Problem 16.

Hardness

Problem 16.

A steel ball falls onto a horizontal surface. If one places a sheet of paper onto the surface with a sheet of carbon paper on top of it, a round trace will be produced after the impact. Propose a hardness scale based on this method.

Introduction

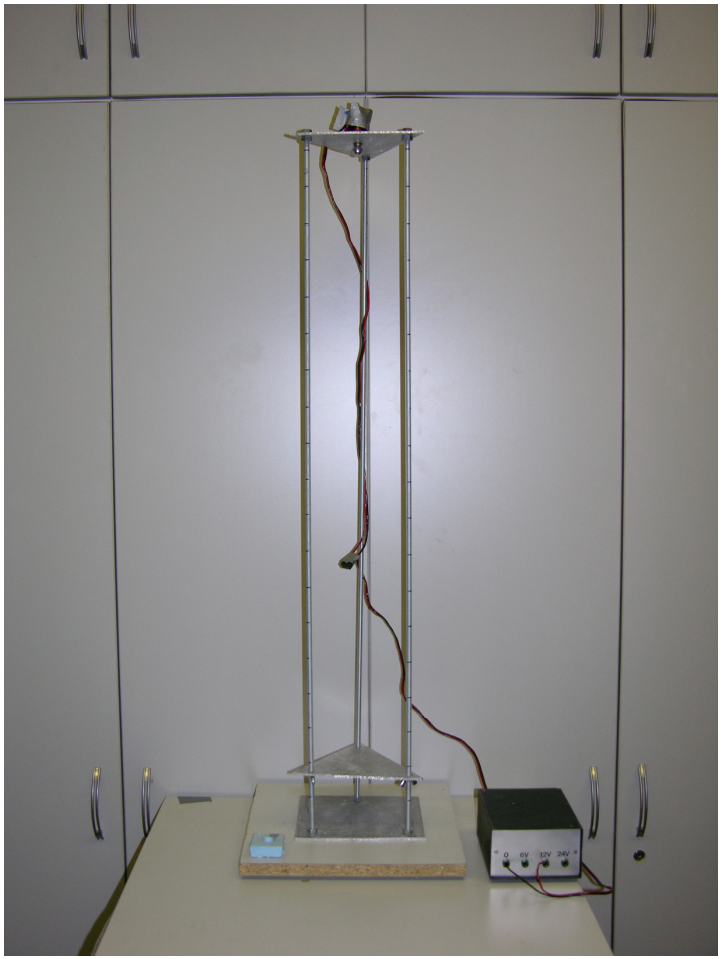
- scale will be based on diameter of trace
- trace is produced because material is deformed after the impact
- impossible to make hardness scale of materials harder than steel (in that case, ball is deforming)

Experimental approach

- apparatus
- measuring
- patterns

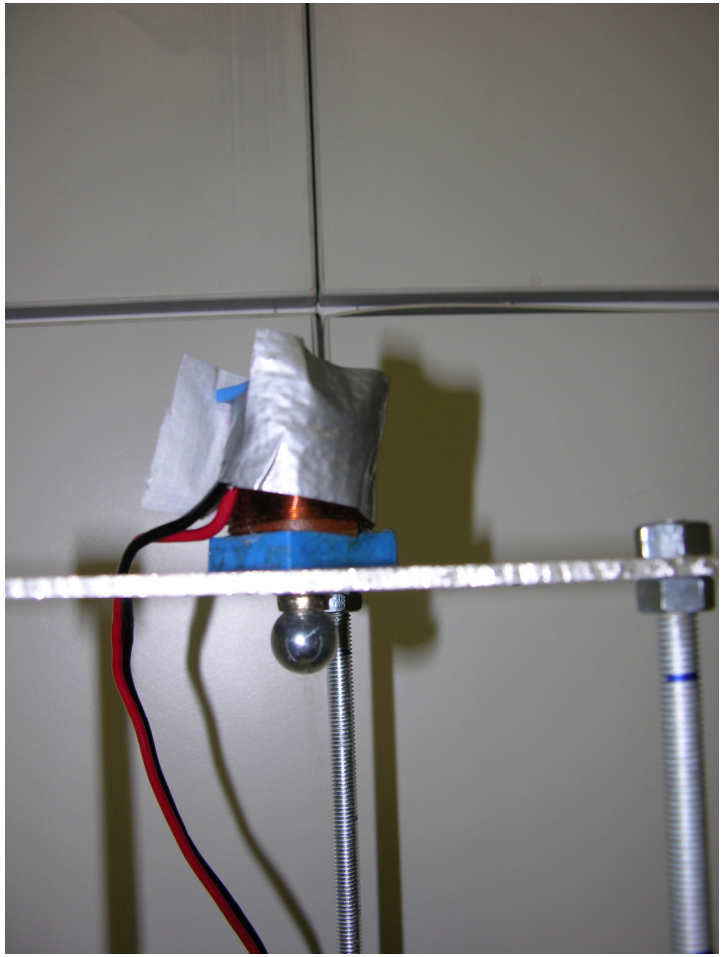
Apparatus

- Aluminium construction with two horisontal plates and three stands
- Electromagnet
- Magnifying glass
- Steel ball (d=1 cm)
- Patterns



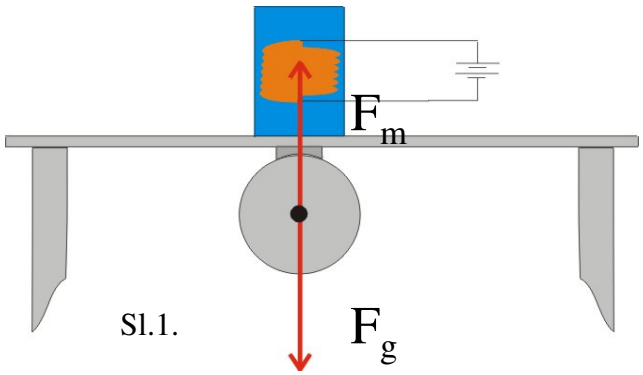
Apparatus cont.



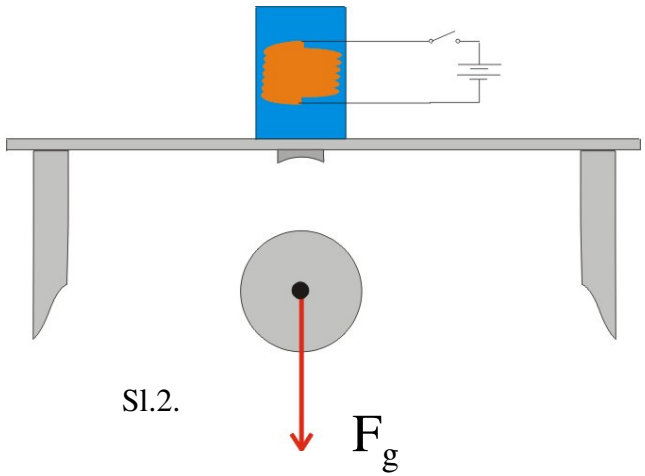


Measuring

- the ball is dropped $\longrightarrow$ magnetic force

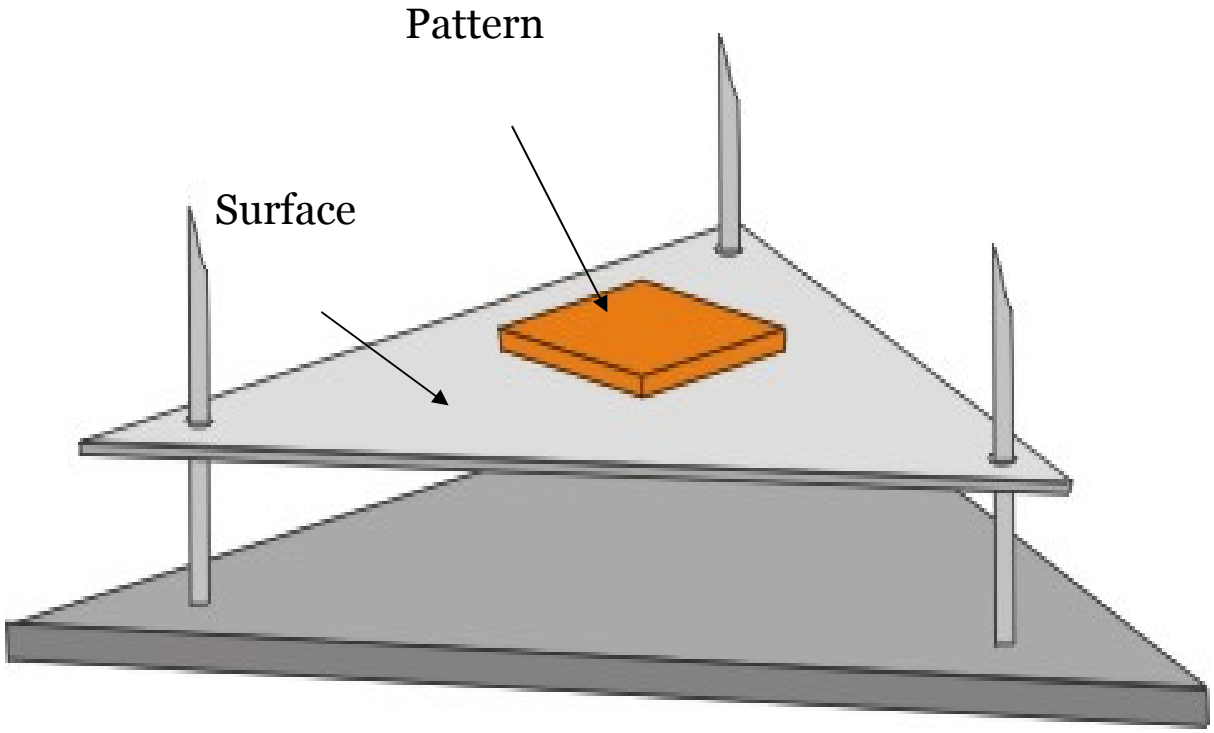


Sl.1.



Sl.2.

Measuring cont.



- the ball is dropped from 1m above the pattern

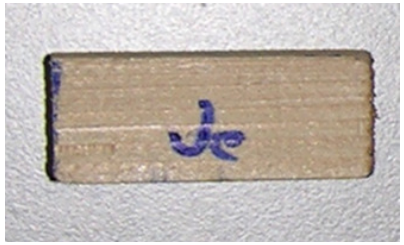
Patterns

- wood, plastics, metals, foams, minerals
- softer than steel
- non-elastic materials



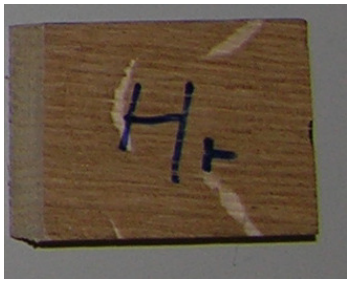
Wood

- pine, maple, teak, oak
- pine- softest material, anisotropy $\longrightarrow$ elliptic trace



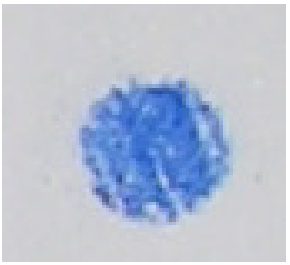


- oak and maple- less anisotropy



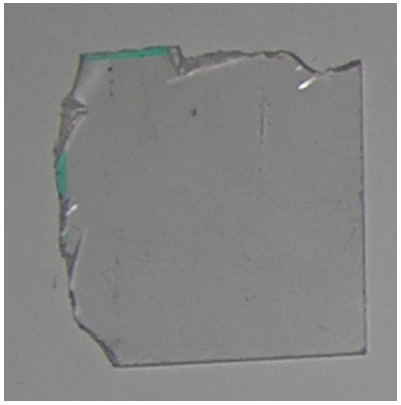






Plastics

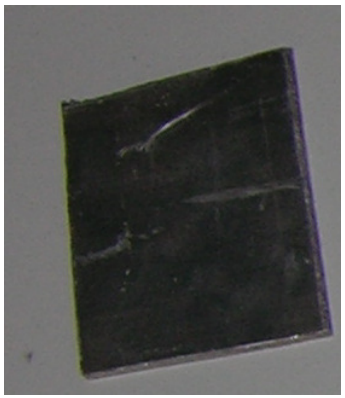
- Polyamide, polymetilmetacrilate (plexyglass), polyvinilcloride (PVC)
- Different characteristics than wood
- Homogeneus and izotropic materials
- Round trace

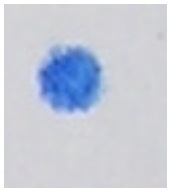




Metals

- aluminium and lead
- metals softer than steel
- round traces $\longrightarrow$ isotropic and homogeneus materials





Foams

- large anisotropy
- the trace is hard to measure
- but by polyuretane and polystirene it is easier to mesure $\longrightarrow$ biggest trace (measured without carbon papper)



Minerals

- in most cases- harder than steel
- exeption- chalk and gypsum

Theoretical approach

- The ball hits the surface with velocity v_o
- It bounces off with smaller velocity
- Some energy goes into deforming surface (pattern)
- Important that surface is softer than ball (steel) – if not, ball is deformed

Theoretical approach cont.

- For the ball – conservation laws:
- Conservation of Energy

$$\frac{1}{2} m v_0^2 (1 - k^2) = \Delta E$$

m – mass of the ball

v_0 – velocity before impact : $v_0 = \sqrt{2gH}$

k – resitution factor

ΔE – energy spend for deforming pattern : $\Delta E = \int_0^h F_d \, dz$

H – height from wich we relese the ball

h – depth of trace

F_d – deformatio n force

- Conservation of momentum

$$m v_0 (1 - k) = \int_0^{\tau} F_d \, dt$$

Theoretical approach cont.

- Approximations used:

$$h << R$$

$$F_d \approx const.$$

- After combinantion conservations we obtain

$$F_d = \frac{mRv_0}{r^2}$$

R – radius of the ball

- base of our hardness scale

Theoretical approach cont.

- Dividing previous equation with surface of trace we get

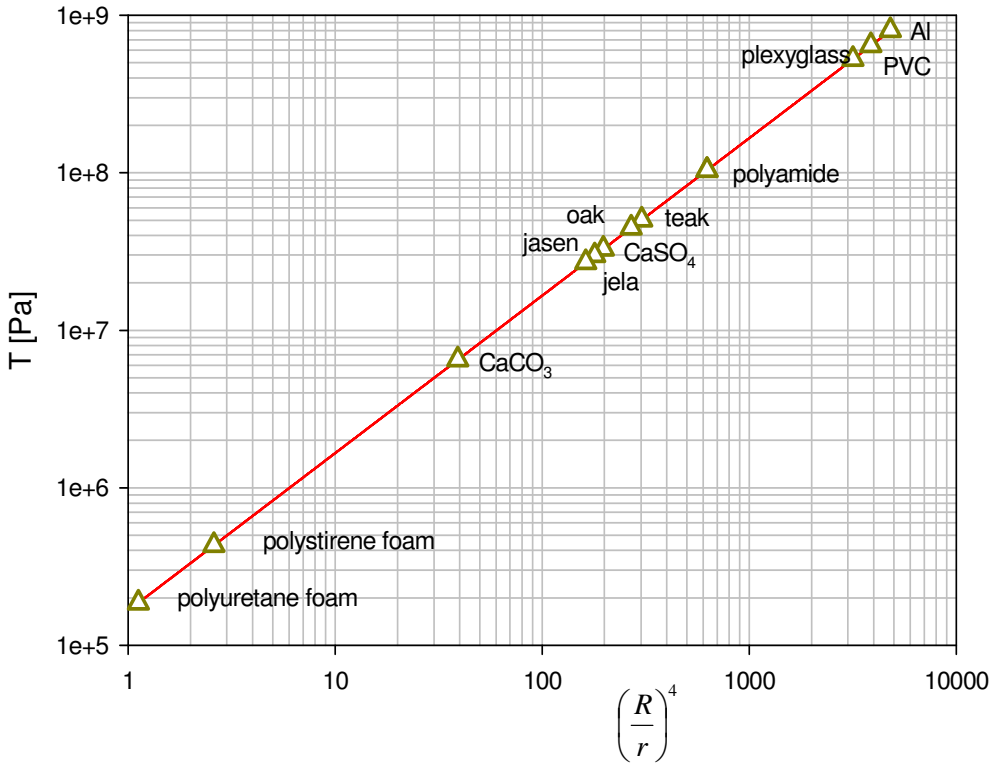
$$T = \frac{8}{3}\rho_b g H \left(\frac{D}{d}\right)^4$$

T – pressure

ρ_b – density of the ball

- Hardness measured in Pascals

Hardness scale



- logaritmic (large) scale
- non-dimensional memeber in order to get linear graph
- polyuretane foam-softest, aluminium-hardest