Problem 4.

Whispering gallery

The problem

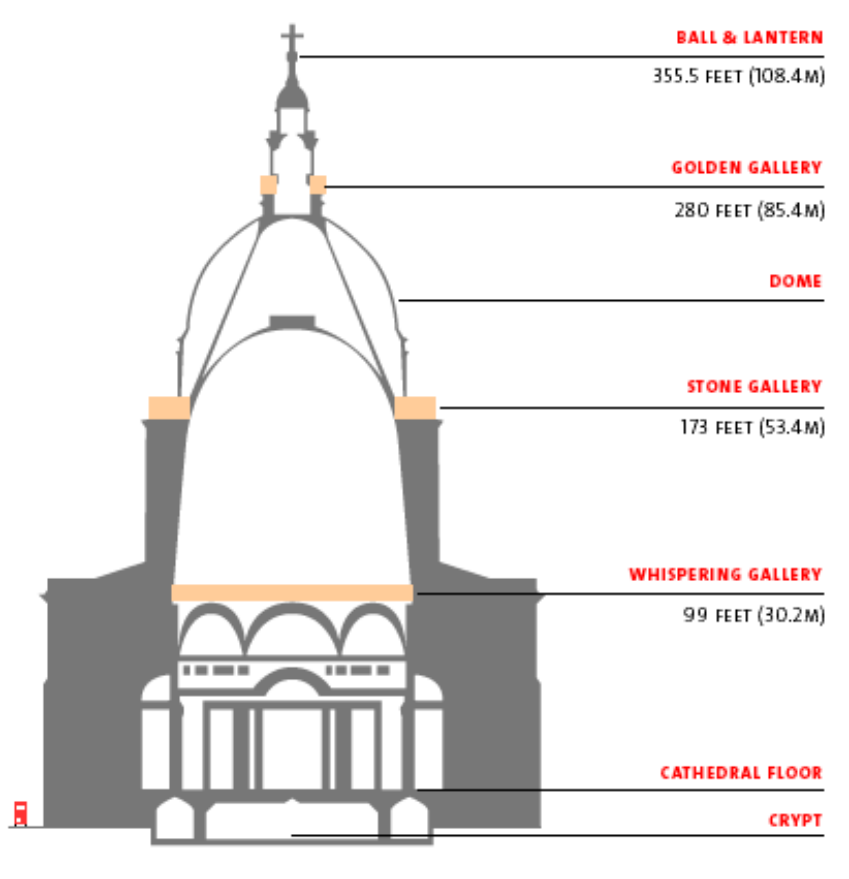
The Whispering Gallery at St Paul's Cathedral in London, for example, is famous for the fact that the construction of the circular gallery makes a whisper against its walls on one side of the gallery audible on the opposite side of the gallery. Investigate this phenomenon.

Outline

- Definitions and scale
- Basic explanation
- Gallery construction
- Measurements and analisys
- Wave model
- Conclusion

Definitions and scale

- **St. Paul's Whispering Gallery in London:**



Circular! No ellipses

High cylinder! – dome influence small

Slightly concave

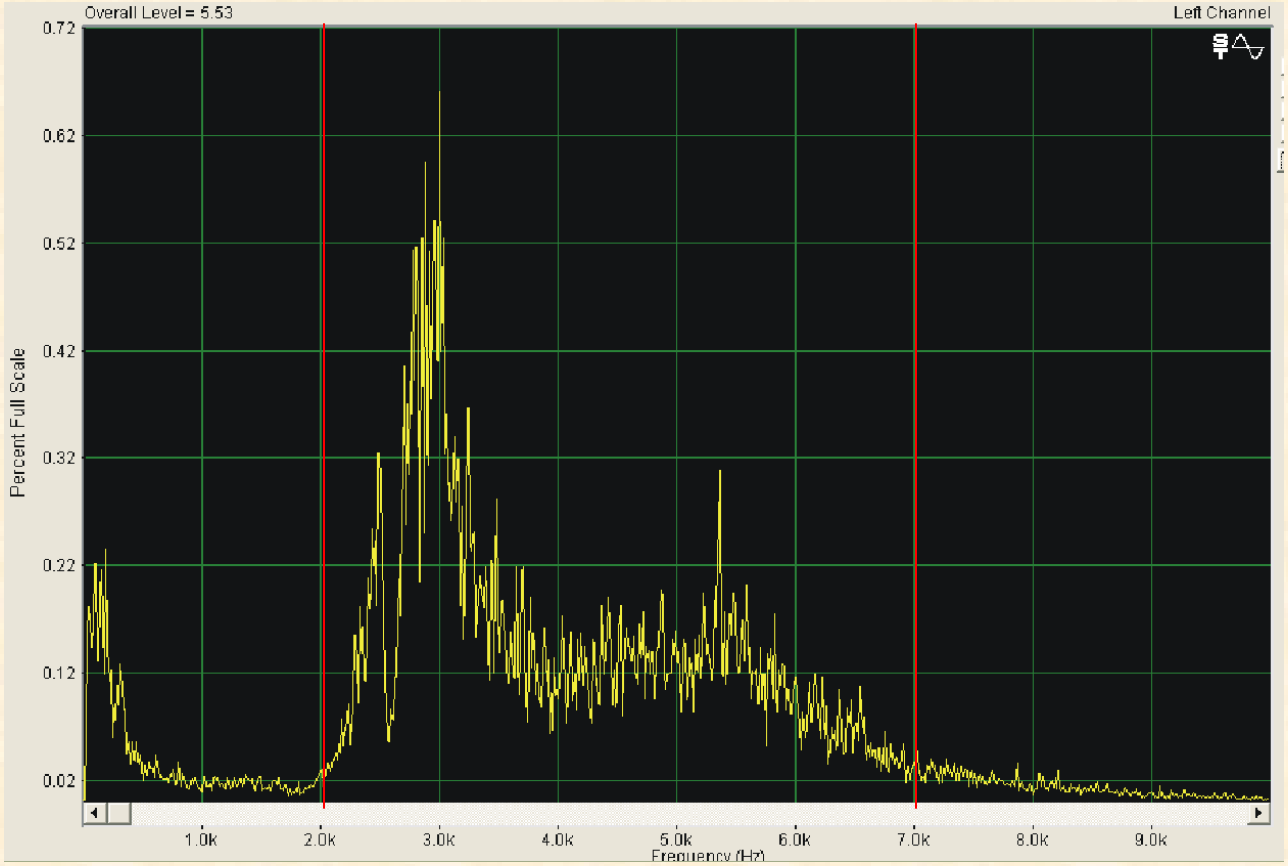
Largest diameter 36 m

No floor nor ceiling

Smooth walls

Definitions and scale cont.

- **Whisper spectrum:**



High frequencies!

Definitions and scale cont.

- **Sound diffraction**
- **Speaker – angular intensity distribution depending on frequency**
- **Diffraction pattern**
- **Measure of angular dispersion:**

$$\frac{\lambda}{D}$$

λ – mean wavelength emitted

D – mouth aperture diameter

$\Rightarrow$ Small wavelengths (large frequencies) better focused – whisper!

Definitions and scale cont.

- **Any model – must scale as:**

$$\frac{\overline{\lambda}}{R}$$

λ – mean wavelength under consideration

R – gallery radius

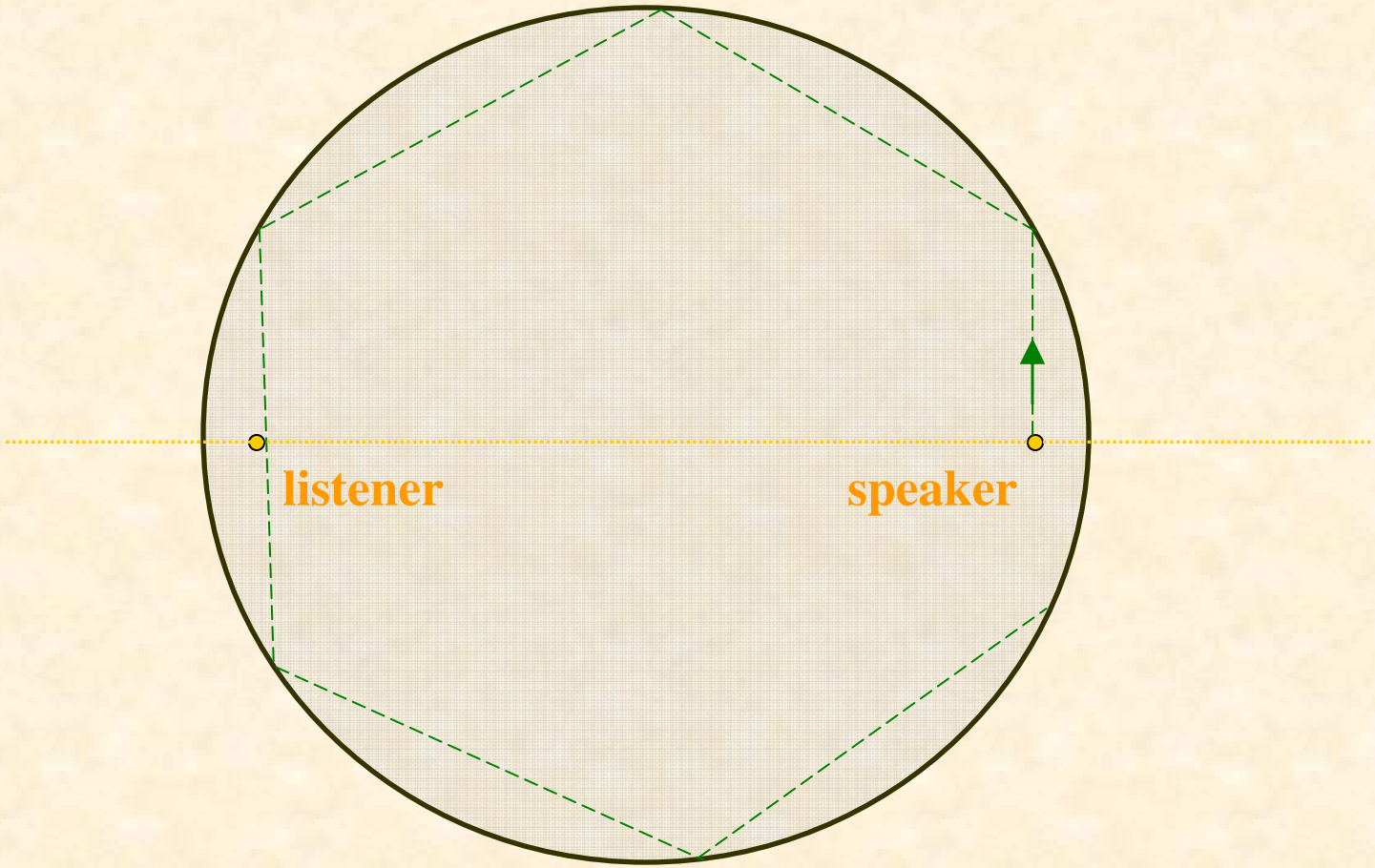
- **Less stringent condition:**

$$\overline{\lambda} << R \quad \Leftrightarrow \text{"geometric sound" condition}$$

- **Basic geometrical explanation follows**

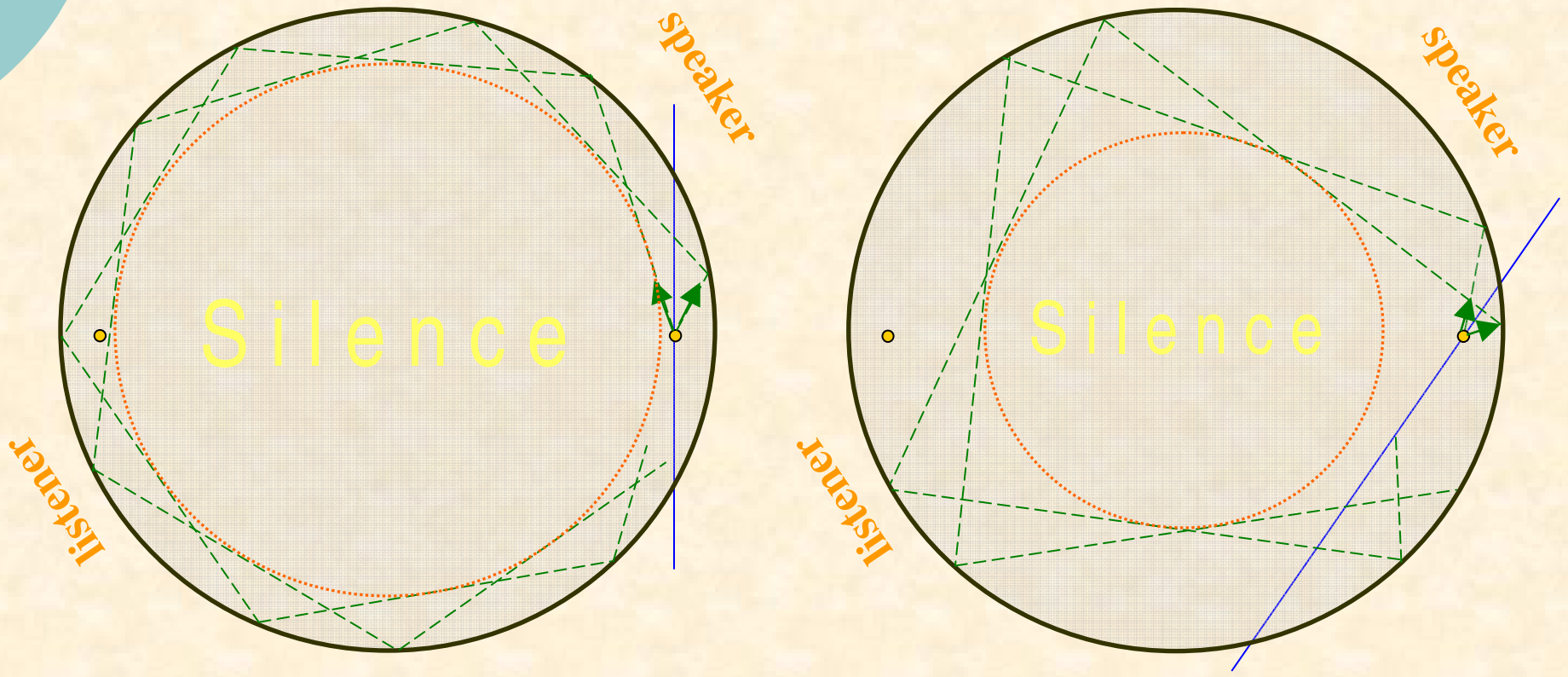
Basic explanation

- **Speaker's waves – bouncing off gallery walls:**



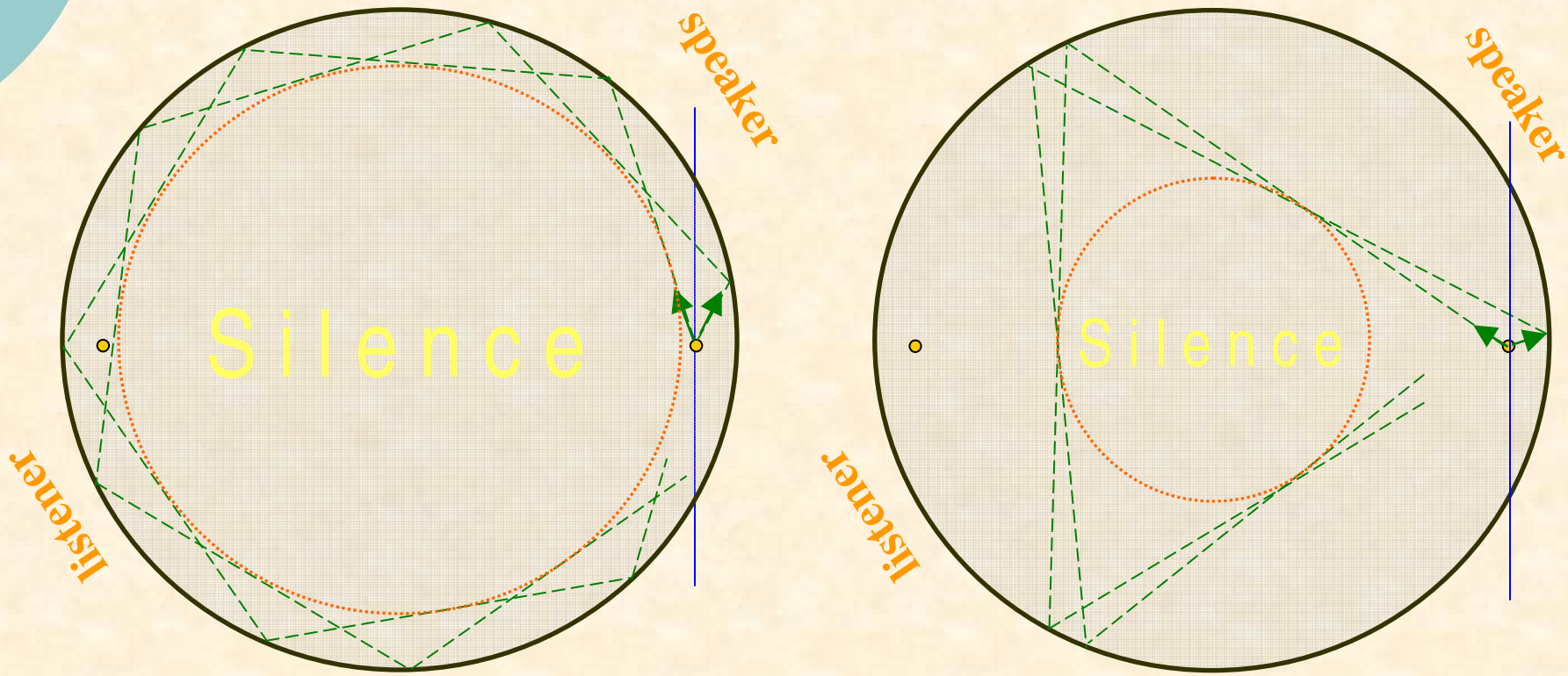
Basic explanation

- **Crucial parameter – angles:**
  - **Speaker orientation**



Basic explanation

- Crucial parameter – angles:
  - Speaker intensity angular distribution



Basic explanation

⇒ Silence region – no sound at all!

⇒ Sound ″ring″ – smaller attenuation than in free air

⇒ Most favourable:

- Speaking direction *paralell* to the wall
- Narrow angular distribution – high frequencies - whisper

- This ray model – generally explains effect occurence

  neglects wave nature

⇒ more detailed research neccesary

Gallery construction

- Model gallery neccesary properties:
  - Rigid smooth walls
  - The larger the better – wavelength scaling
  - Floor & ceiling eliminated
- Our model gallery:

  1 mm steel strip wall

  1 m diameter

  Sponge layered floor, no ceiling

Gallery construction



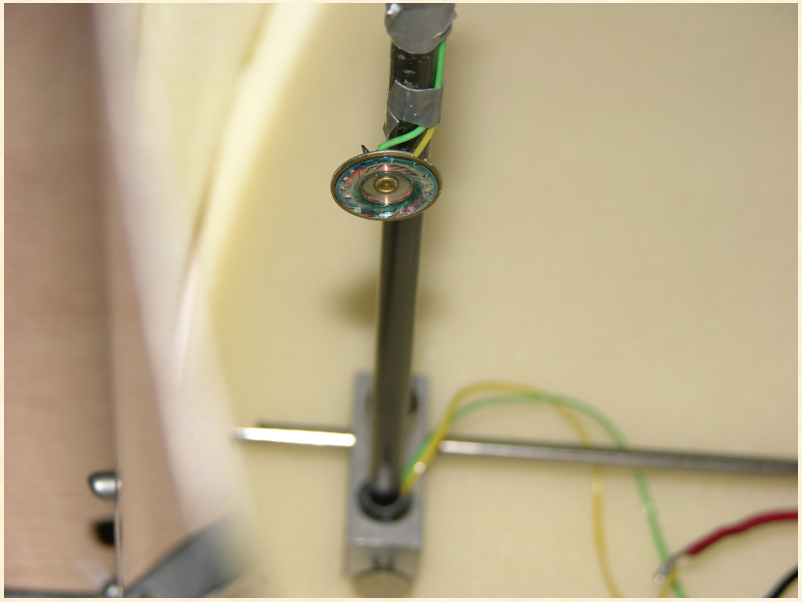
Measurement

- **Speaker:**

  Ex - headphone with good frequency characteristic

- **Receiver:**

  Piezo transducer



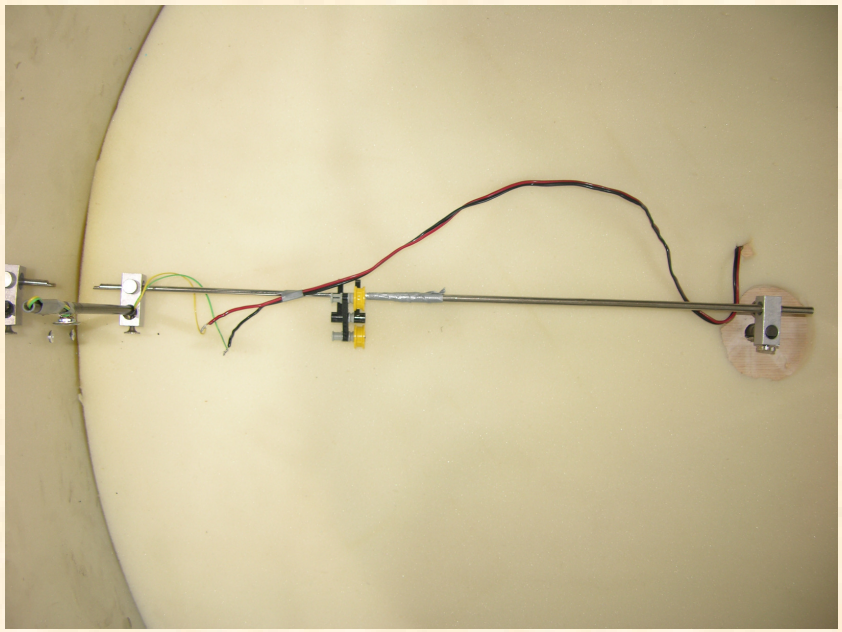


Measurement

- Measured:
  - Radial amplitude distribution on opposite side of speaker
  - Angular amplitude distribution on constant distance from wall
- Changed:
  - Wavelength
  - Speaker position
  - Speaker angle to wall

Measurement

- **Speaker – on moving arm for angular measurements:**



- **Receiver – on 0.1 mm translator, for radial measurements:**



Measurement

- Acoustic signal – precision function generator
- Measurement – AC voltmeter with band pass filter





Measurement

Measurement

- **Results:**
  - **Near wall – always large amplitude**
  - **Peaked structure**
  - **Wavelength dependent**
  - **Speaker distance from wall – influence on structure**
  - **Small dependence on speaker orientation**

Wave model

- Results – mainly unexplainable by simplistic ray theory
- Wave model – takes wave nature of sound into account
- Theoretical bases
  - Wall boundary condition
  - Gallery normal modes
  - Fourier analisys

Wall conditions

- Air clings to wall due to molecular forces
- "No slip boundary condition"

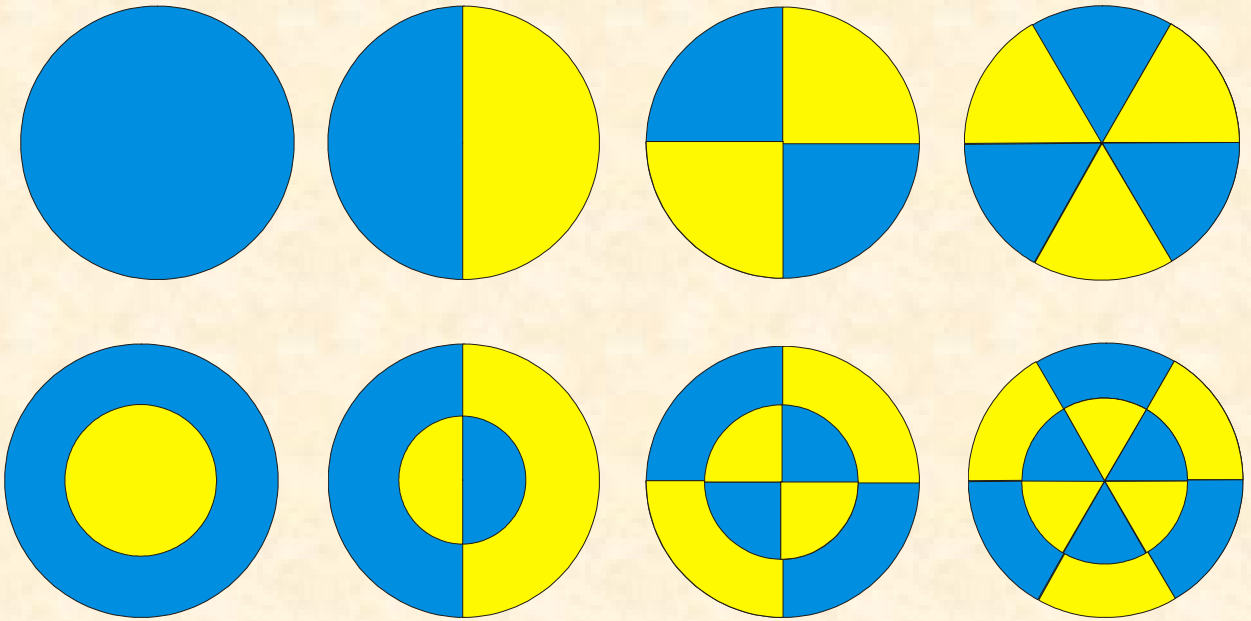
  $\Rightarrow$Acoustic air velocity at wall zero

  $\Rightarrow$Pressure – in contraphase with velocity – largest oscillations at wall!

- Boundary condition gives rise to resonant normal modes:

Normal modes

- Air in gallery supports only discrete frequency modes



Normal modes

- **Two "degrees of freedom":**
  - **Order *m* – number of waves fitting full circle**
  - **For each *m* – degree *n* – number of waves fitting radially**
- **Mathematically – Bessel functions:**

$$\psi_{mn} = a_{mn} \cos m\varphi J_m(k_n r)$$

ψ – wavefunction (here – acoustic velocity)

a_{mn} – amplitude of *nm* th mode

φ – polar angle

J_m – Bessel function of *m* – th order

K_n – wavenumber of *n* – th harmonic

r – radial coordinate

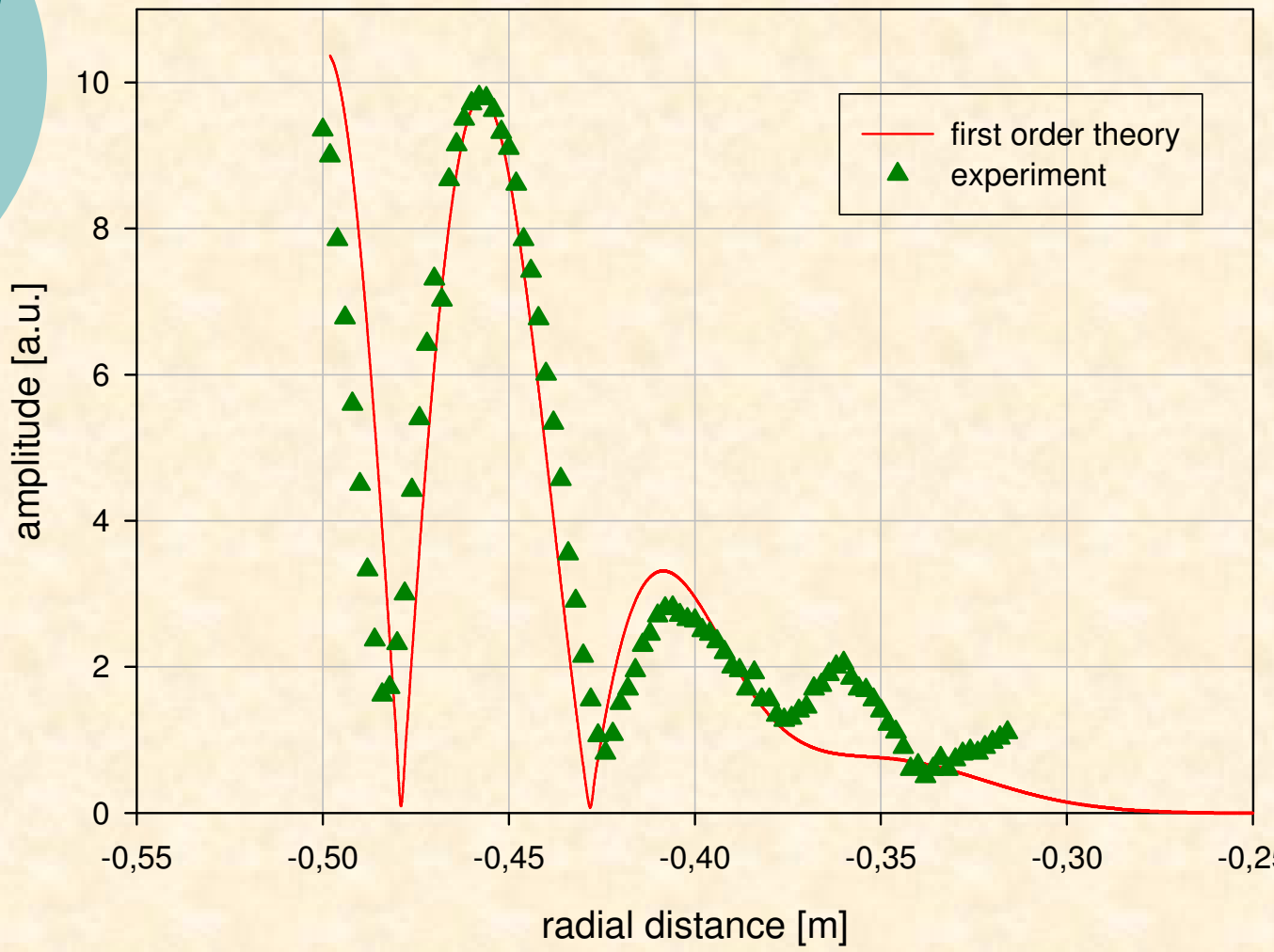
Normal modes

- Speaker waves – excite many modes
- Resultant amplitude distribution – superposition of all excited modes!
    - Peaked structure
    - Boundary condition explains high pressure amplitude at walls
    - Speaker orientation doesn't change structure much
    - Speaker position changes excited modes

Normal modes

- Calculation – approximative:
    - Supposed – only one order m is excited
- Wavefunction becomes simply analysable
- Fourier series
- Only first four to five coefficients calculated – experimental justification

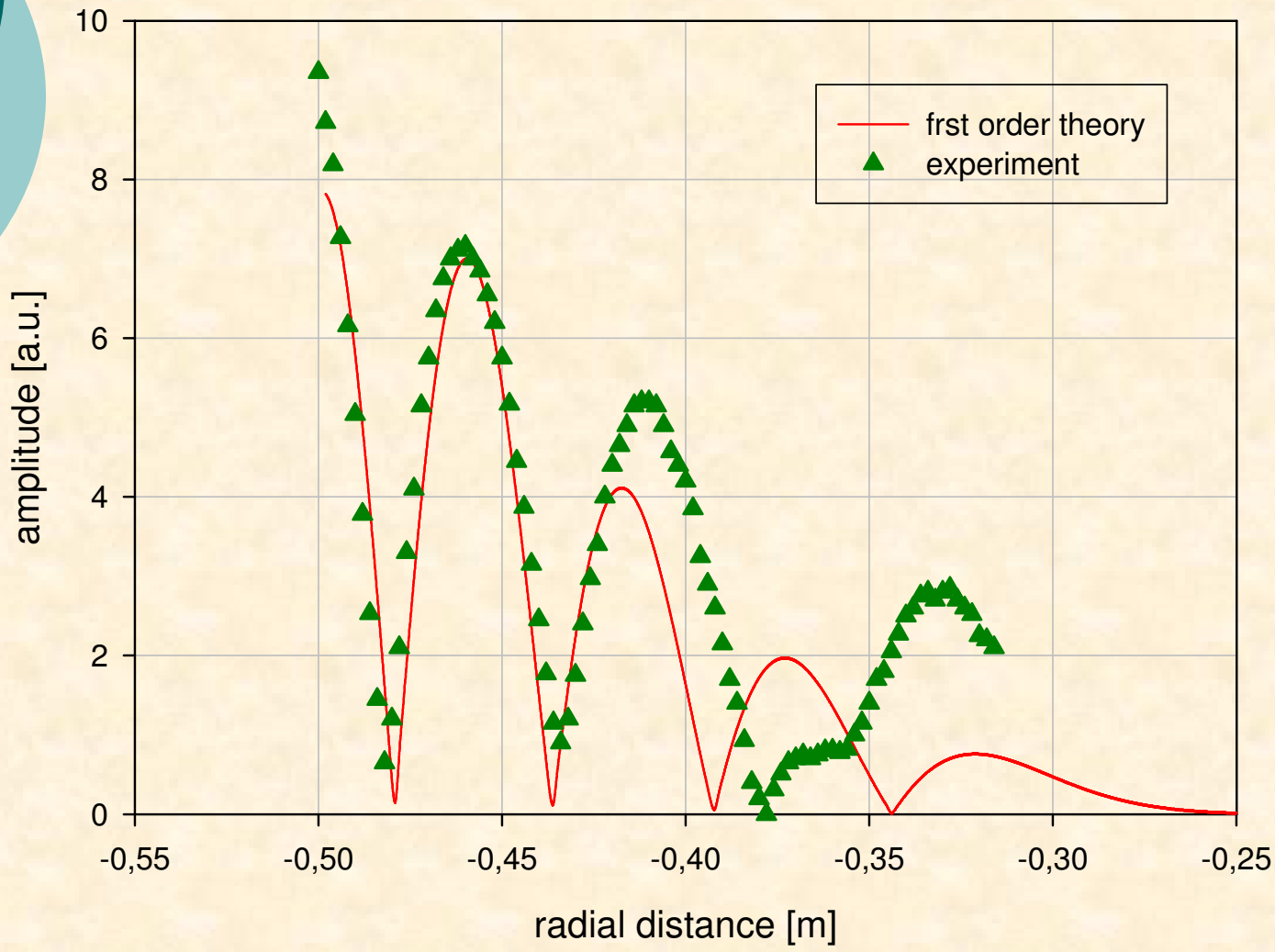
Predicted



Speaker distance from wall

5.1 cm

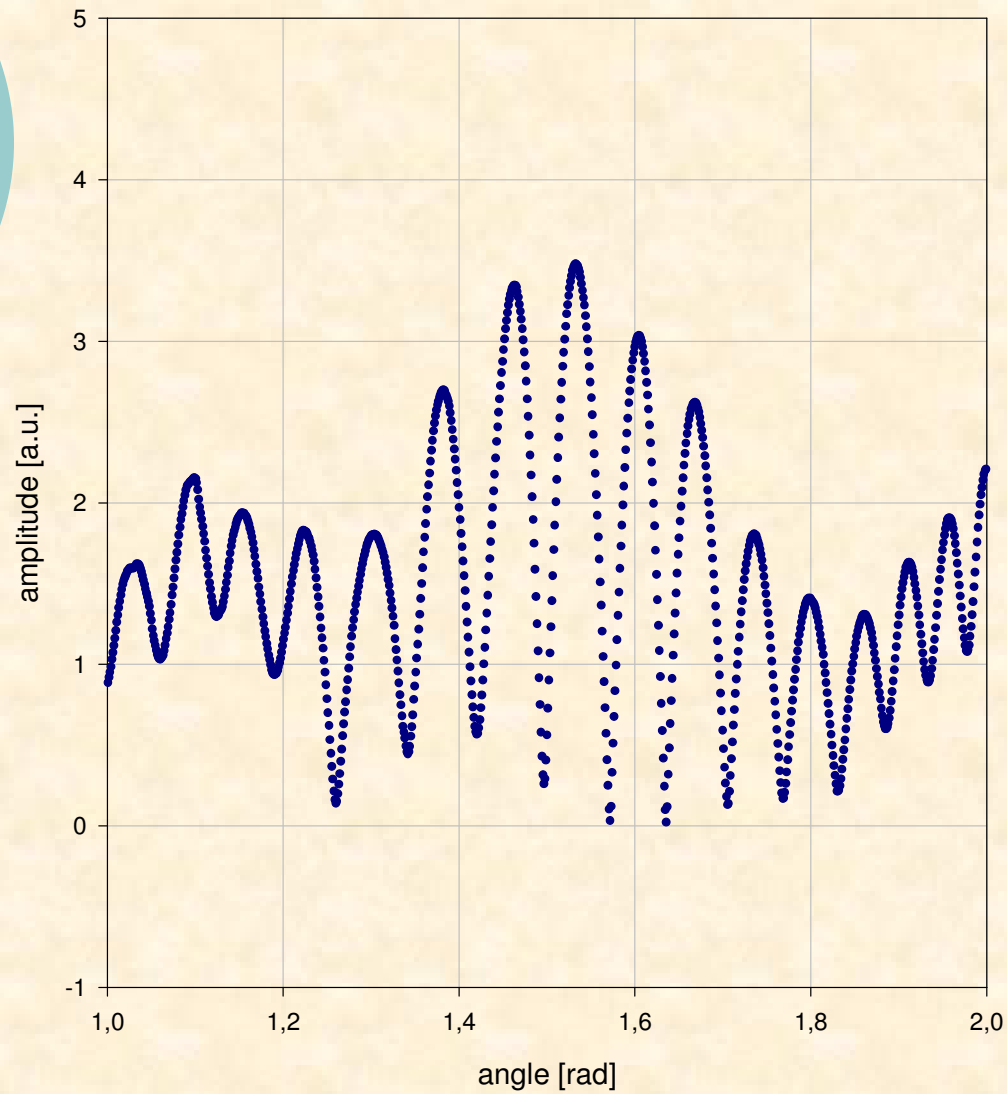
Predicted



Speaker distance from wall

3.1 cm

Predicted



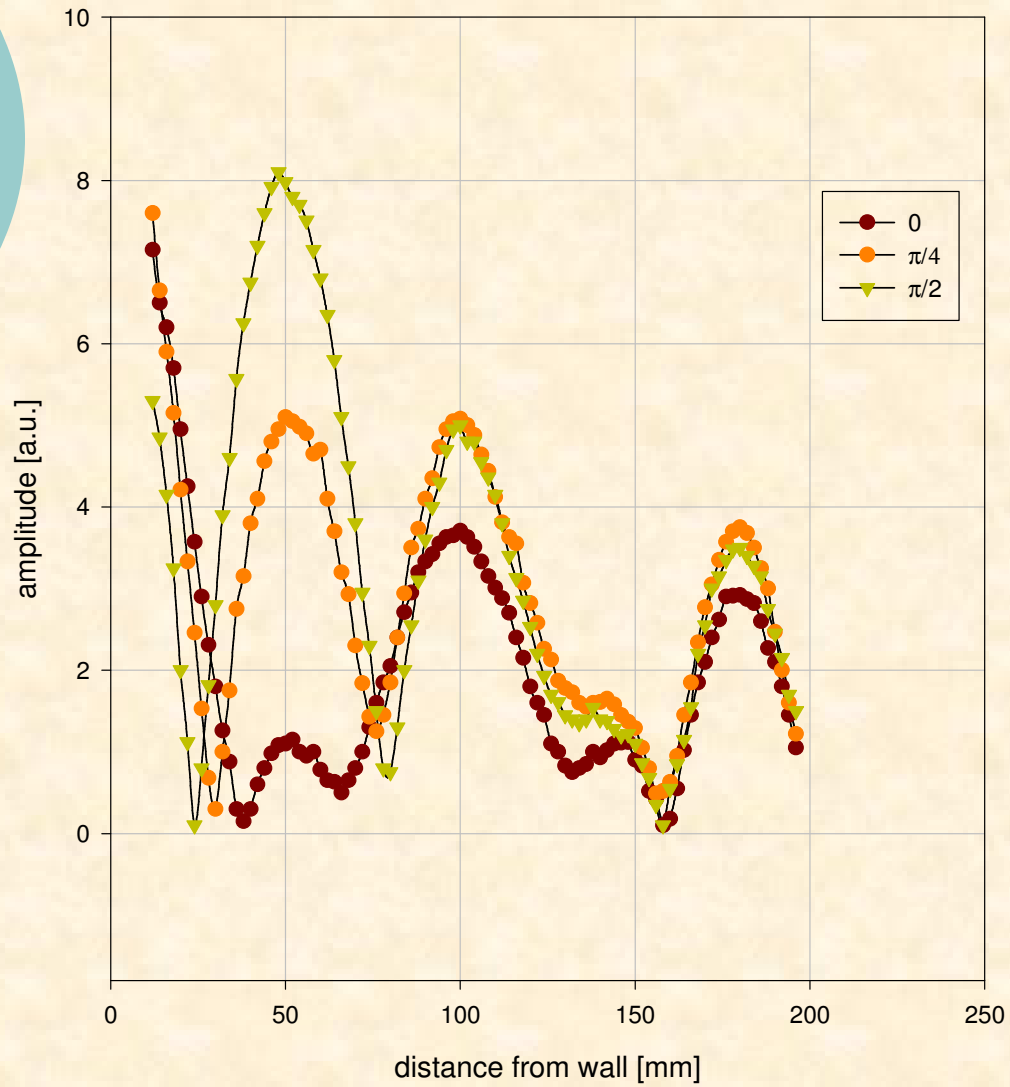
Angular distribution detail

Speaker distance from wall 5.0 cm

Standing wave pattern – proof of mode existence

Measured order – 46

Predicted



Speaker orientation dependence

Speaker distance from wall 2.1 cm

No strong dependence, similar structure

Structure as predicted

First peak growth – floor effect?

Not predicted

- At some speaker distances/orientations – parasite minima
    - Floor effects?
    - Secondary speaker emmision maxima?
- Complete angular distribution – many orders and parasite effects, hard to predict