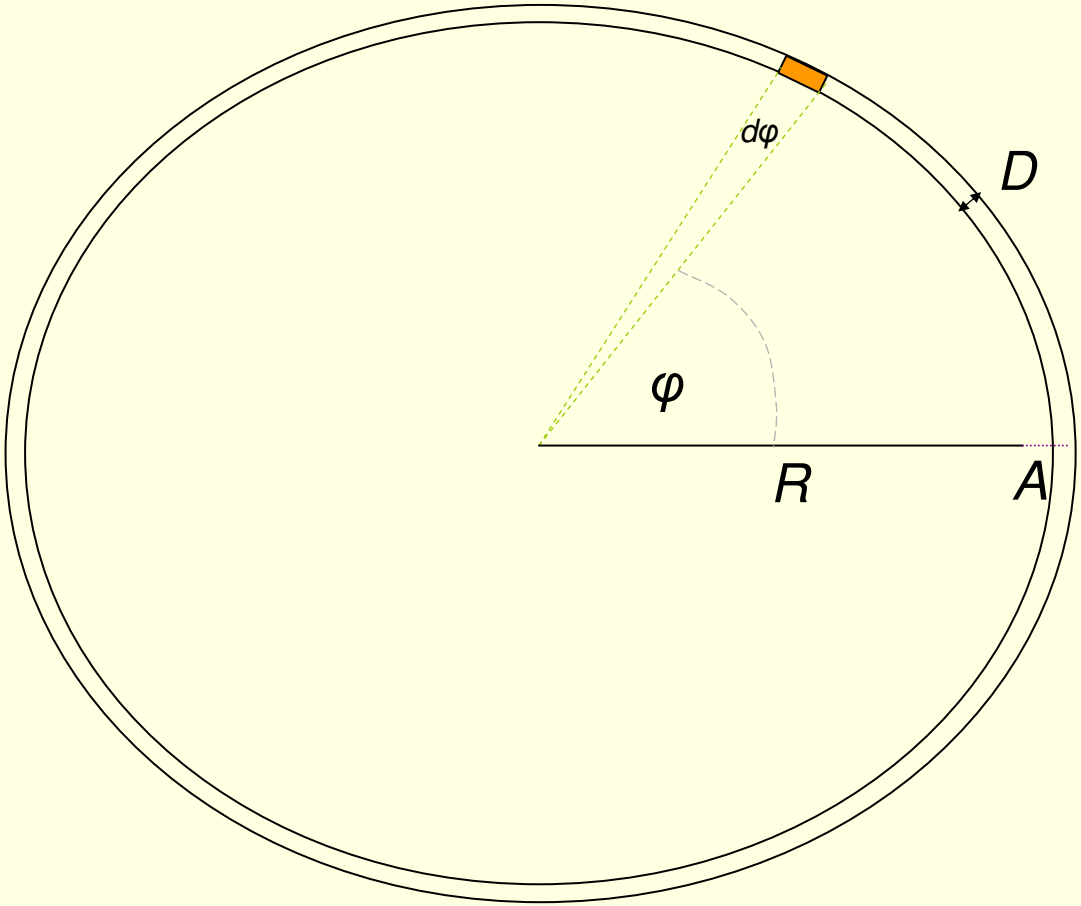
$$\overline{E_p} = \frac{1}{4} k A^2$$

k – elastic coefficient

A – amplitude

1. Waves on the empty glass cont.

- Kinetic energy geometry of infinitesimal mass:



1. Waves on the empty glass cont.

- Kinetic energy:

$$dE_k = \frac{1}{2}\left(\frac{z}{L} A\omega_0 \sin\omega_0 t \cos 2\varphi\right)^2 \rho_g RDd\varphi dz$$

(velocity)^2 mass

z – hight from the bottom

L – overall height of the glass

A – amplitude

ω_0 – circular frequency

φ – polar angle

ρ_g – density of the glass

R – radius of the glass

D – thickness of the glass

1. Waves on the empty glass cont.

$$\Rightarrow \quad k = \frac{1}{3} R \pi L D \rho_g \omega_0^2$$

- We can mesure the value of elastic coefficient over the frequency of empty glass
- It is hardly changed when we add water

2. Glass with water

- Energy conservation (we disregard hight change of water):

$$\overline{E_{kg} + E_{kw}} \approx \overline{E_{pg}}$$

E_{kg} – kinetic energy of the glass

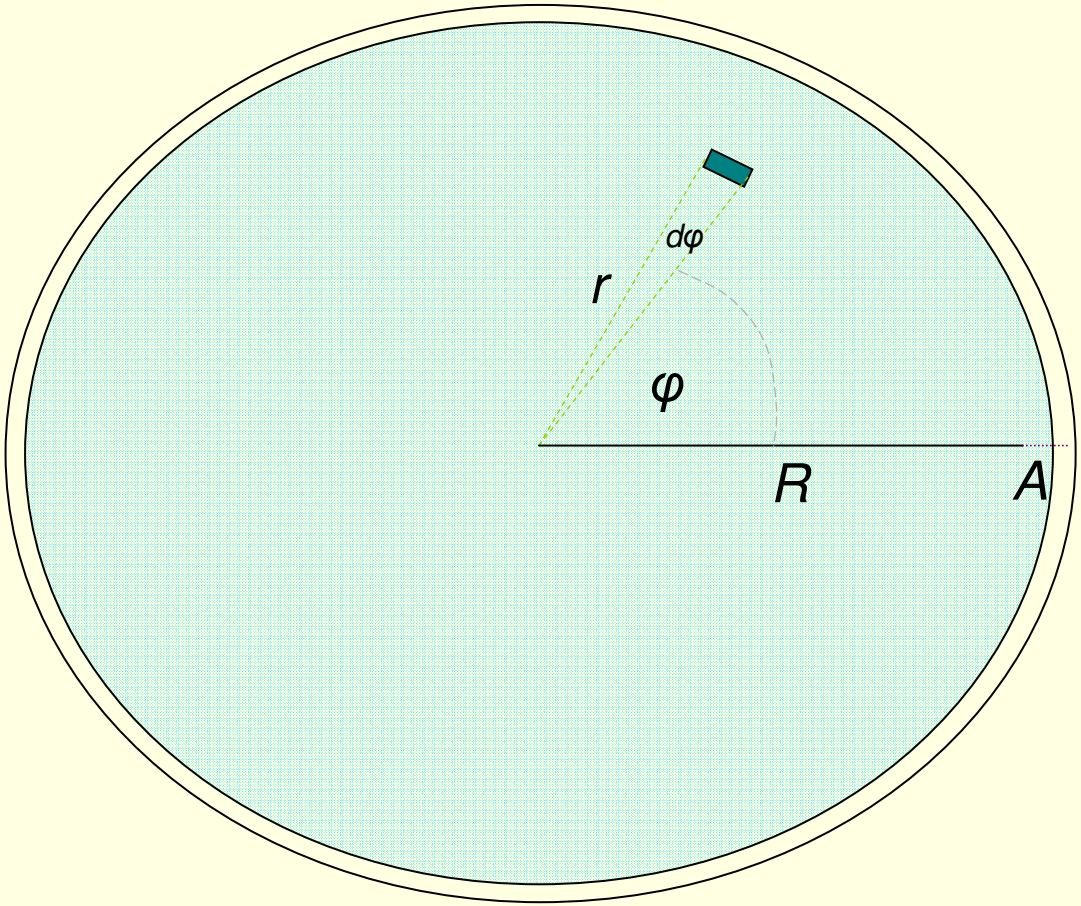
E_{pg} – elastic potential energy

E_{kw} – kinetic energy of water

- Energy of the glass it the same as in the previous solution

2. Glass with water cont.

- Kinetic energy geometry of infinitesimal water:



2. Glass with water cont.

- Kinetic energy:

$$dE_{kg} = \frac{1}{2}\left(\frac{z}{L}\frac{r}{R}A\omega\sin\omega t\cos 2\varphi\right)^2 \rho_w r dr d\varphi dz$$

(velocity)^2 mass

z – hight from the bottom

L – overall height of the glass

A – amplitude

ω – circular frequency

φ – polar angle

ρ_w – density of water

R – radius of the glass

r – distance from axis

2. Glass with water cont.

- Energy integration goes to *l* (*l* = level of water)
- Insertion in conservation:

$$\left(\frac{\omega_0}{\omega}\right)^2 - 1 = \frac{1}{4}\frac{\rho_w}{\rho_g}\frac{R}{D}\left(\frac{l}{L}\right)^3$$

ω_0 – circular frequency of empty glass

ρ_w – density of water

ρ_g – density of glass

ω – circular frequency of glass with water

- We will use the same equation with salted water by changing ρ_w element

3. Salted water

- Density is changed by adding of salt
- Change is linear to the concentration of salt
- Frequency equation takes the following form:

$$\left(\frac{\omega}{\omega_s}\right)^2 - 1 = \frac{1}{4}\frac{\rho_{So}}{\rho_g}\frac{R}{D}\left(\frac{l}{L}\right)^3$$

ω_s – circular frequency of glass with water and salt

ρ_{So} – density of solution

3. Salted water cont.

- Neglected:
  - Appearance of waves
  - Oscilation of the bottom of the glass
  - Non-linear effects (large amplitudes)
  - Modes in z – direction
  - Changes of elasticity due to the water/salt
  - Higher harmonics...

Cosiderating all…

- Fit with experiment is relatively good!
- Evaluation per hight and density resultes with similar, good result
- Numerical analysis: comparison of constant

$$\left(\frac{\omega}{\omega_s}\right)^2 - 1 = \eta \frac{\rho_{So}}{\rho_g} \frac{R}{D} \left(\frac{l}{L}\right)^3$$

Theoretical η: 0,25

Numerical analysis

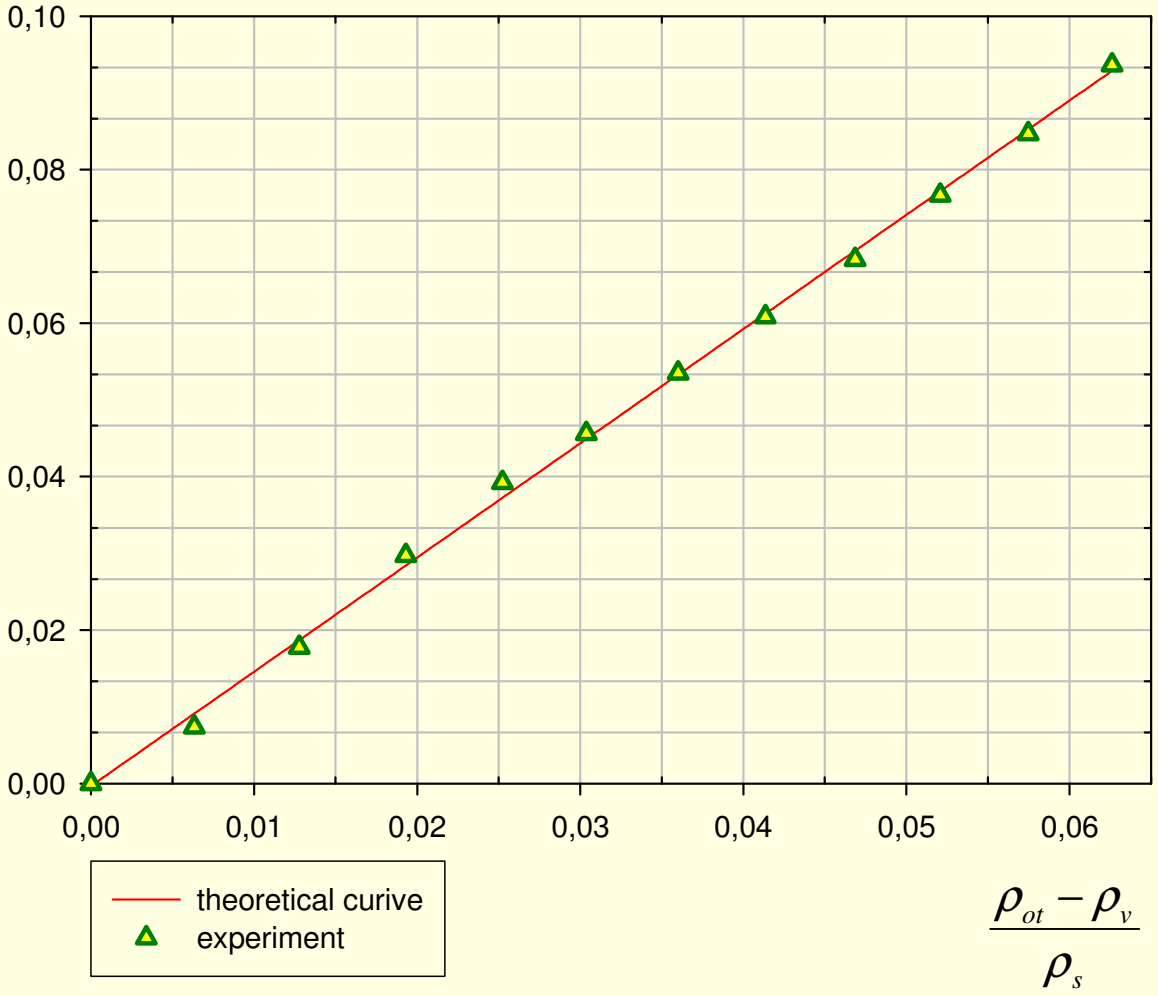
L	55,2 cm
2R	35,9 cm
D	3 mm
ρ_g	2,6 g/cm^3
A	1,49 $\pm$0,01
B	5,07 $\pm$0,02
η (ρ_s)	0,26
η (L)	0,22

L – overall height of the glass
R – radius
D – thickness of the glass
ρ_g – density of the glass
A – experimental constant
B – experimental constant

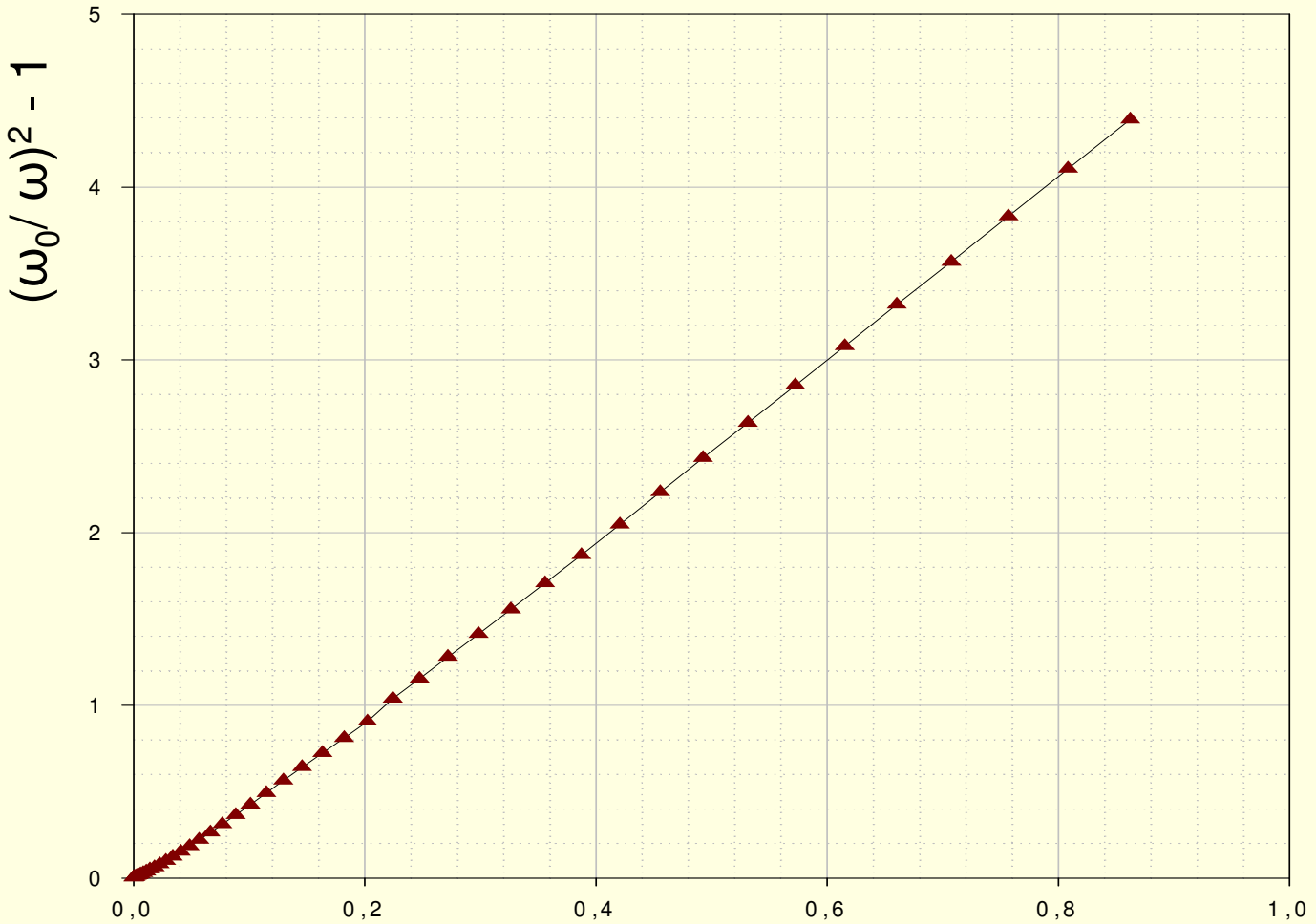
Theory predicts 0,25

Density evaluation

$$\left(\frac{\omega}{\omega_s}\right)^2 - 1$$



Hight evaluation



Conclusion

- Despite roughness our model gives a good description of frequency diminution for an oscilating glass filed with salted water
- We have an answer to the problem:

  As the salt dissolves, density increses which increses effective oscillating mass and frequency drop.