Problem 12. – Ball Spin

Spin can be used to alter the flight path of balls in sport. Investigate the motion of a spinning ball, for example a table-tennis or tennis ball, in order to determine the effect of the relevant parameters.

Effect

Looking from above:



Explanation

We are looking from center of a mass reference system.

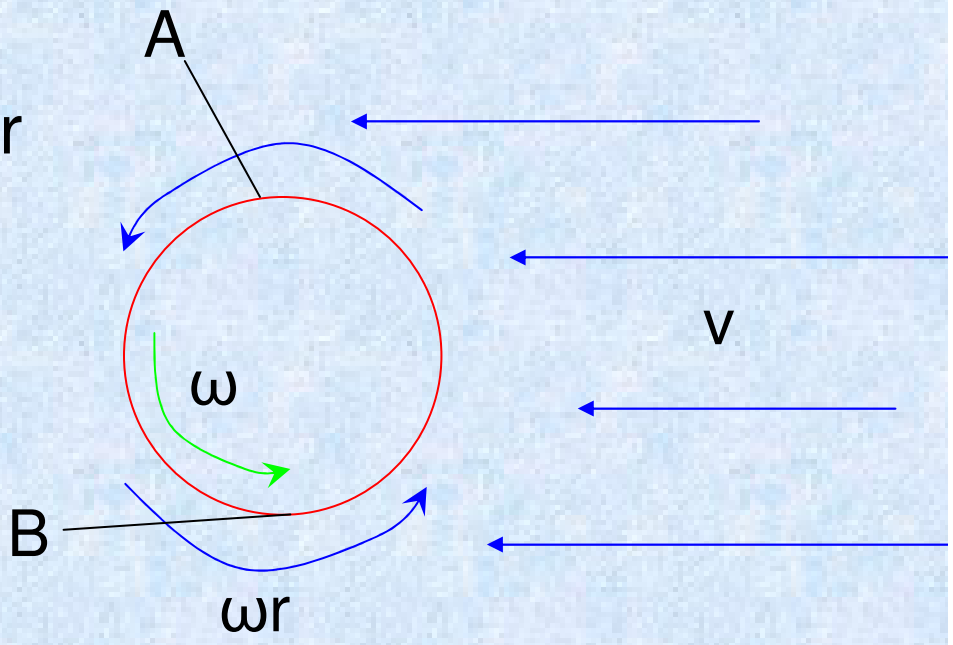
- existence of boundary layer

- pressure difference between A and B (Bernoulli):

$$P_B - P_A = \frac{1}{2}\rho\left(v_A^2 - v_B^2\right)$$

v_A – air speed in A

v_B – air speed in B



ω – ball angular speed

r – ball radius

v – ball and air relative speed

Explanation

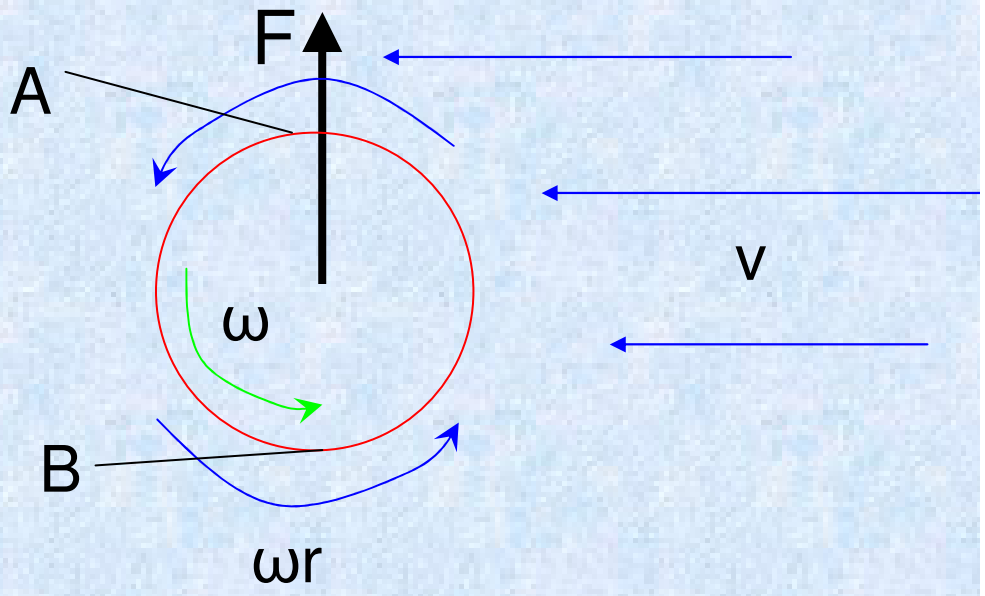
$$v_A = v + \omega r$$

$$v_B = v - \omega r$$

$$\Rightarrow$$

$$P_B - P_A = \rho \omega r v$$

$$F \propto \Delta P$$



There is force on the side!!!

v_A – air speed in A

v_B – air speed in B

ω – ball angular speed

r – ball radius

v – ball and air relative speed

Theoretical introduction

- there is force on an object moving trough fluid called aerodinamic force

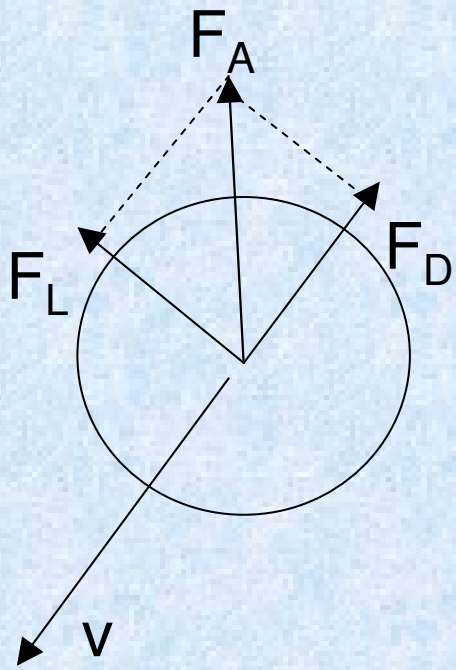
- it can be resolved in two components called lift (perpendicular to the object speed) and drag (parallel to the object speed)

- to describe motion of a ball one should know dependence of drag and lift on:

  - air and ball relative speed

  - ball angular velocity

  - other (ball size, ball surface...)



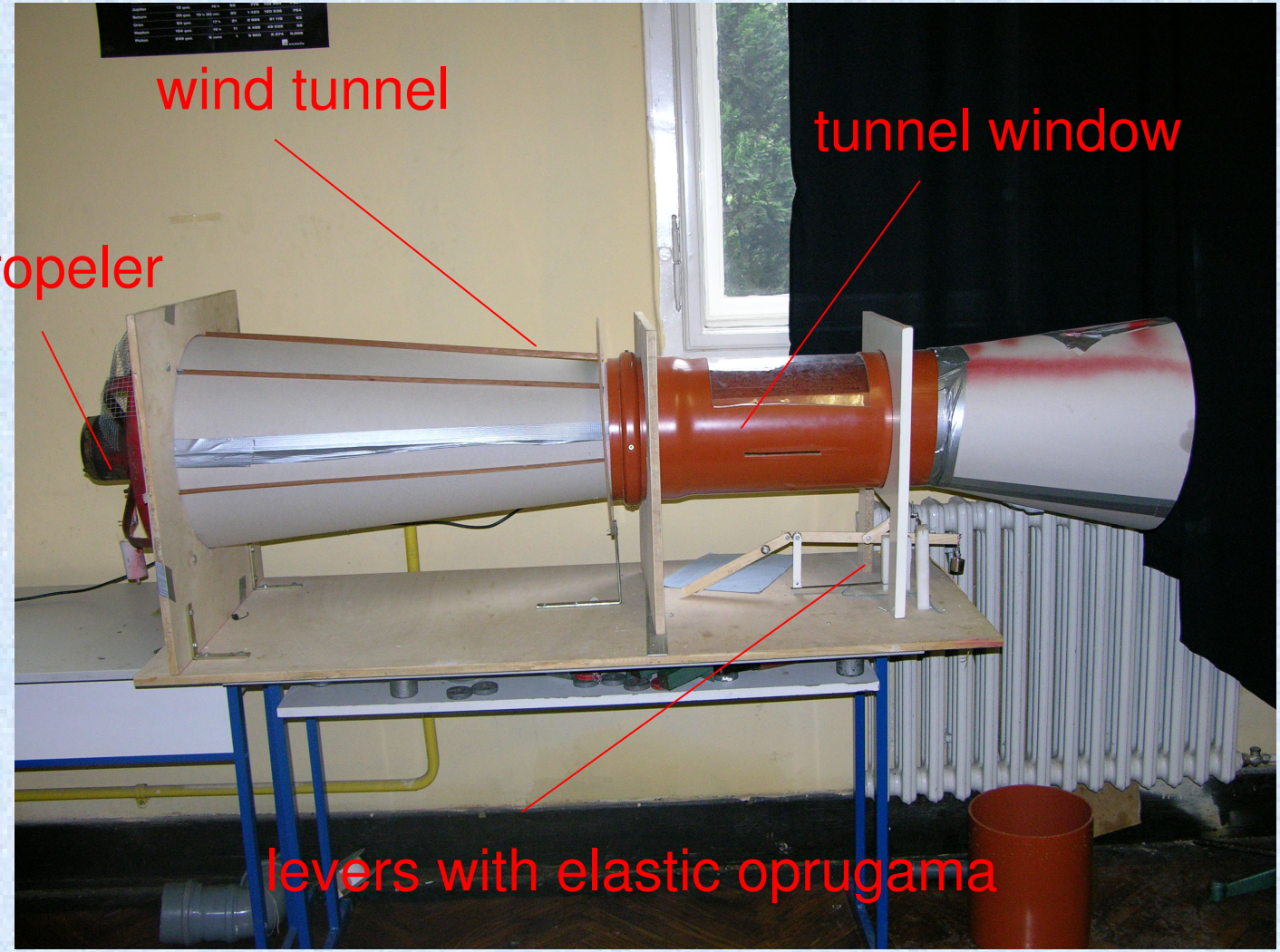
F_A – aerodinmical force

F_L – lift

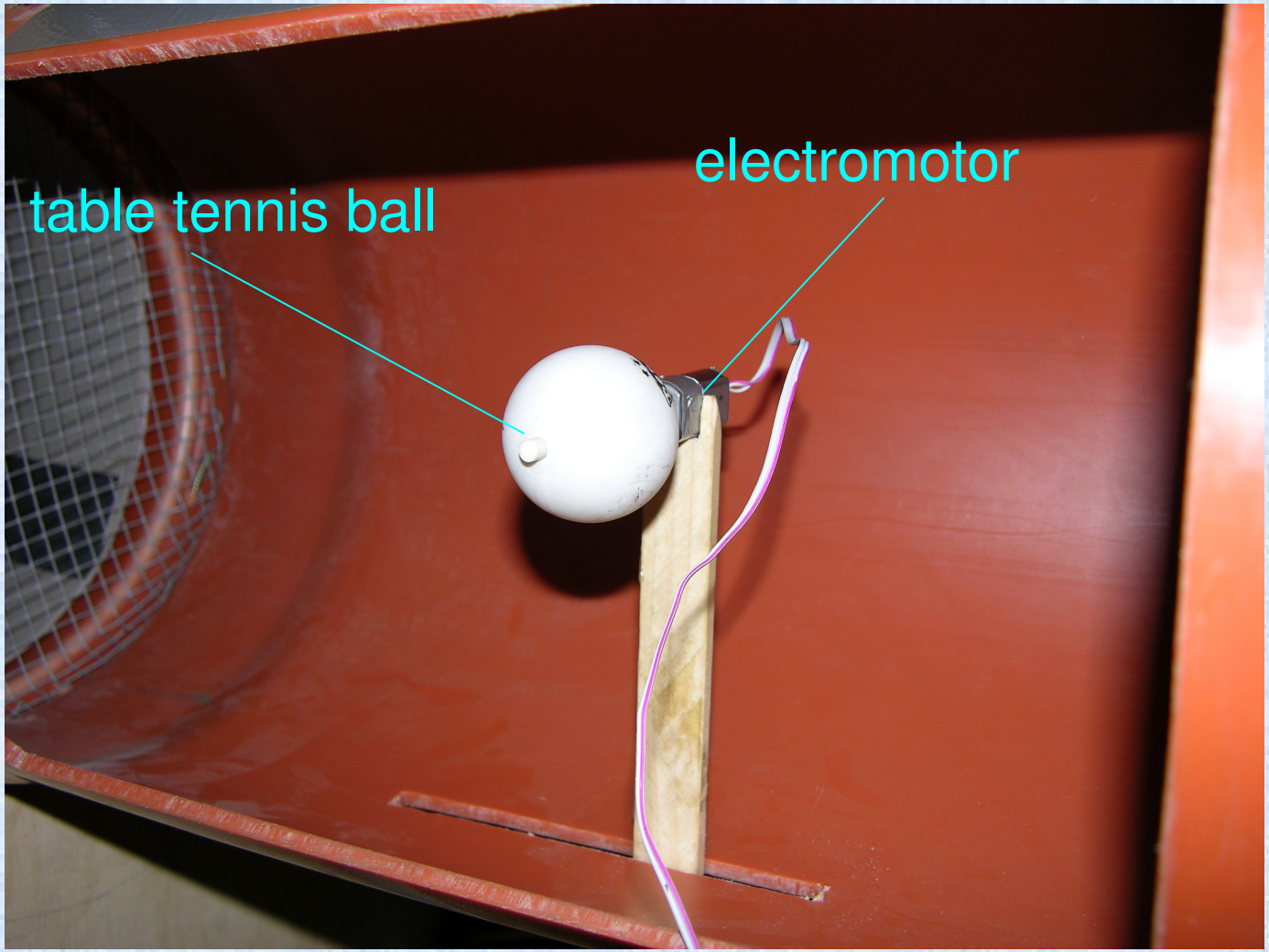
F_D – drag

v – object speed

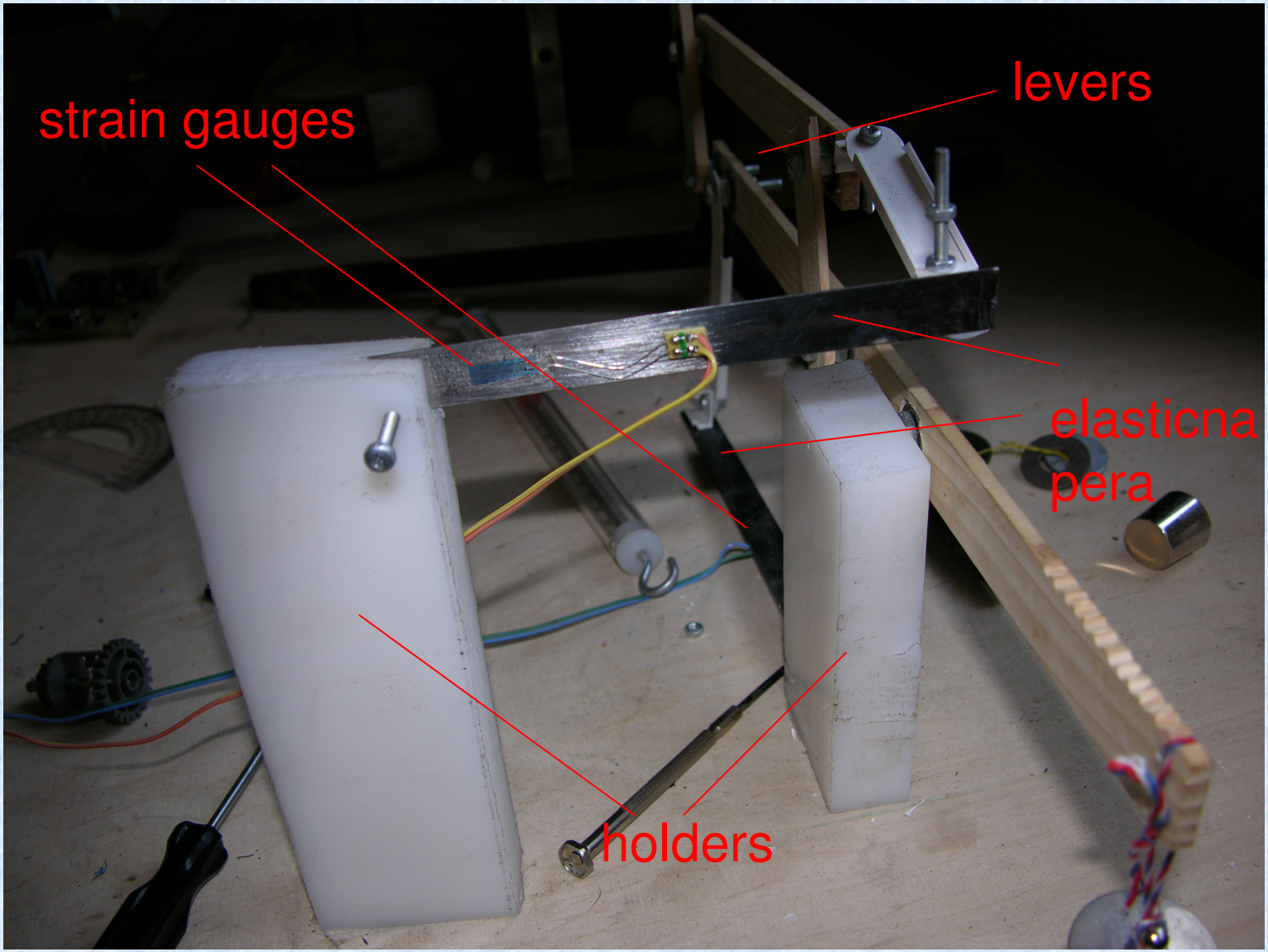
Aparature


wind tunnel
tunnel window
propeler
levers with elastic oprugama

Aparature



Aparature



Aparature

- change of strain gauge resistance is proportional to its deformation

- as they are on elastic deformation is proportional to the force: $F \propto \Delta R$

- we had to calibrate strain gauges to be able to measure lift and drag

- to measure change in resistance we used Wheatstone bridge

- ball angular velocity was measured with stroboscope

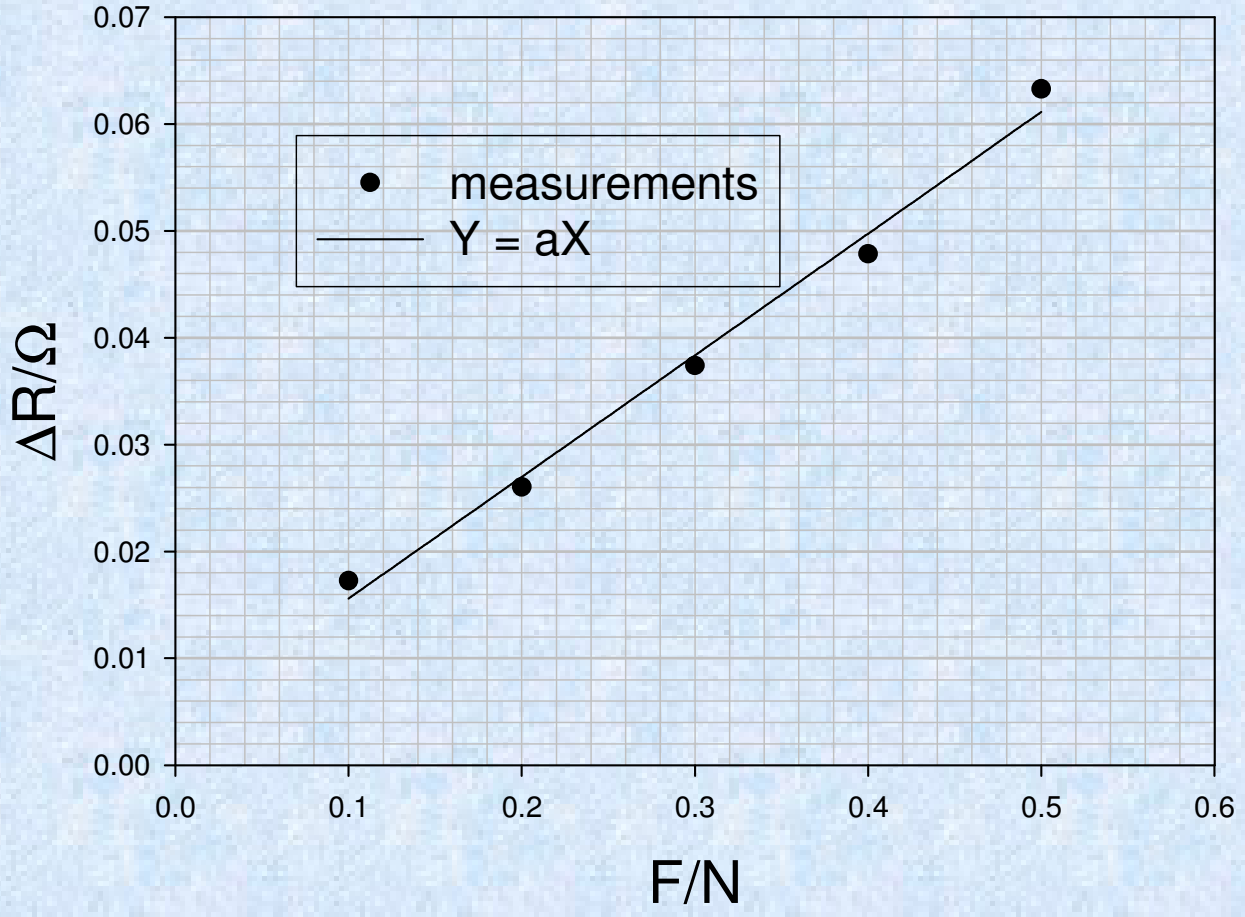
Calibration of strain gauges

- for drag:

$$\Delta R = 0.114 F$$

ΔR – strain gauge resistance change

F – drag



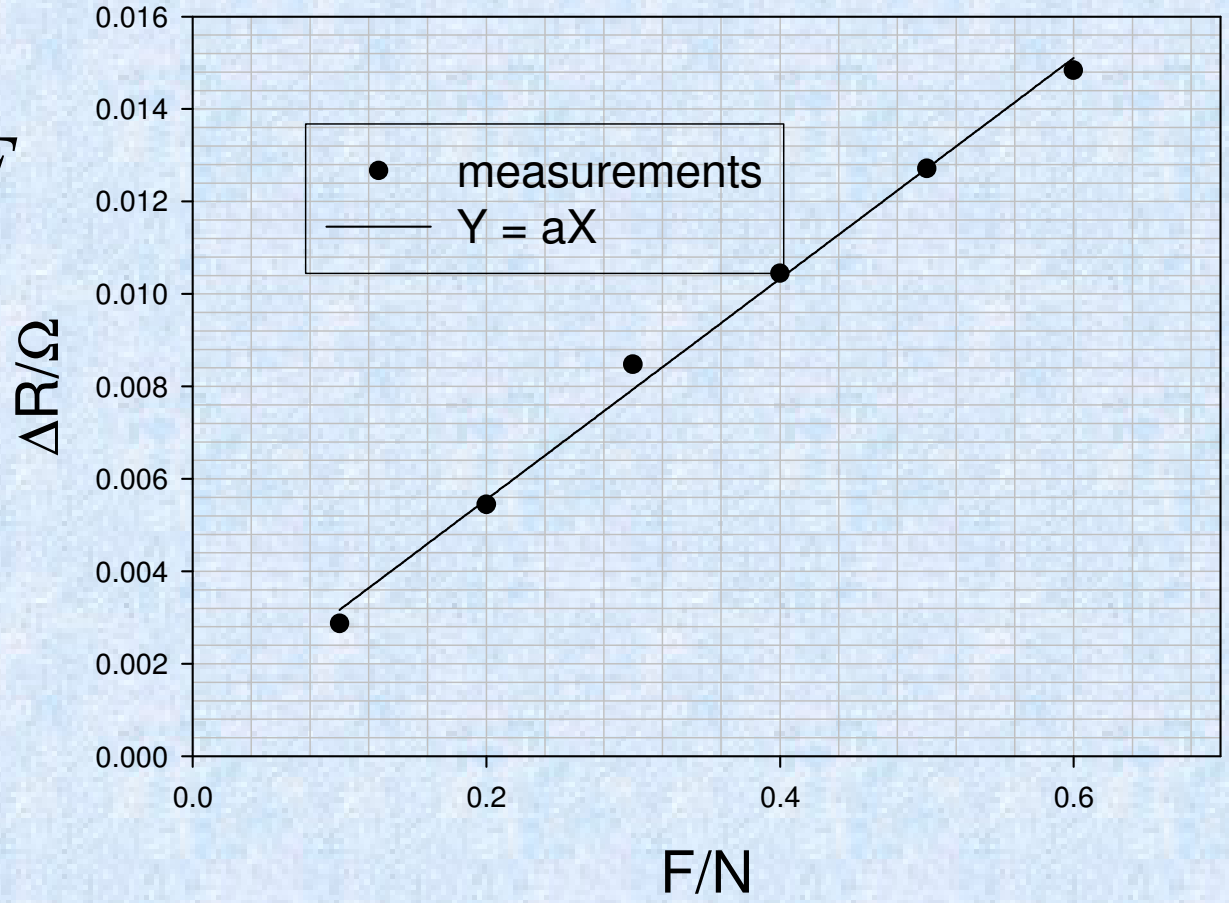
Calibration of strain gauges

- for lift:

$$\Delta R = 0.0239 F$$

ΔR – strain gauge resistance change

F – lift

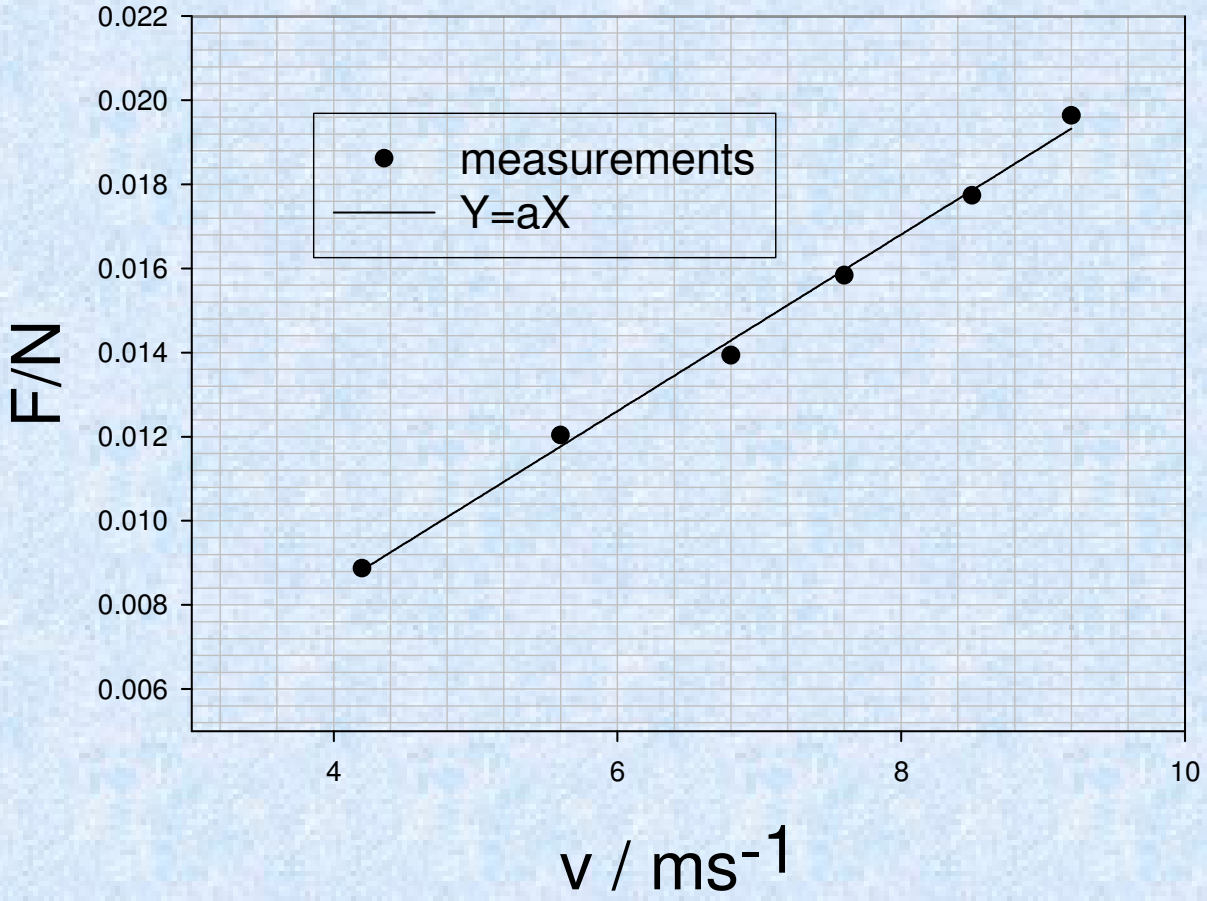


Lift dependence on speed (ω = 374,35 rad/s)

$$F_L = 0.021 \cdot v$$

F_L – drag

v – ball and air relative speed

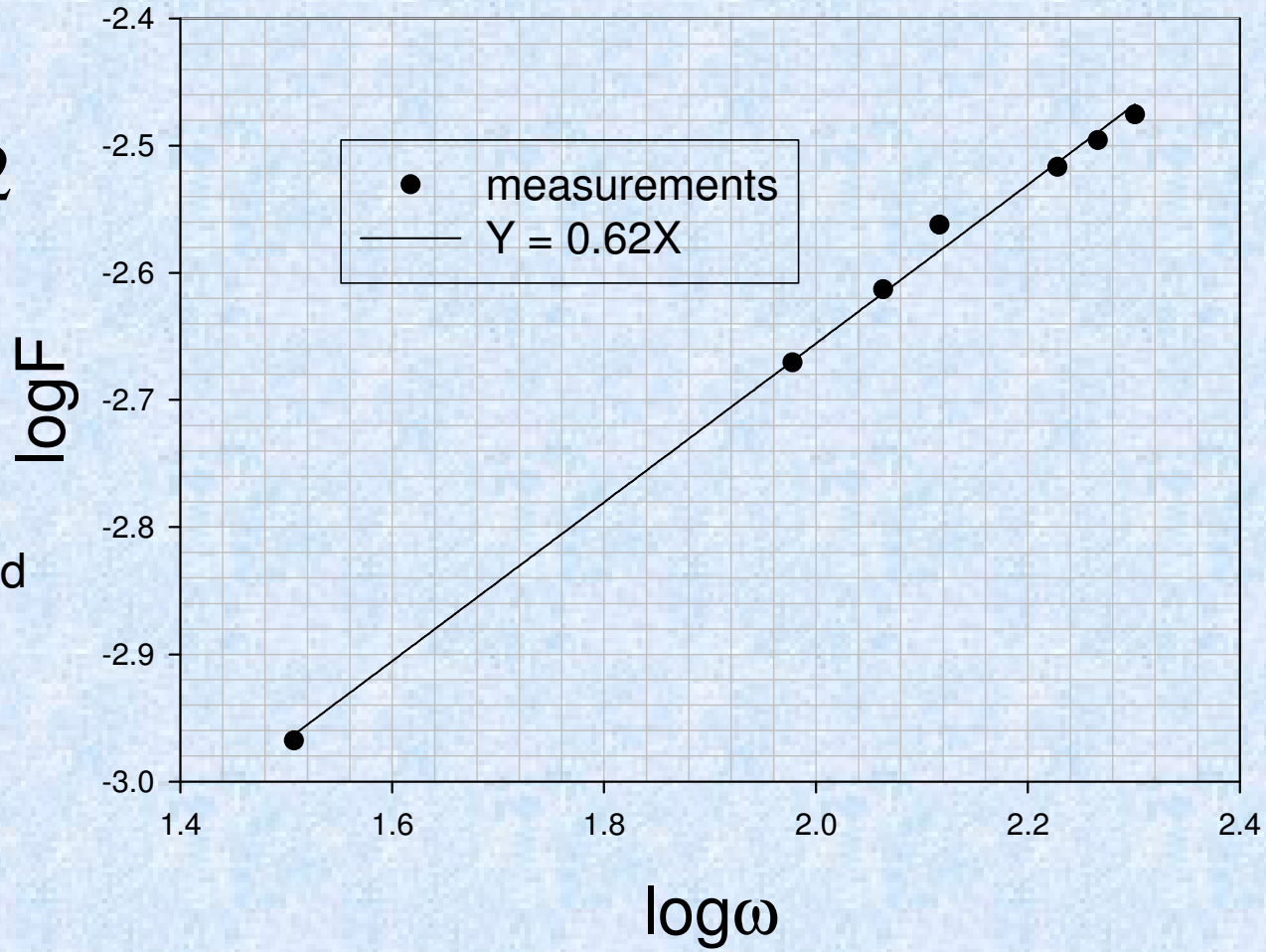


Lift dependence on angular speed ($v = 8.8\ ms^{-1}$)

$$F_L \propto \omega^{0.62}$$

F_L – lift

ω – ball angular speed

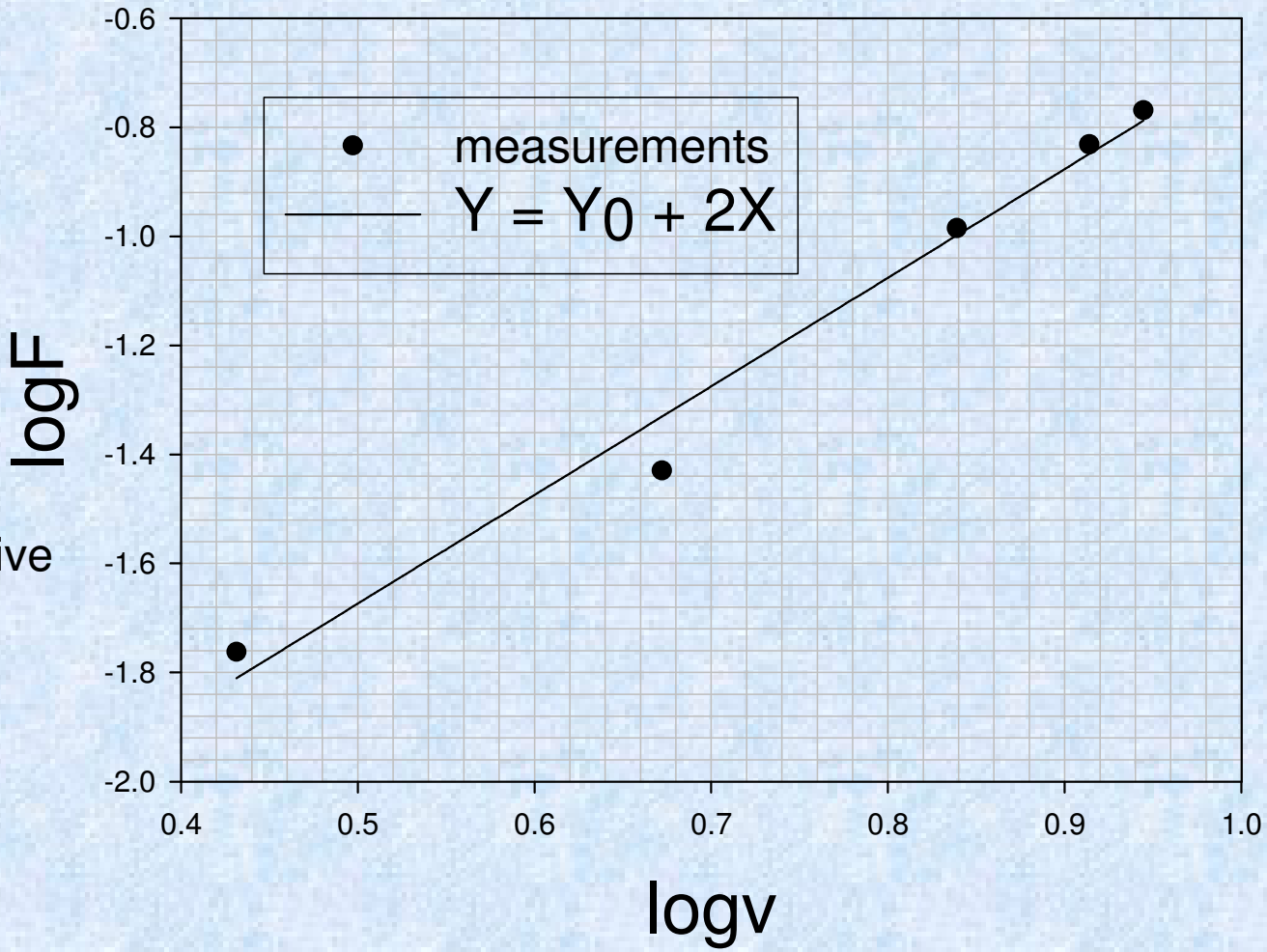


Drag dependence on speed

$$F_D \propto v^2$$

F_D – drag

v – ball and air relative speed

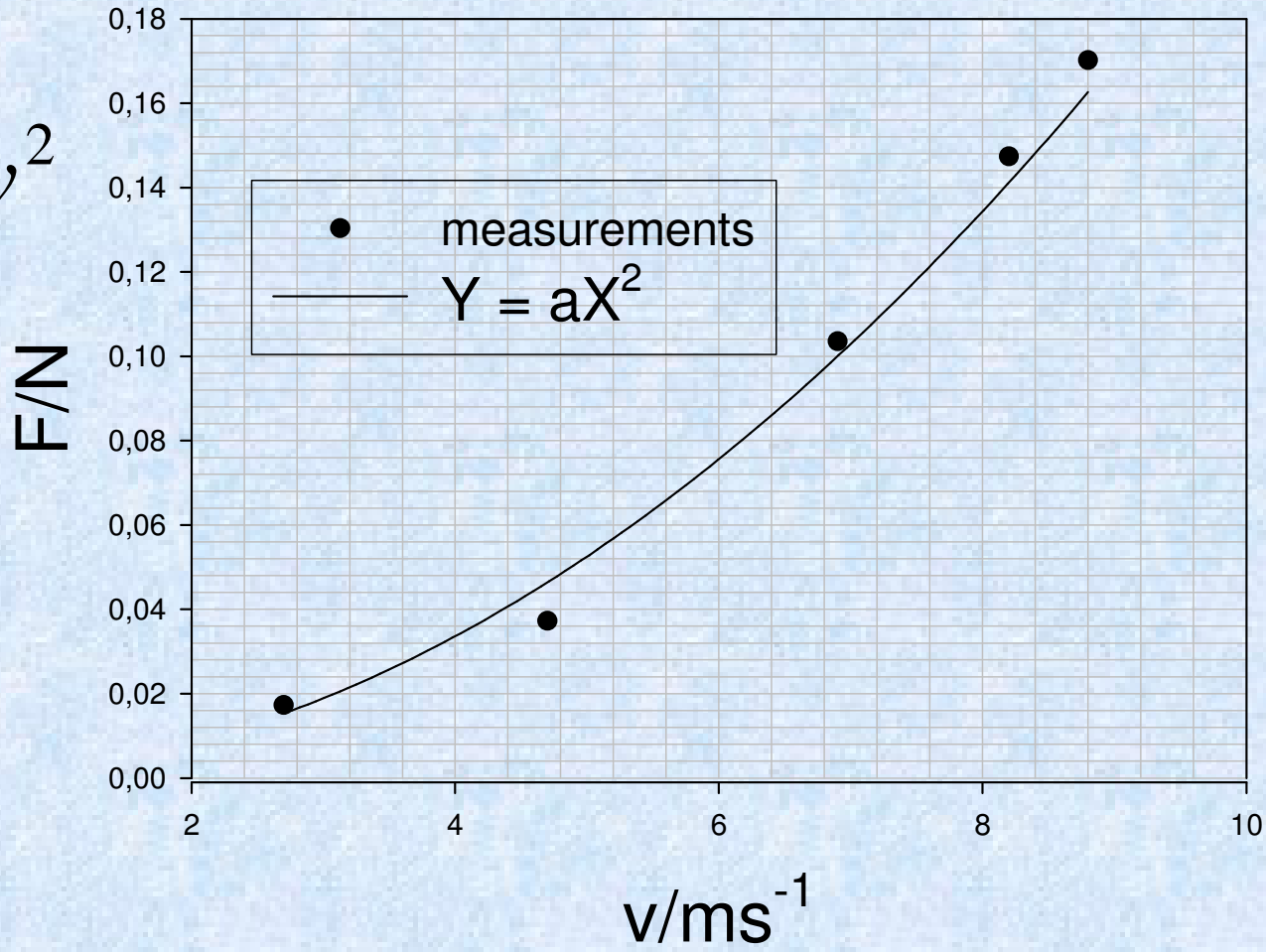


Drag dependence on speed

$$F_D = 0{,}0021v^2$$

F_D – drag

v – ball and air relative speed



Results

$$F_D = k_D v^2 \qquad k_D = 2{,}1 \cdot 10^{-3} \frac{kg}{m}$$

$$F_L = k_L v \omega^{0,62} \qquad k_L = 5{,}33 \cdot 10^{-5} \frac{kg}{s^{0,38}}$$

Reynolds numbers we worked with:

$$5 \cdot 10^3 \leq \mathrm{Re} \leq 1.5 \cdot 10^4$$

As drag is in direction opposite of ball speed we should write:

$$F_D = -k_D v^2$$

Izvod putanje

Equation of motion (for lift and drag):

$$F_D = m\frac{dv}{dt} = -k_D v^2$$

$$F_L = m\frac{v^2}{R} = k_L v$$

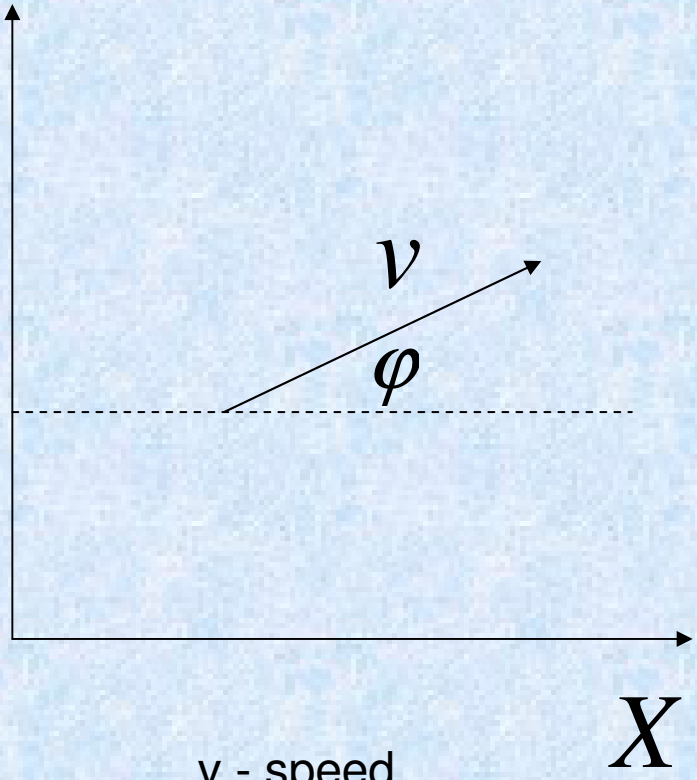
From geometry:

$$d\varphi = \frac{ds}{R}$$

$$dx = ds \cdot \cos\varphi$$

$$dy = ds \cdot \sin\varphi$$

.....



F_D – drag

F_L – lift

m – ball mass

R – radius of curvature of putanje

φ – angle of deflection

v - speed

ds – path differential

dx – infinitesimal change of x

dy – infinitesimal change of y

Izvod putanje

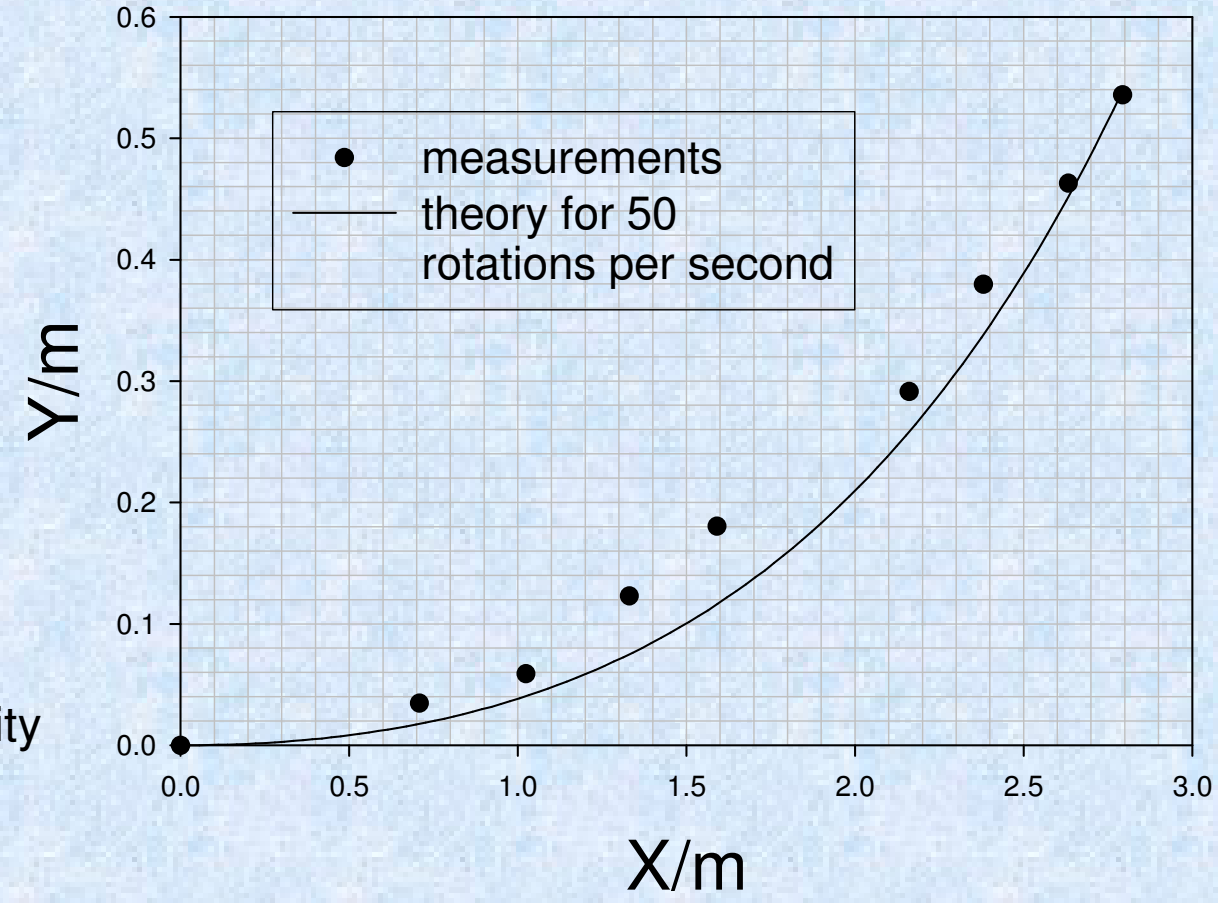
$$y = \frac{mk_L\omega^{0,62}}{v_0 k_D^2}\left(e^{\frac{k_D}{m}x} - \frac{k_D}{m}x - 1\right)$$

Value of ω was estimated to be 50 rotations per second.

m – ball mass

v_0 – initial speed

k_L, k_D – proportionality constants



Conclusion

- effect was achieved

- we studied parameters of aerodinamical force

$$F_D = k_D v^2 \qquad k_D = 2{,}1 \cdot 10^{-3} \frac{kg}{m}$$

$$F_L = k_L v \omega^{0{,}62} \qquad k_L = 5{,}33 \cdot 10^{-5} \frac{kg}{s^{0{,}38}}$$

- for values of Reynolds number: $5 \cdot 10^3 \leq \mathrm{Re} \leq 1.5 \cdot 10^4$

- we constructed mathematical model which gives good results