Problem 10.

Inverted Pendulum

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It is possible to stabilise an inverted pendulum. It is even possible to stabilise inverted multiple pendulum (one pendulum on the top of the other). Demonstrate the stabilisation an determine on which parameters this depends.

Introduction

- Inverted pendulum - center of mass is above its point of suspension
- Achieving stabilisation – pendulum suspension point vibrating!
- Principal parameters:
    - lenght
    - frequency
    - amplitude

Introduction cont.

Experimental approach

- Apparatus
- Construction
- Measurements:
    - Pendulum angle in time
    - Stabilisation conditions:
        amplitude vs. pendulum length
        amplitude vs. frequency
- Double pendulum

Apparatus

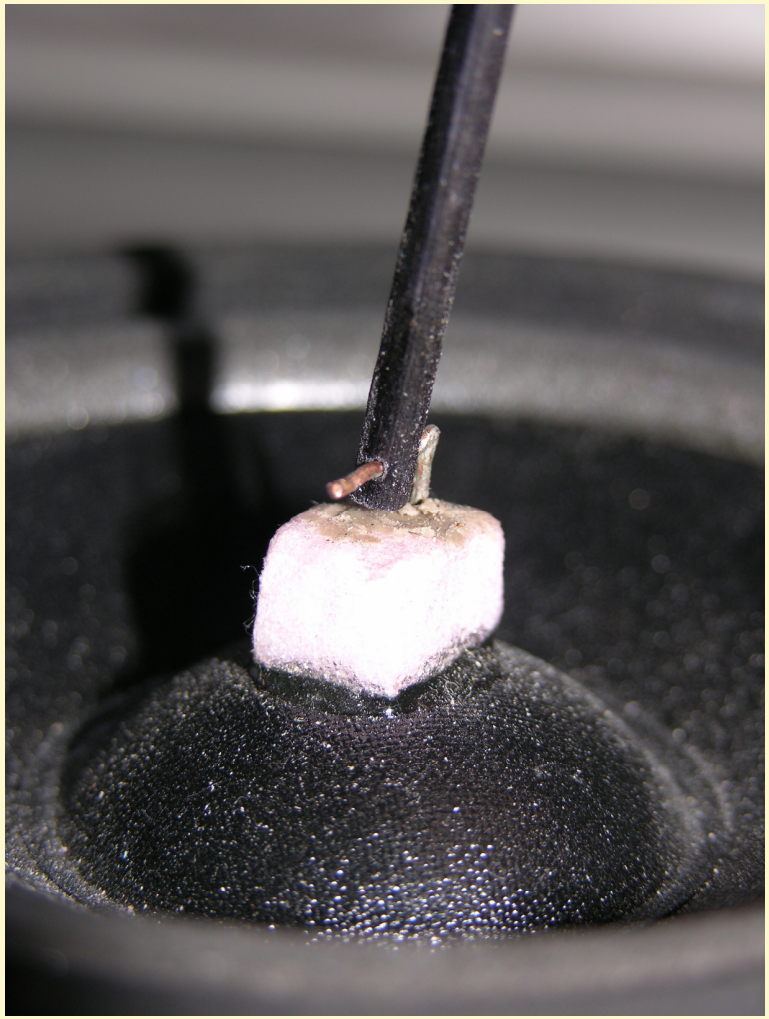
- Speaker (subwoofer)
- Function generator
- Amplifier
- Stroboscope
- Pendula (wooden)

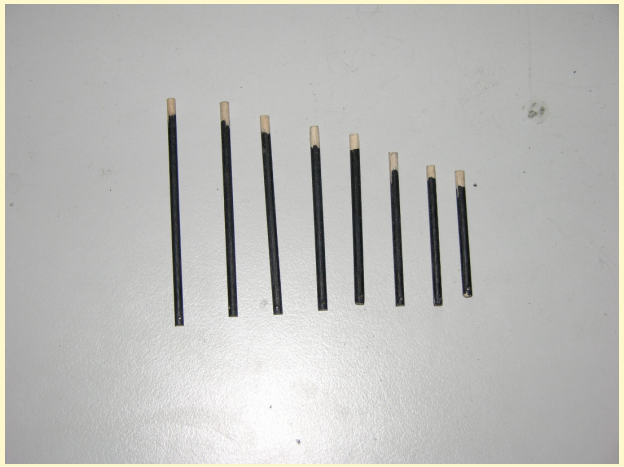


Apparatus cont.

- Speaker – low harmonics generation
- Audio range amplifier
- Stroboscope – accurate frequency measurement
- Point of support amplitude measured with (šubler)
- Multiple measurements for error determination

Construction





Lengths [cm]:

4	4.5	5	5.5	6	6.5	7	7.5

Density [kg/m^3]:

626

Measurements

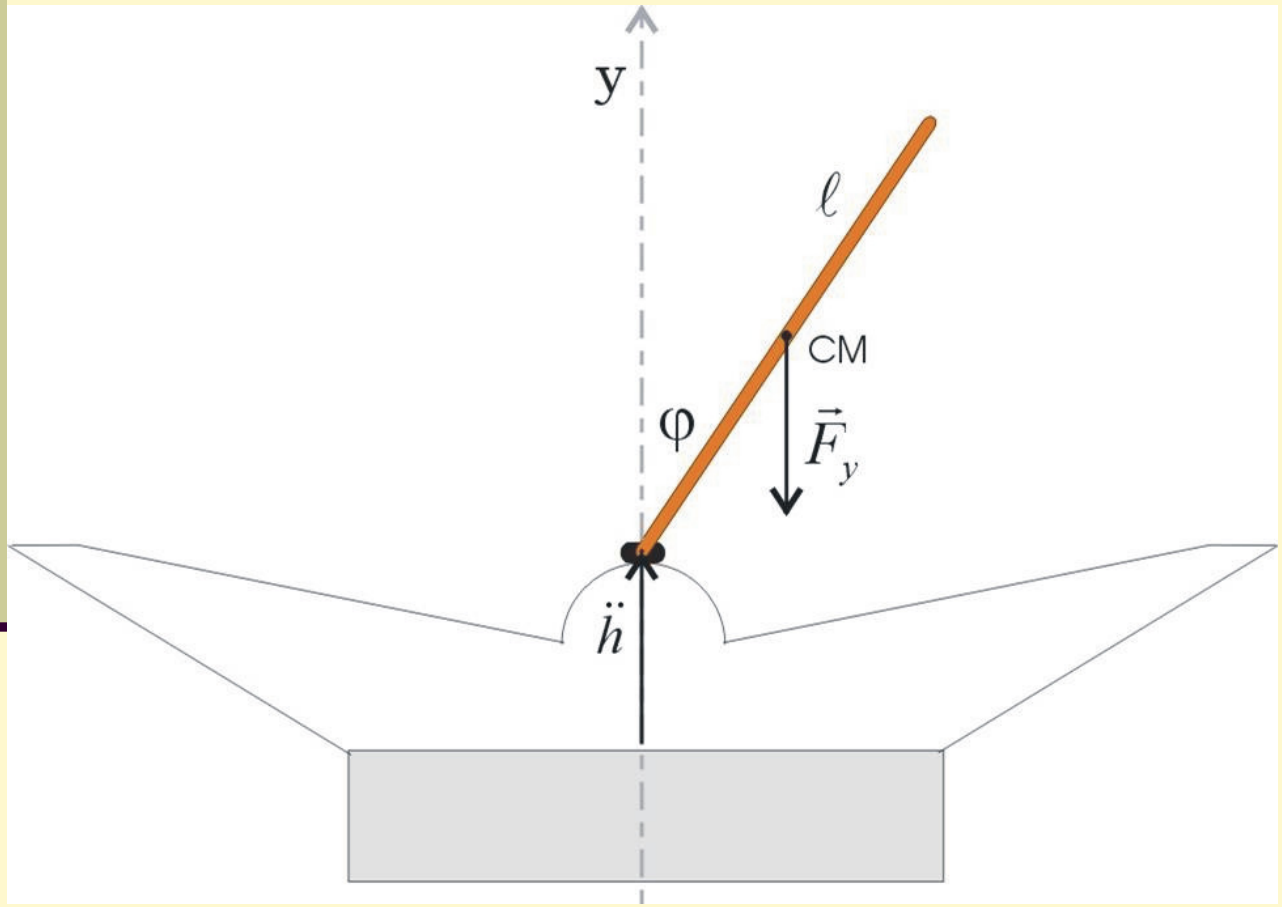
- Stability – pendulum returns to upward orientation
- measurements of boundary conditions:
  - frequency vs. amplitude
  - length vs. amplitude
  - angle in time (two cases);
    - inverted pendulum
    - “inverted” inverted pendulum – for drag determination

Double pendulum

Theoretical approach

- Pendulum – tends to state of minimal energy
- Upward stabilisation possible if enough energy is given at the right time
- Formalism – two possibilities:
    - equation of motion
    - energy equation – Lagrangian formalism
- Forces approach – more intuitive:

Forces on pendulum



l – pendulum lenght

φ – angle between pendulum and y axis

$\ddot{h}$ – acceleration of suspension point

F_r – resistance

F_y – inertial force acting on the center of mass

Equation of motion

- In noninertial pendulum system:

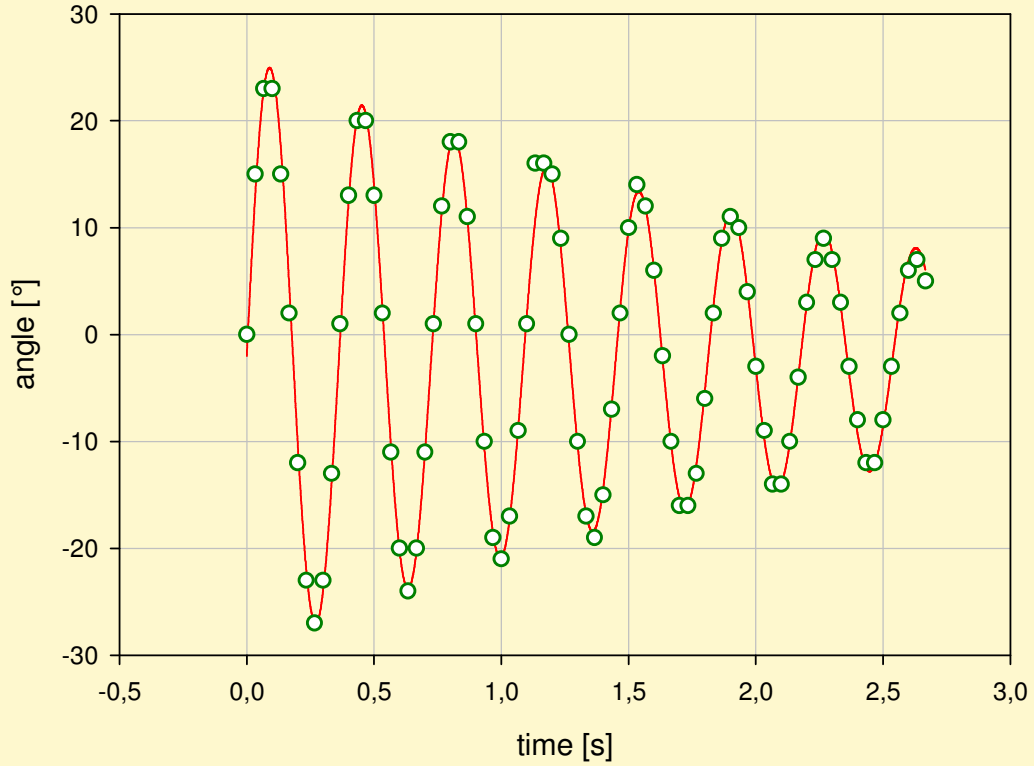
$$I_s \ddot{\varphi} = -\frac{1}{2} F_y l \sin\varphi - \frac{1}{2} F_r l$$

l – pendulum lenght
φ – angle between pendulum and y axis
I_s – pendulum moment of inertia
F_r – resistance
F_y – inertial force acting on the center of mass

- Inertial acceleration:
  - gravity component
  - periodical acceleration of suspension point

Equation of motion cont.

- Resistance force – estimated to be linear to angular velocity
- "inverted" inverted pendulum measurements



$$\varphi_{\max} \sim e^{-\frac{\beta_{eff}}{2}t}$$

β_{eff} – damping coefficien t

$\beta_{eff} = 3.0\ s^{-1}$

$\varphi_{\max}$ – angular amplitude

Equation of motion cont.

$\Rightarrow$ equation of motion:

$$\ddot{\varphi} + \beta_{eff}\dot{\varphi} + \omega_0^2\left(1 - \frac{A\omega^2}{g}\sin\omega t\right)\sin\varphi = 0$$

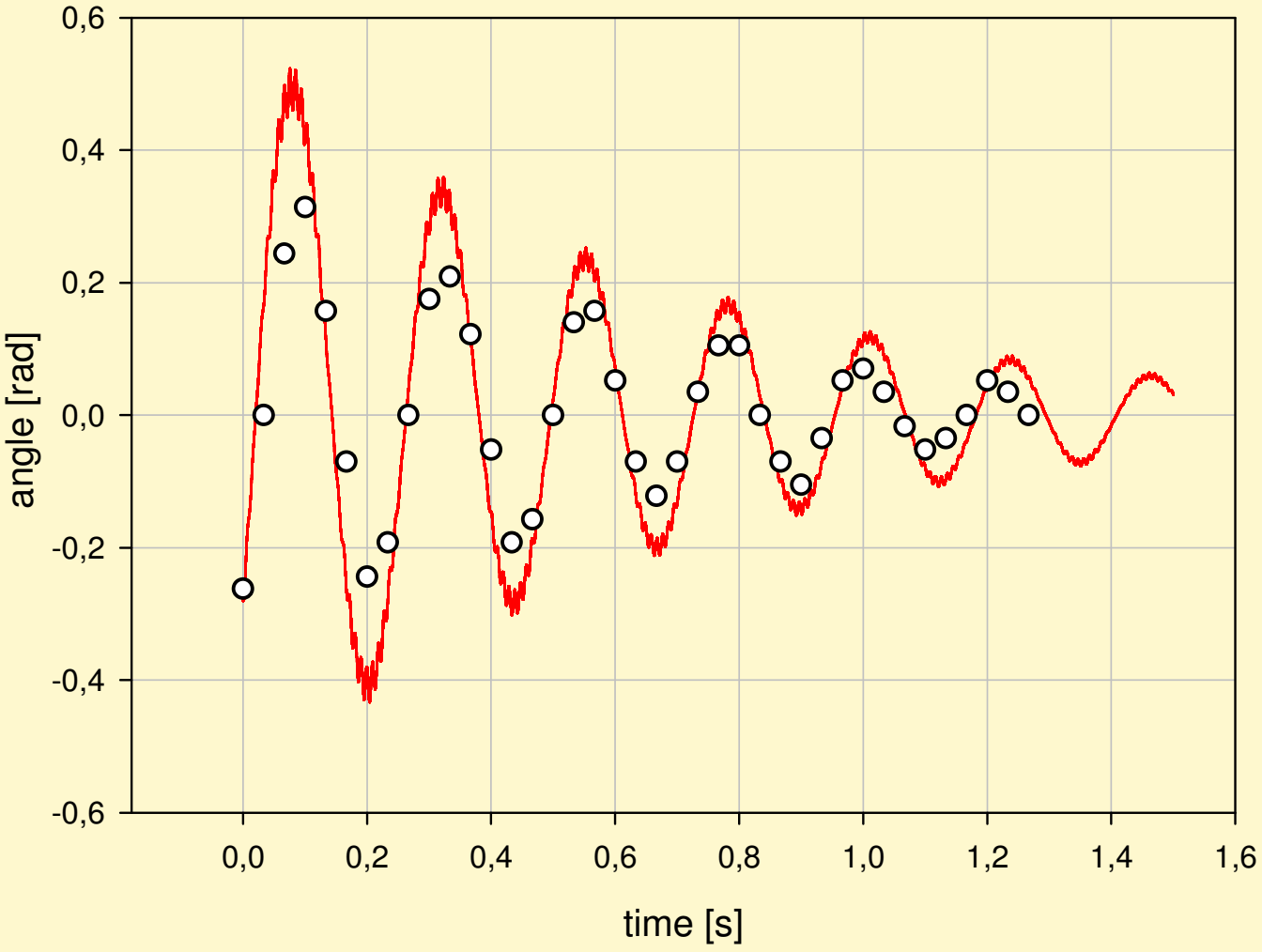
ω_0^2 – parameter : $\omega_0^2 = \frac{3}{2}\frac{l}{g}$

A – suspension point amplitude

ω – suspension point angular frequency

- Analytical solution very difficult
- Numerical solution – Euler method

Equation of motion cont.



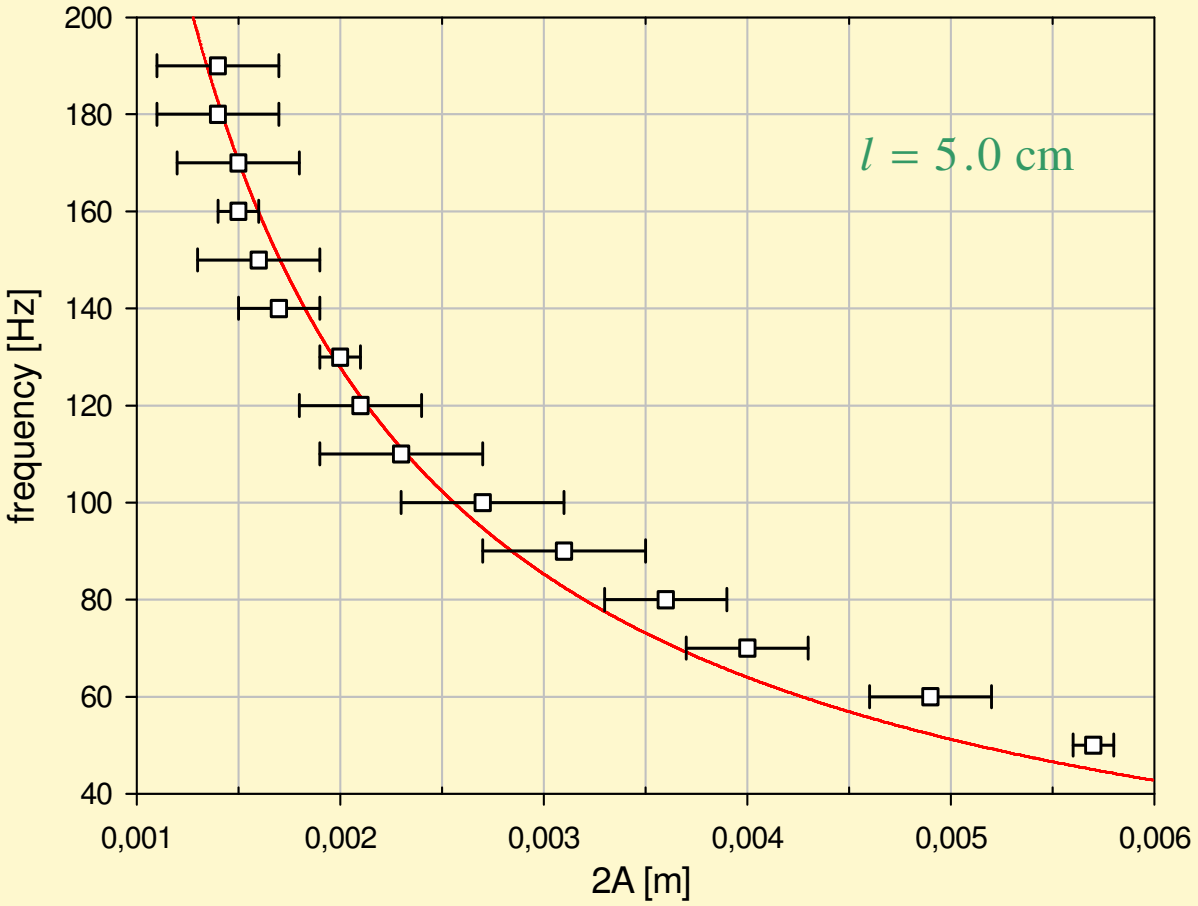
$l = 5.0\,\text{cm}$

$\omega = 685\,\text{rad/s}$

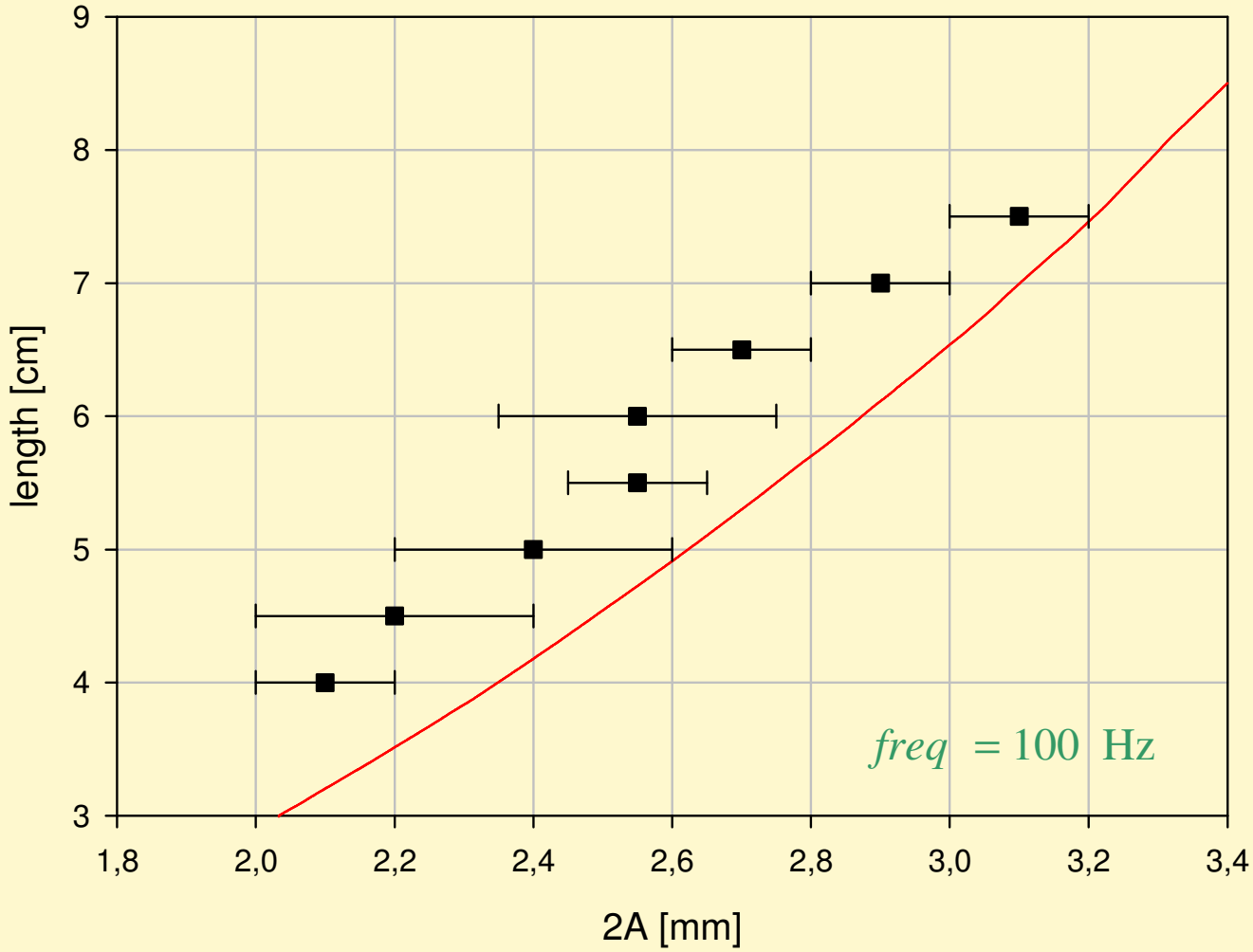
$2A = 4.5\,\text{mm}$

Stability conditions

⇒From equation of motion solutions stability determination:



Stability conditions cont.



Stability conditions cont.

- Agreement between model and measurements relatively good
- Discrepancies due to:
  - errors in small amplitude measurements
  - speaker characteristics (higher harmonics generation)
  - nonlinear damping...

Conclusion

- we determined and experimentaly prove stability parameters
- mass is not a parameter
- theoretical analisis match with results
- we managed to stabilise multiple inverted pendulum