## Problem 15.

# Optical Tunnelling

# Problem

Take two glass prisms separated by a small gap. Investigate under what conditions light incident at angles greater than the critical angle is not totally internally reflected.

# Experiment

- Two measurement ranges:
  - Centimeter waves – accurate measuring
  - Visile light – obtaining the effect
- Parameters:
  - Waveelngth of the light used
  - Refraction index of prisms and medium in the gap
  - Polarization
  - Distance between prisms

# 1. Centimeter waves

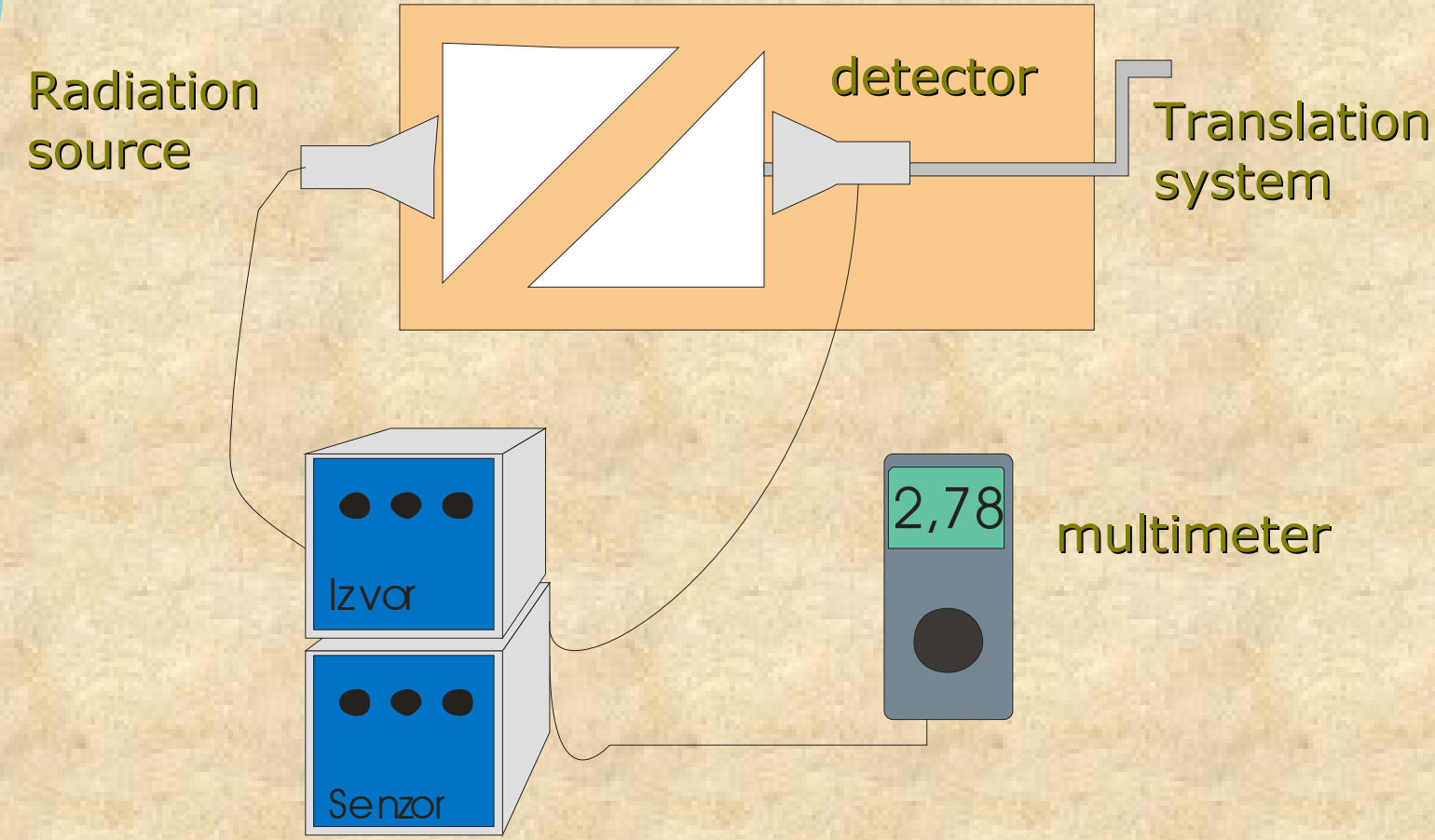
- Wavelength: 3 cm
- Polarization: linear, electrical field perpendicular to plane of propagation
- Prism refraction index: 1.5 (paraffin)
- Measurements:
    - Intensity of tunneled waves
    - Intenzity of reflected waves

in dependence on prism distance

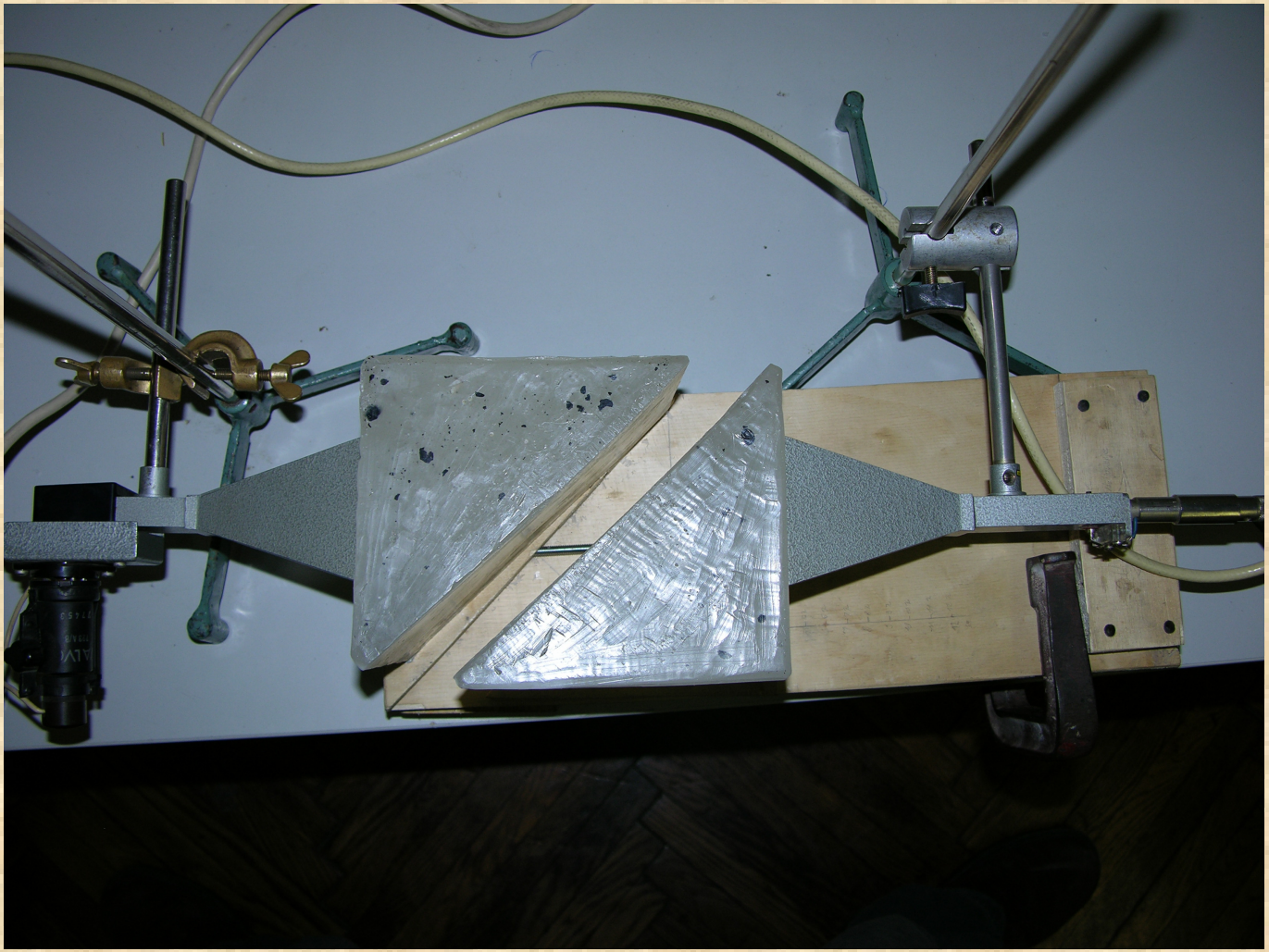
- Measured – voltage in the detector
- Voltage – proportional to field!

# Centimeter waves cont.

- Apparatus schematic:

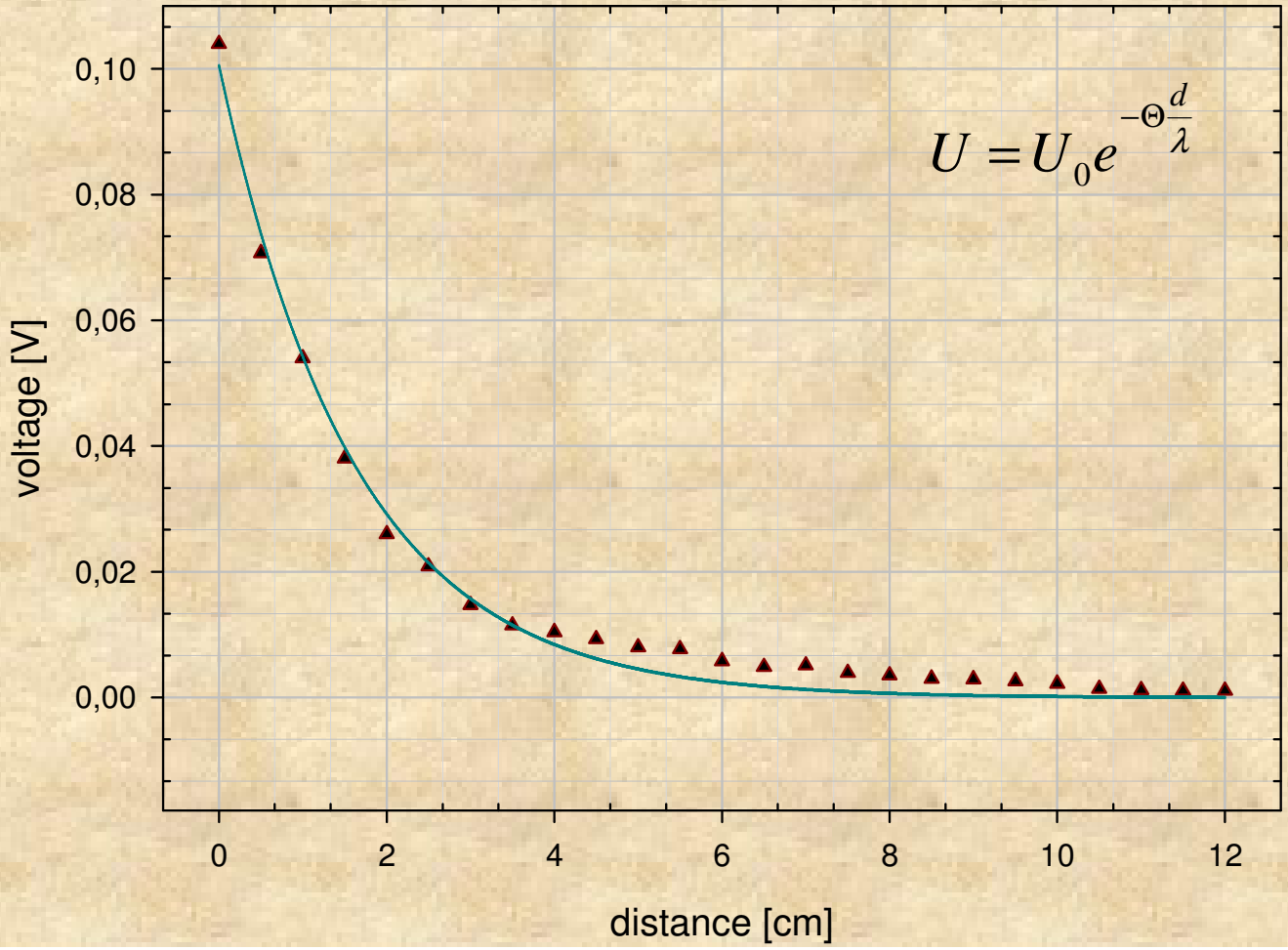


# Centimeter waves cont.



# Centimeter waves cont.

Tunelled field:



# 2. Visible light

- Wavelenght: 780 nm
- Polarization: linear
    - electric field perpendicular to plane of propagation
- Prism index of refraction: 1.48 (measured)
- Measurements:
    - Intensity of tunneled waves
    - Intensity of reflected waves
- Intensity measurement: photodiode
- Voltage – proportional to square of field!

# 2. Visible light

- Measurement in time
  - A slow motor (0.5 r/min) moves the translator
  - Voltage sampling at the diode every 1/50 of a second
- The signal grows in time
- Change of prism distance:

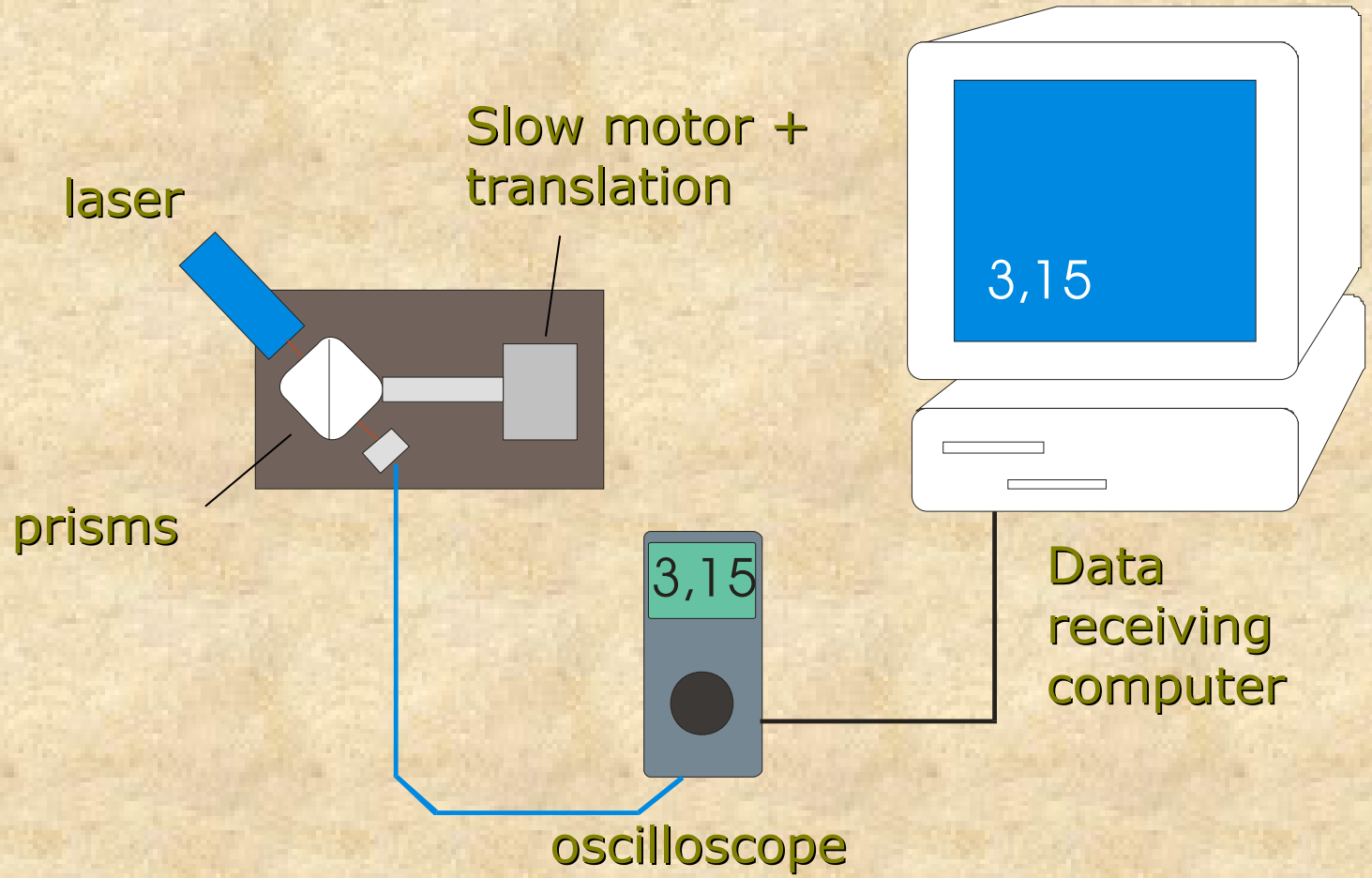
$$\Delta d = -vt$$

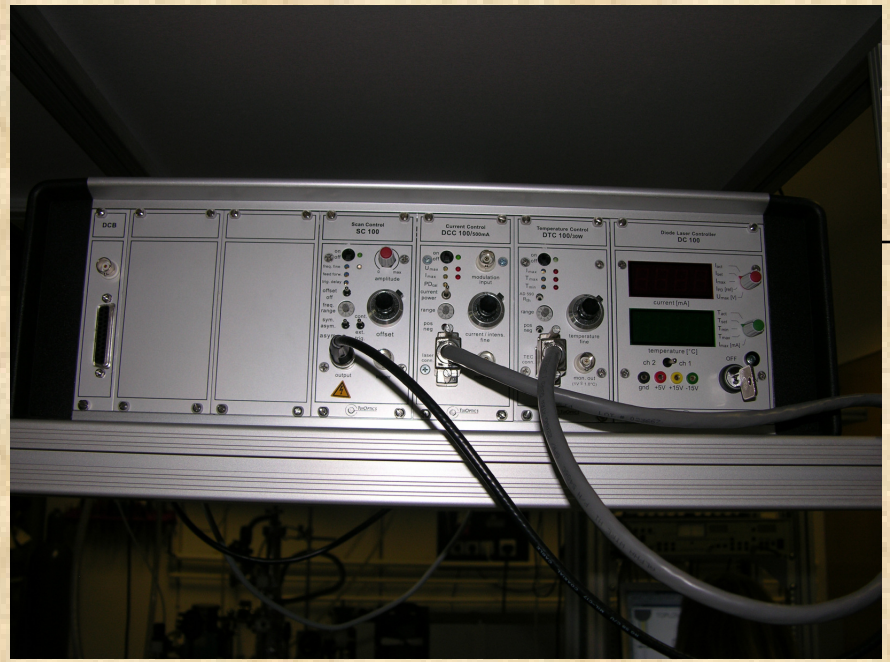
$v$ – translator speed

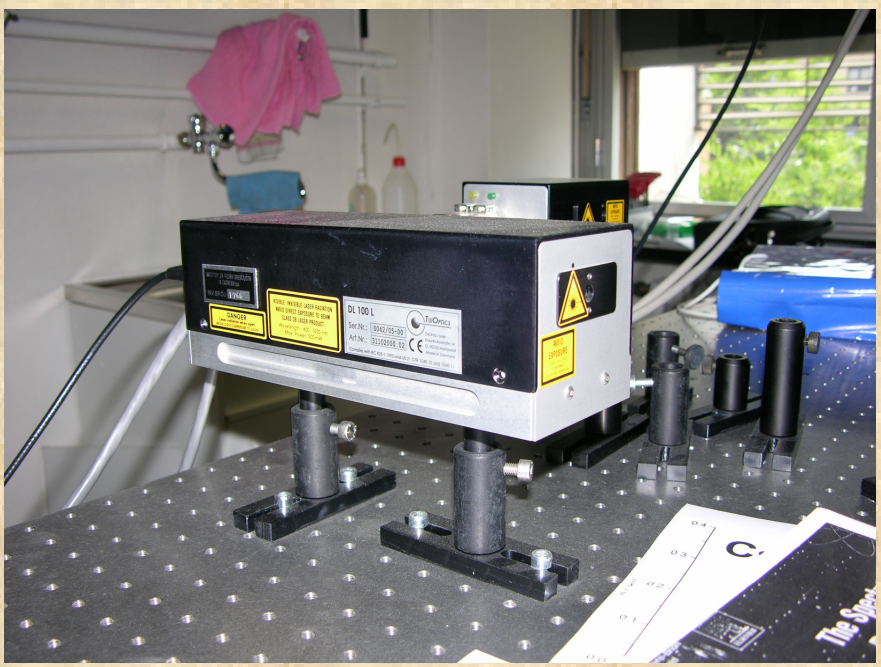
$t$ – elapsed time

# Visible light cont.

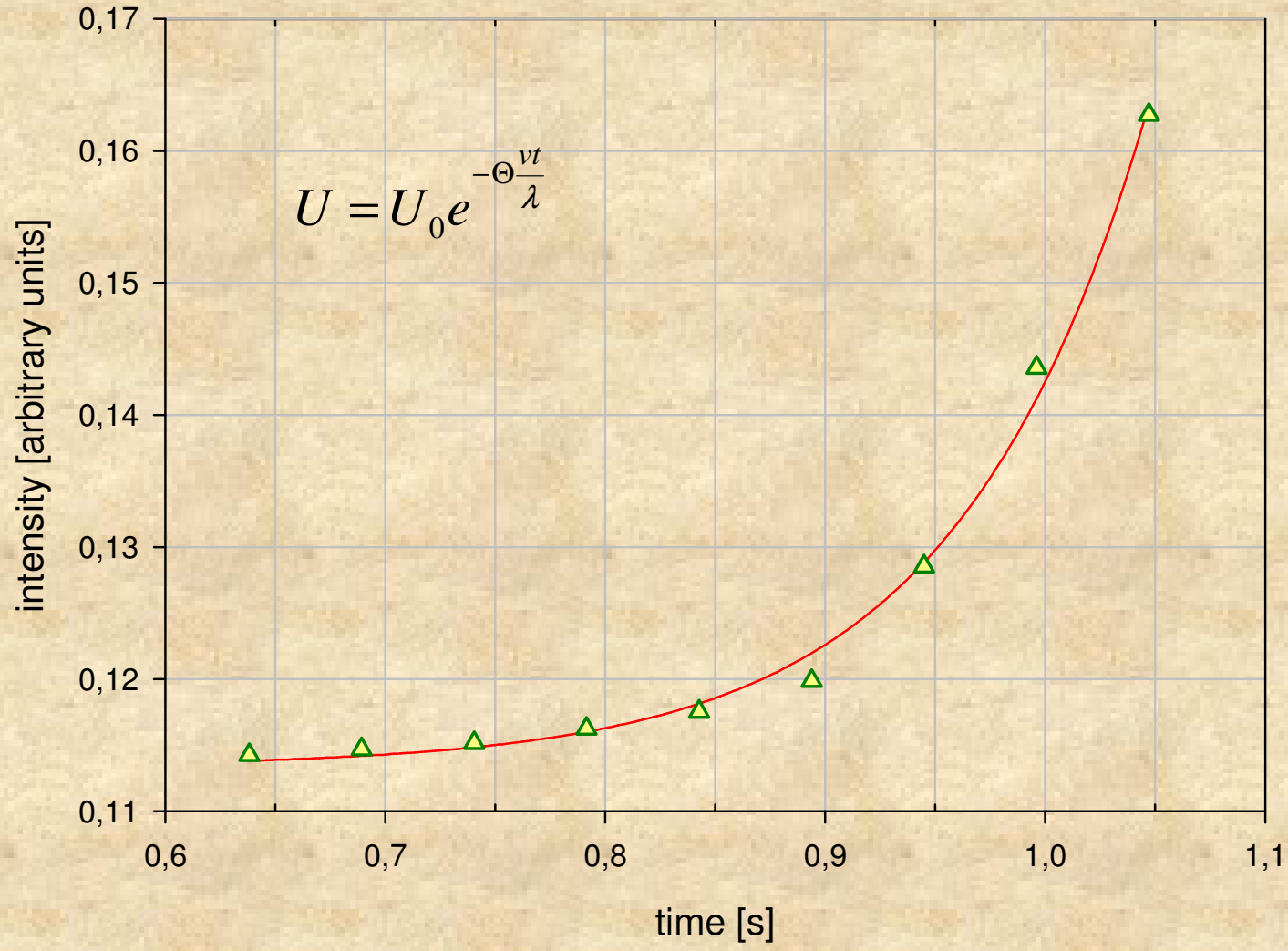
- Apparatus schematic:




Scan Control
SC 100
Current Control
DCC 100/500mA
Temperature Control
DTC 100/10W
Diode Laser Controller
DC 100
current [mA]
temperature [°C]
ch 2
ch 1
gnd +5V +15V -15V


DL 100 L
TuiOptics

# Visible light cont.



# Explanation

- Huygens principle:

Every atom ″through″ which light passes is a source of light identical to the incident light

⇒ Electromagnetic waves in dielectrics – the resultant of interference of the initial wave and all scattered waves

# Explanation cont.

- At total reflection – the reflected ray is the only interference maximum
- Behind the reflection plane – destructive interference, but only *far away* from the plane
- Close to the plane (distances of the order of the wavelength) the waves haven't interfered completely and a decaying field exists

# Explanation cont.

- That field decays fast due to interference
- If a prism is put into the field – a new interference maximum can be formed in the prism
- A new, tunnelled wave is formed in the prism
- The energy of the reflected wave becomes smaller

# Maxwell equations

$$\nabla \mathbf{E} = -\frac{1}{\varepsilon_0}\nabla \mathbf{P}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \mathbf{B} = 0$$

$$c^2 \nabla \times \mathbf{B} = \frac{1}{\varepsilon_0}\frac{\partial \mathbf{P}}{\partial t} + \frac{\partial \mathbf{E}}{\partial t}$$

**E** – electric field

**B** – magnetic field induction

P **–** polarization

*c* – speed of light in a vacuum

$\varepsilon_0$ – vacuum permittivity

# Plane wave solutions

Electrical field:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\omega t - \mathbf{kr})}$$

Magnetic field:

$$\mathbf{B} = \frac{1}{\omega}\mathbf{k} \times \mathbf{E}$$

$\mathbf{E}_0$ – amplitude

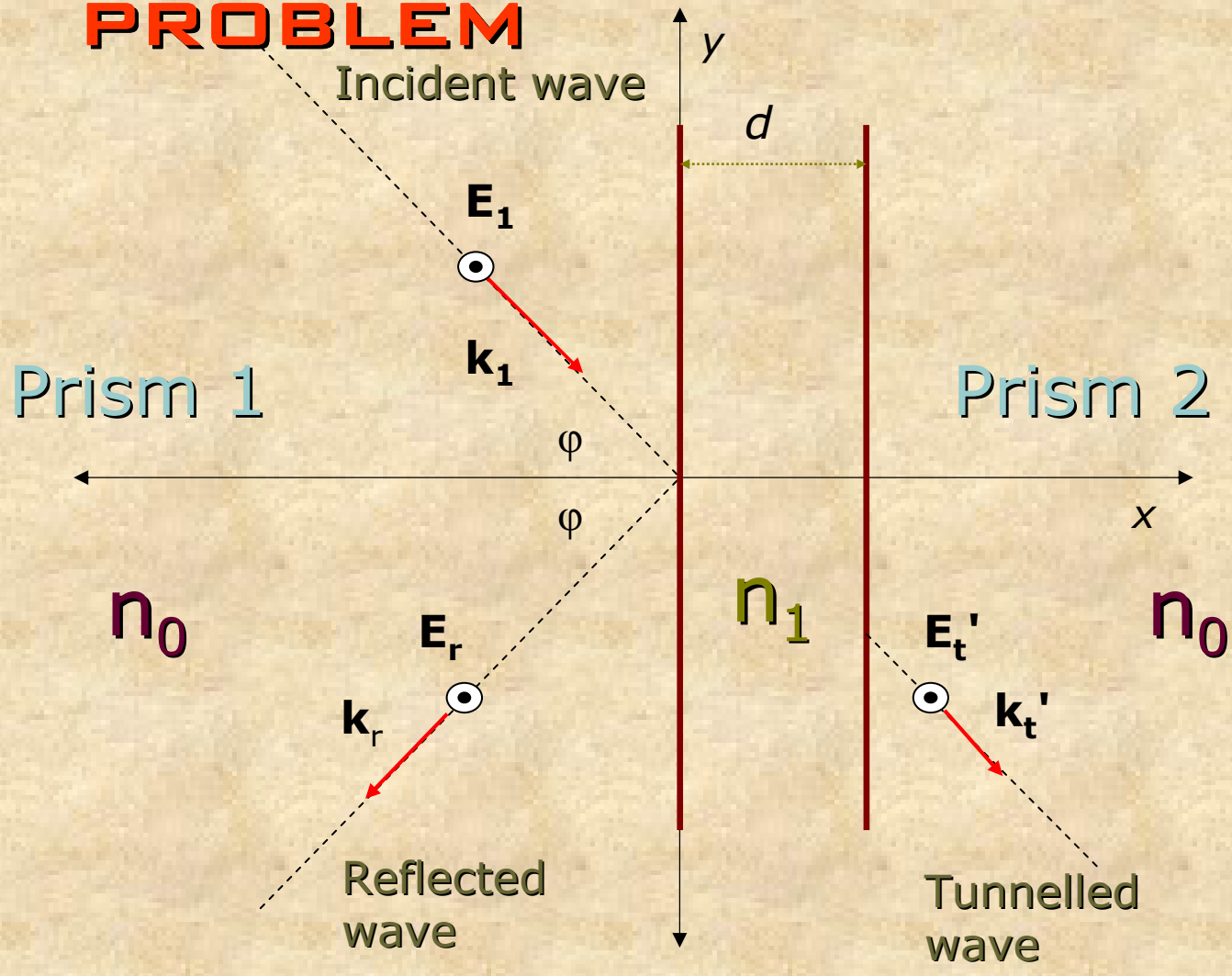
$\omega$ – frequency

$t$ – time

**k** – wave vector

**r** - radiusvector

# Geometry of the problem



# Boundary conditions

- If the electric field is perpendicular to the wave vector plane:

$$E_{10} + E_{r0} = E_{t0}$$

$E_{10}$, $E_{r0}$, $E_{t0}$– amplitudes of the incident, reflected and transmitted waves

$k_{1x}$, $k_{rx}$, $k_{tx}$– x components of the wave vectors of the incident, reflected and transmitted waves

$$k_{1x}E_{10} + k_{rx}E_{r0} = k_{tx}E_{t0}$$

# Boundary conditions cont.

- For the wave vectors:

$$k_{ty} = k_{1y}$$

$$k_{tx}^2 = \left(\frac{n_1}{n_0}\right)^2 k_1^2 - k_{1y}^2$$

$k_{1y}$, $k_{ty}$– y components of the incident and transmitted wave vectors

$n_0$ – prism index of refraction

$n_1$ – medium between prisms index of refraction

# Solution

- If the incident angle is greater than the reflection angle, Snell's law gives

$$k_{tx} = ik_1 \frac{n_1}{n_0} \sqrt{\left(\frac{n_0}{n_1}\right)^2 \sin^2 \varphi - 1}$$

φ – incident angle

- *x* – component of the wave vector is a pure imaginary => the wave propagates along the plane

=> the amplitude decays exponentially

# Solution cont.

- The field in the second prism:

$$E_t' = E_{10} e^{-2\pi n_1 \frac{d}{\lambda}\sqrt{\left(\frac{n_0}{n_1}\right)^2 \sin^2\varphi - 1}} = E_{10} e^{-\Theta\frac{d}{\lambda}}$$

$d$ – prism distance

$\lambda$ – vacuum wavelegth of incident light

$\Theta$ – decay coefficient

# Comparation – decay coefficient

- Centimeter waves:

| | |
|---|---|
| Experimental | 2.5±0.1 |
| Theoretical | 2.22 |

- Optical range:

| | |
|---|---|
| Experimental | 1.1±0.1 |
| Theoretical | 1.94 |

# Comparation cont.

- Agreement is relatively good
- Error causes:
  - Imprecise prism refraction index values
  - In optical range:
    - Prism surface defects and dust
    - Motor precision ...

# Conclusion

- We have obtained, measured and modelled optical tunnelling
- It may be said:

The only condition for light incident on a prism plane with an angle greater than the critical angle not reflecting completely is to put another prism plane next to the original plane to a distance of the order of the wavelength used