Problem 12.

Rolling Magnets

Problem

Investigate the motion of a magnet as it rolls down an inclined plane.

Outline

- Only rolling motion investigated!
- Two distinct cases:
    - Nonconducting plane
    - Conducting plane
        - Quasiinfinite plane
        - Finite plane
- Common parameters:
    - Magnet properties
    - Plane inclination

The magnets

- Permanent $Nd_2Fe_{14}B$ magnets
- Field of magnetization 1.4 T
- Density 7500 kg/m^3
- 3 different sizes:

Diameter [cm]	2.54	1.00	0.95
Thickness [cm]	2.54	0.50	0.63





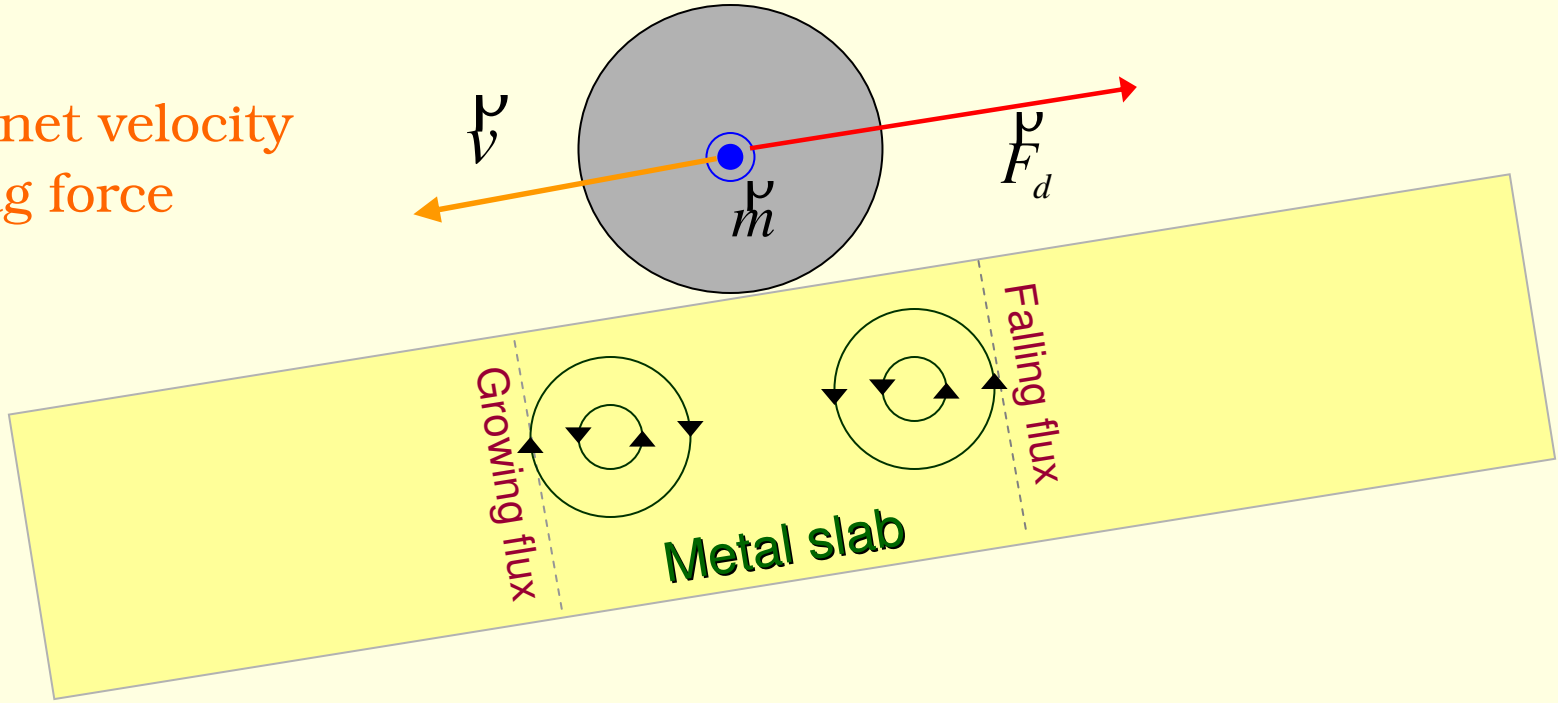
Case 1 – Nonconducting plate

- Wooden/plastic plate
- Magnet influenced only by the Earth field
- Curved trajectory
- Parameters:
  - Plane inclination
  - Magnet properties
- Much less appealing than second case – not studied in detail

Case 2 – Conducting plate

- In conducting plate – eddy currents induced due to time-changing field flux
- Eddy current field gradient – velocity-dependent drag on magnet

v – magnet velocity
F_d – drag force



Conducting plate cont.

Two subcases:

- Magnet moving far from the plate edges - ″infinite″ plate

Conducting plate cont.

- Magnet getting near the edges – boundary effects

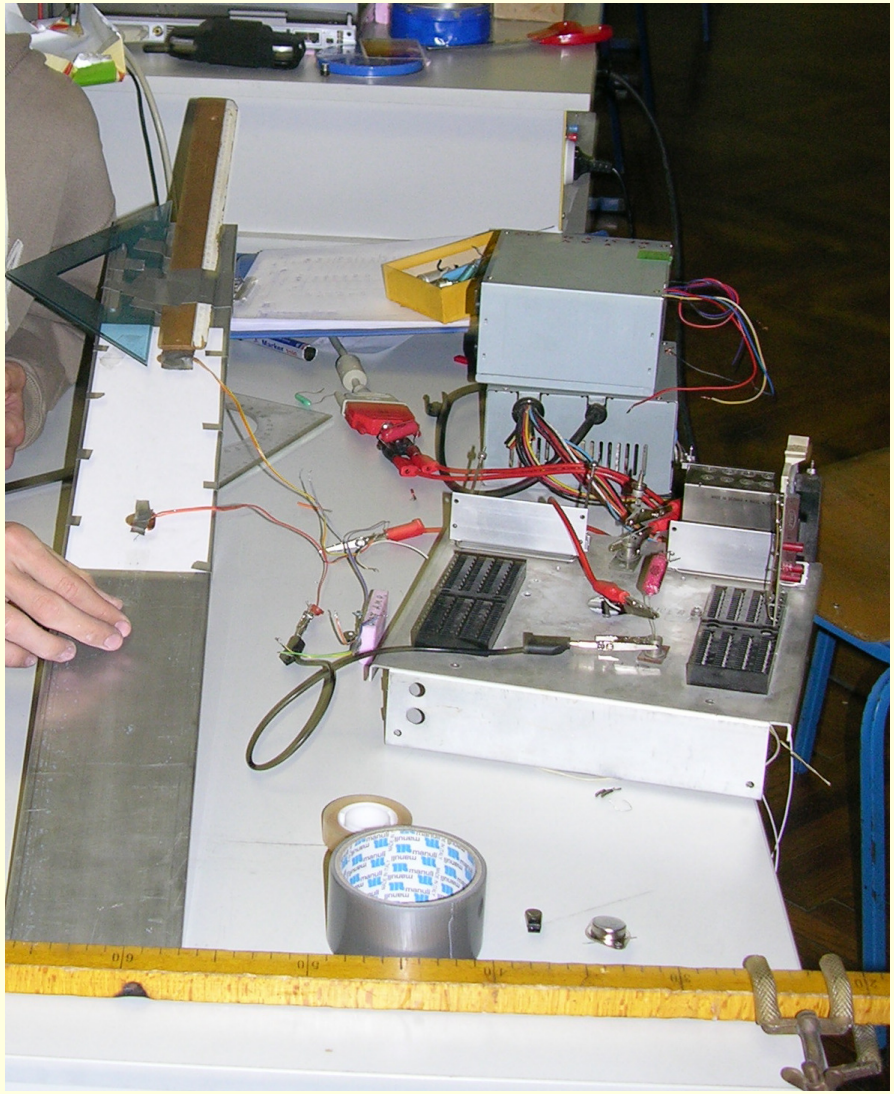
1. Infinite conducting plate

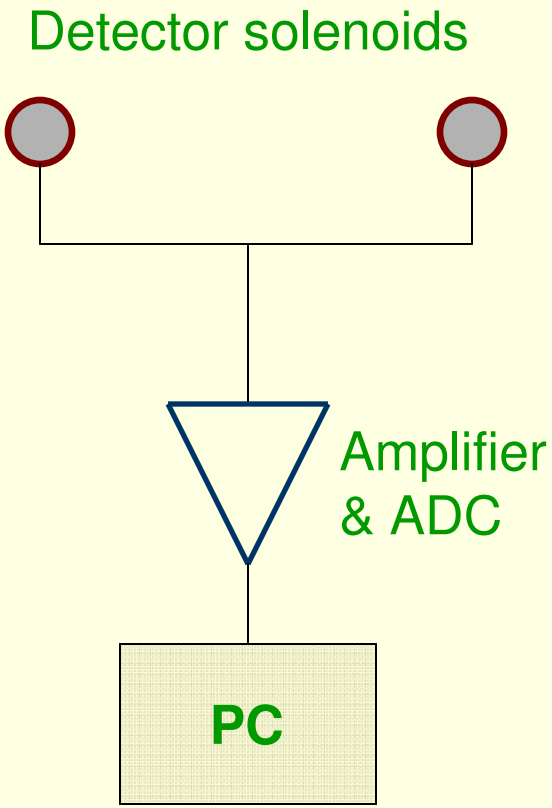
- First case much simpler:
  - Linear motion
  - Constant velocity (drag balances gravity) – simple reference system switching
- Main parameters:
  - Magnet dimensions and magnetization
  - Plate inclination
  - Plate conductivity

Experiment

- Measurements:
  - Dependence of terminal velocity on plate inclination for several magnets
  - Dependence of terminal velocity on plate conductivity
- Aluminium plate
- Velocity measurement – solenoid system
- Conductivity modification – temperature change

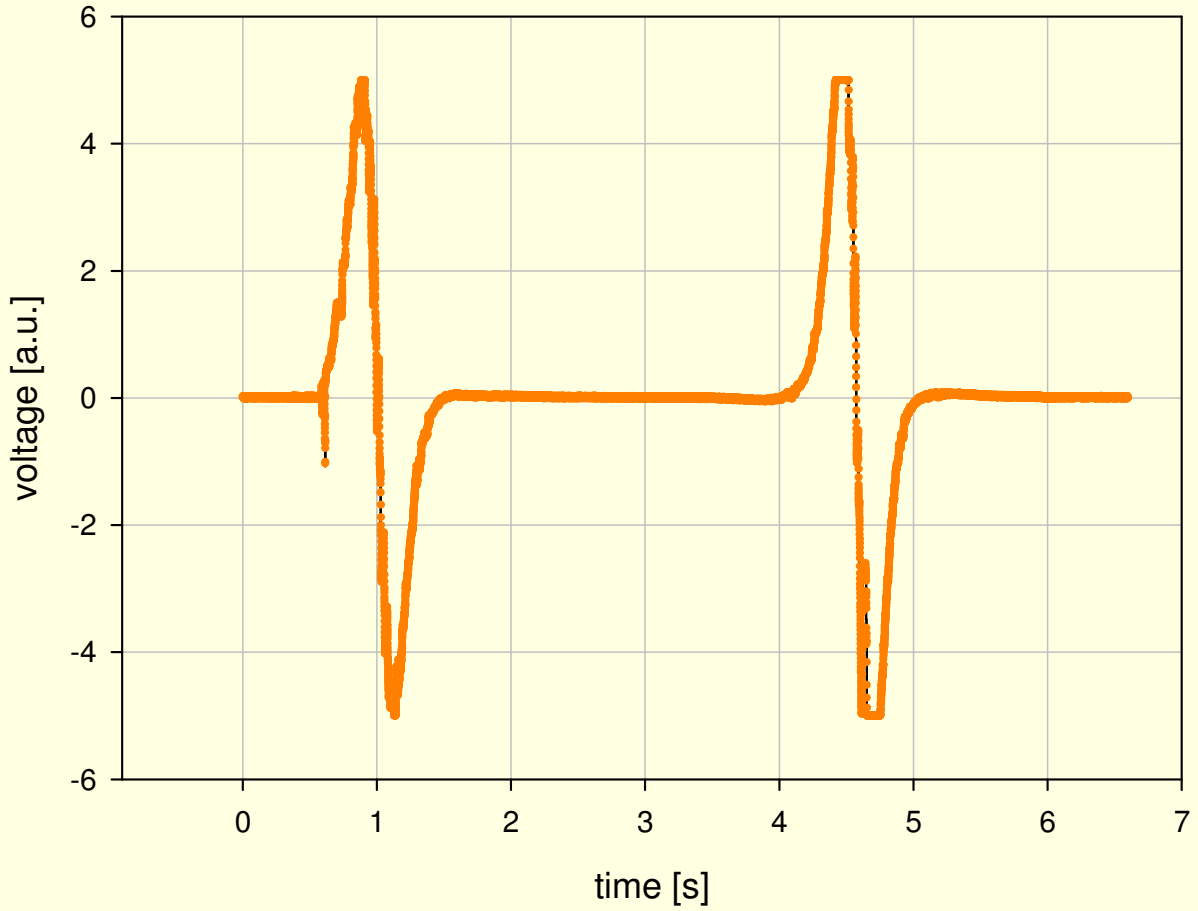
1. Velocity – inclination cont.





1. Velocity – inclination cont.

- Velocity measurement – solenoids detect passing magnet due to induction:



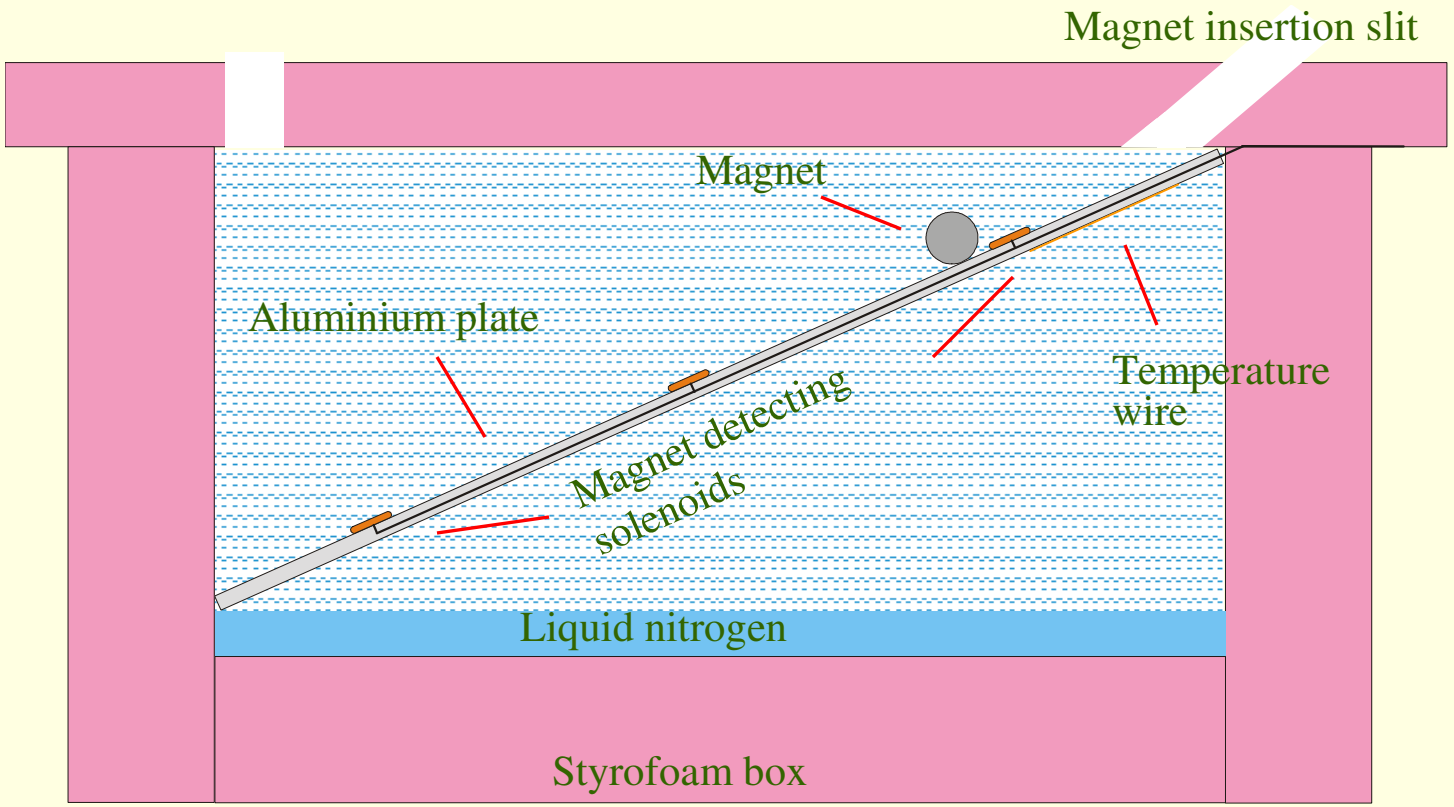
2. Velocity - conductivity

- Conductivity change:
  - Cooling plate in insulating box to 73 K with liquid N_2
  - As plate warms up magnet is released and velocity measured
  - Conductivity measured directly – resistance of wire attached to plate

Temperature range	73 – 200 K
Conductivity range	37 – 200 MS

Velocity – conductivity cont.

- Apparatus shematic:



Velocity – conductivity cont.



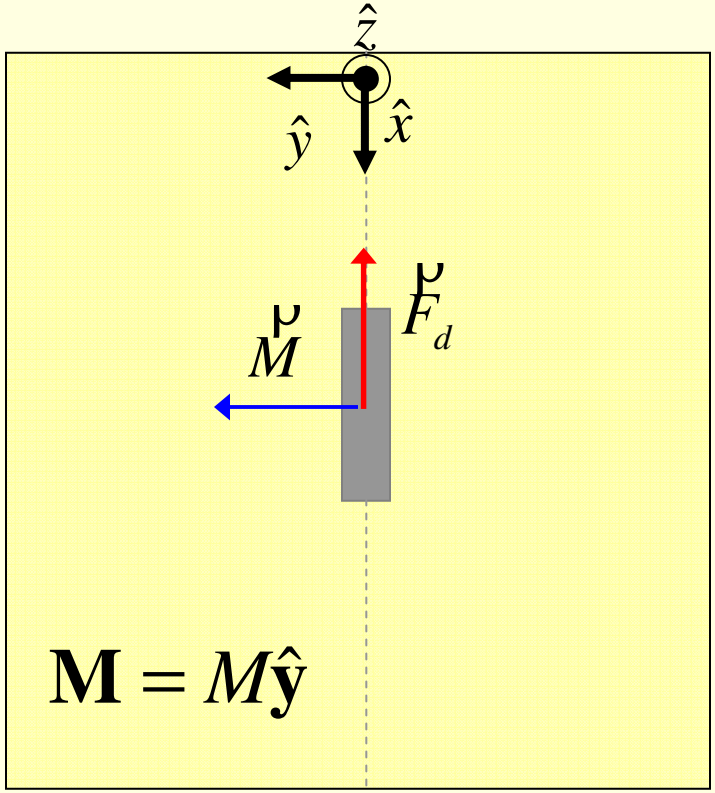
Box inside with plate and solenoids



The box

Theory

- The geometry in magnet reference system:



M – magnetization vector

F_d – drag force

x,y,z – magnet reference system

x',y',z' – plate reference system

t - time

$$\hat{\mathbf{x}} = \hat{\mathbf{x}}' - \mathbf{v}t$$

$$\hat{\mathbf{y}} = \hat{\mathbf{y}}'$$

$$\hat{\mathbf{z}} = \hat{\mathbf{z}}'$$

Theory cont.

- Induced field – from Maxwell equations in magnet reference system
- For small velocities - field equation:

$$\nabla^2 \mathbf{B}_{in} + \mu_0 \sigma v \frac{\partial \mathbf{B}_{in}}{\partial x} = -\mu_0 \sigma v \frac{\partial \mathbf{B}_0}{\partial x}$$

Induced field

Source term – magnet field

j – current density

σ – plate conductivity

B_0 – field of magnet

μ_0 – permeability of vacuum

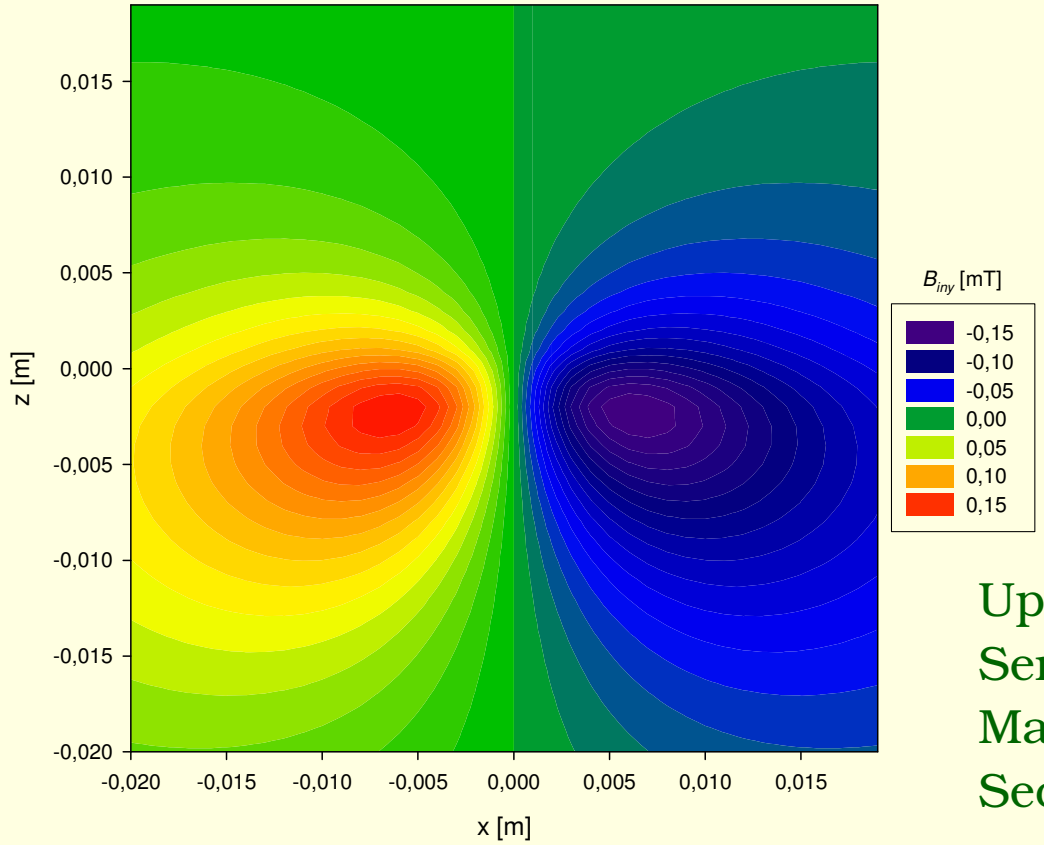
v – magnet velocity

$\Rightarrow$ Solution – power series in $\mu_0 \sigma v$

- For small velocities – linear first term dominates!

Theory cont.

- Needed for force – y - component
- Numerical integration yields:

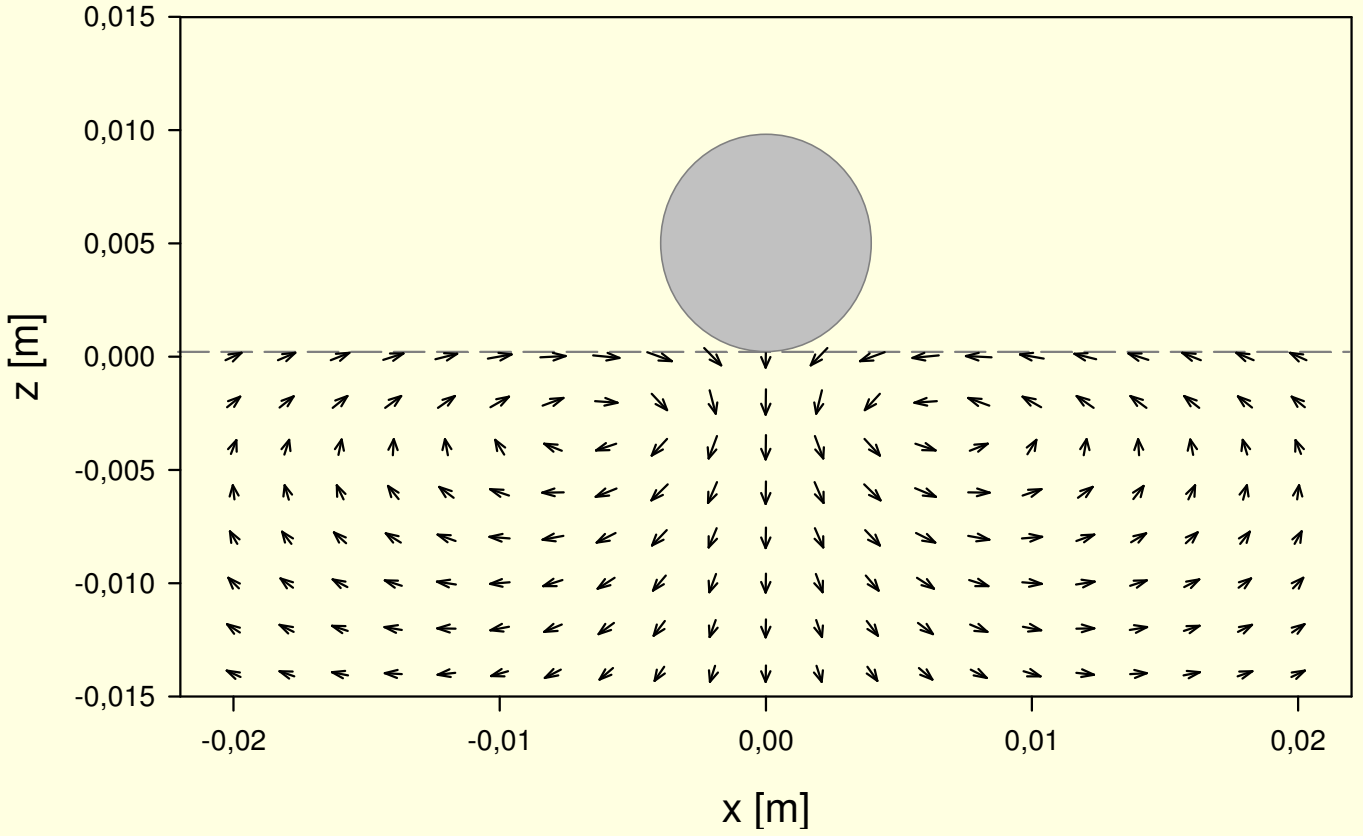


Magnet radius [cm]	0.5
Magnet thickness [cm]	0.5
Counductivity [MS]	29.85

Upper plate boundary $z = 0$
Semiinfinite plate
Magnet centre of mass $z = 0.5$ cm
Section $y = 0$

Theory cont.

- The currents are obtained by differentiation:



Theory cont.

- Drag force – for small velocities

$$F_d = -\Lambda \sigma v$$

Λ – calculated constant
σ – plate conductivity
v – magnet velocity

Diameter [cm]	2.54	1.00	0.95
Thickness [cm]	2.54	0.50	0.63
$\Lambda \cdot 10^9$ [kg/sS]	52.8(4)	1.19(3)	2.95(2)

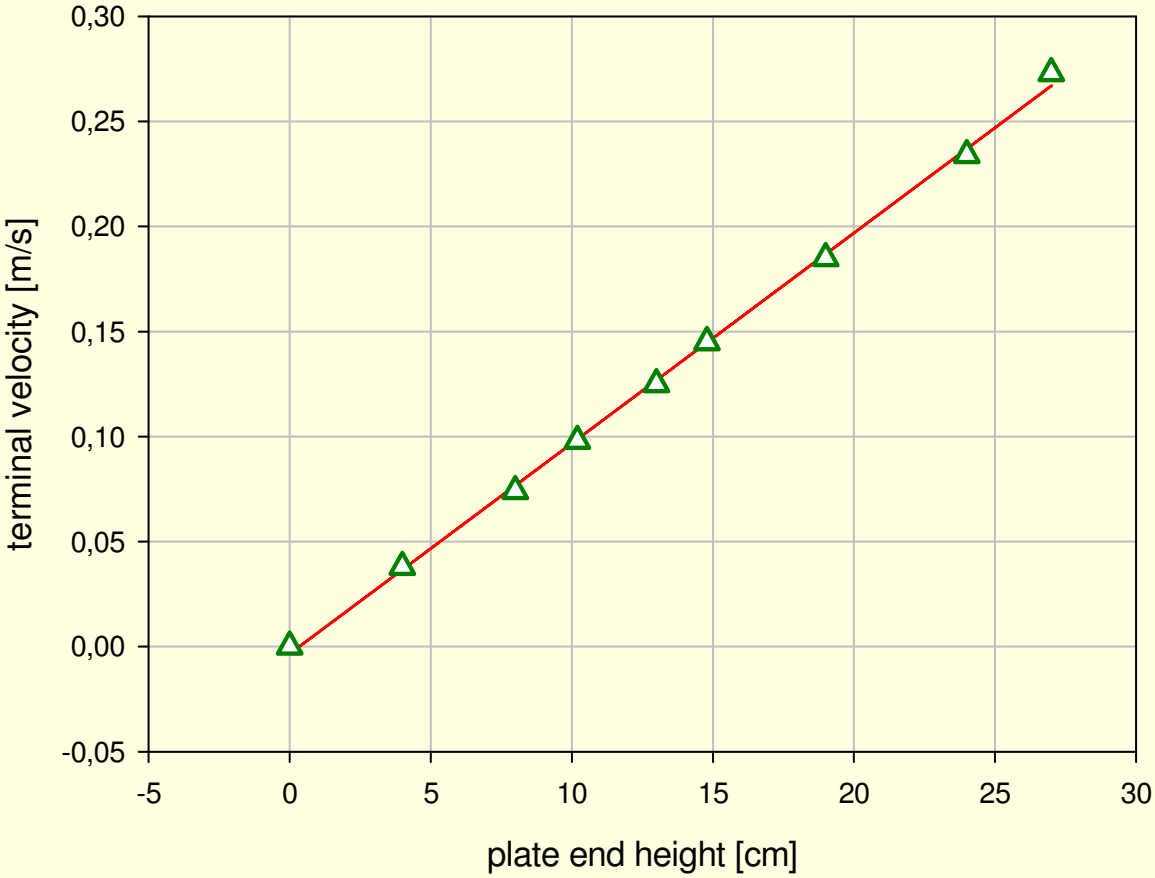
- Terminal state – balance between gravity and drag force:

$$v_T = \frac{\zeta g}{\sigma \Lambda} \sin \varphi$$

ζ – magnet mass
g – acceleration of gravity
φ – plate inclination

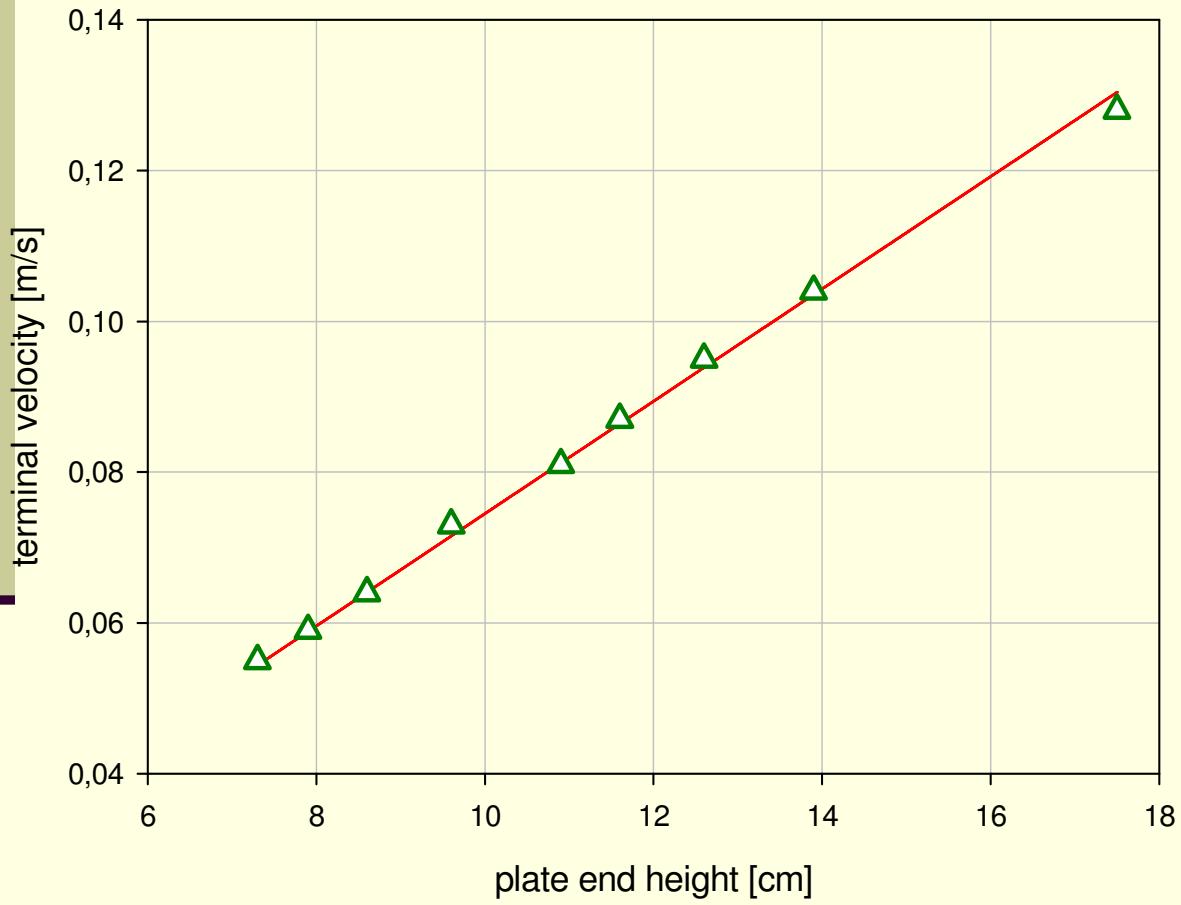
Results and comparation cont.

- For two magnets – dependence of terminal velocity on sin φ linear!



Diameter [cm]	1.0
Thickness [cm]	0.5
Counductivity [MS]	29.85
Plate thickness [cm]	1.0

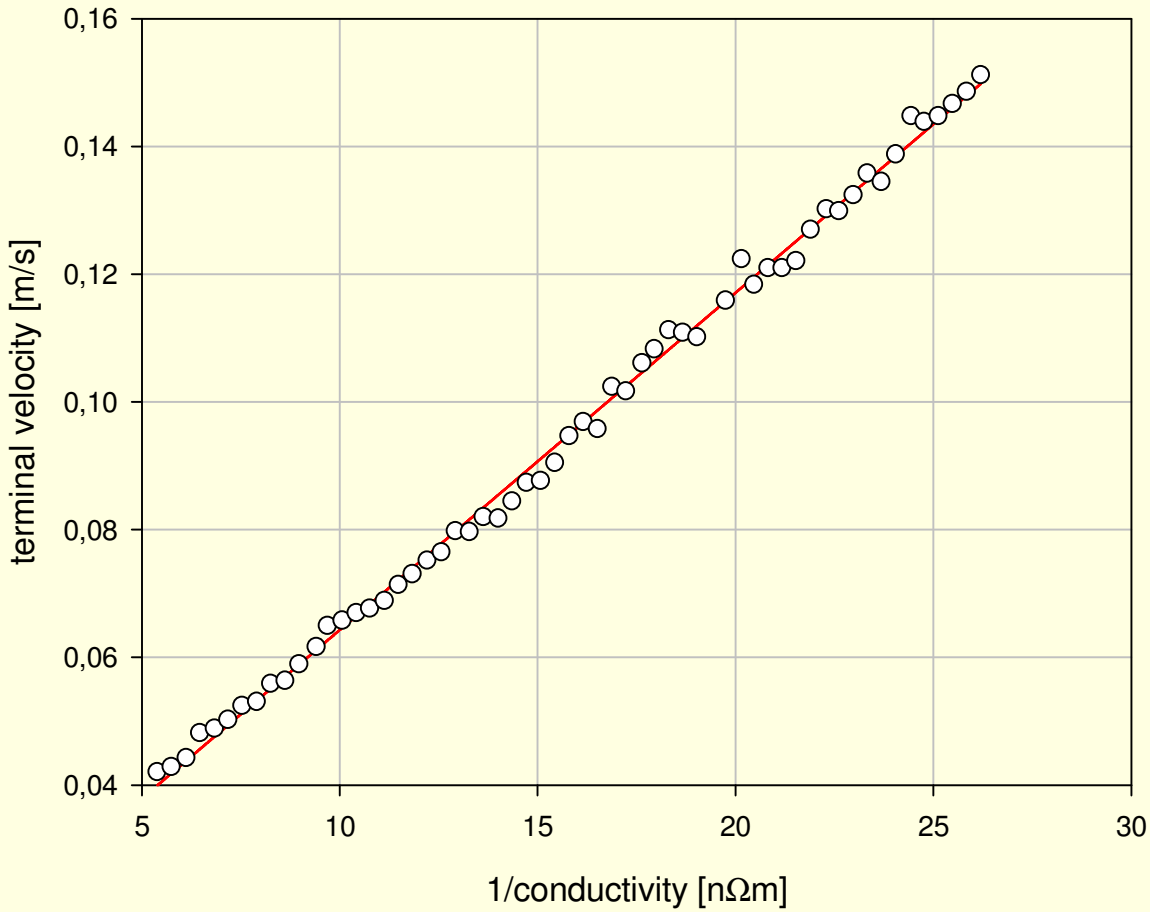
Results and comparation cont.



Diameter [cm]	2.54
Thickness [cm]	2.54
Counductivity [MS]	29.85
Plate thickness [cm]	1.0

Results and comparation cont.

- For third magnet – dependence of terminal velocity on 1/conductivity linear:



Diameter [cm]	0.95
Thickness [cm]	0.63
Plate angle [°]	28.5
Plate thickness [cm]	1.0

Results and comparation cont.

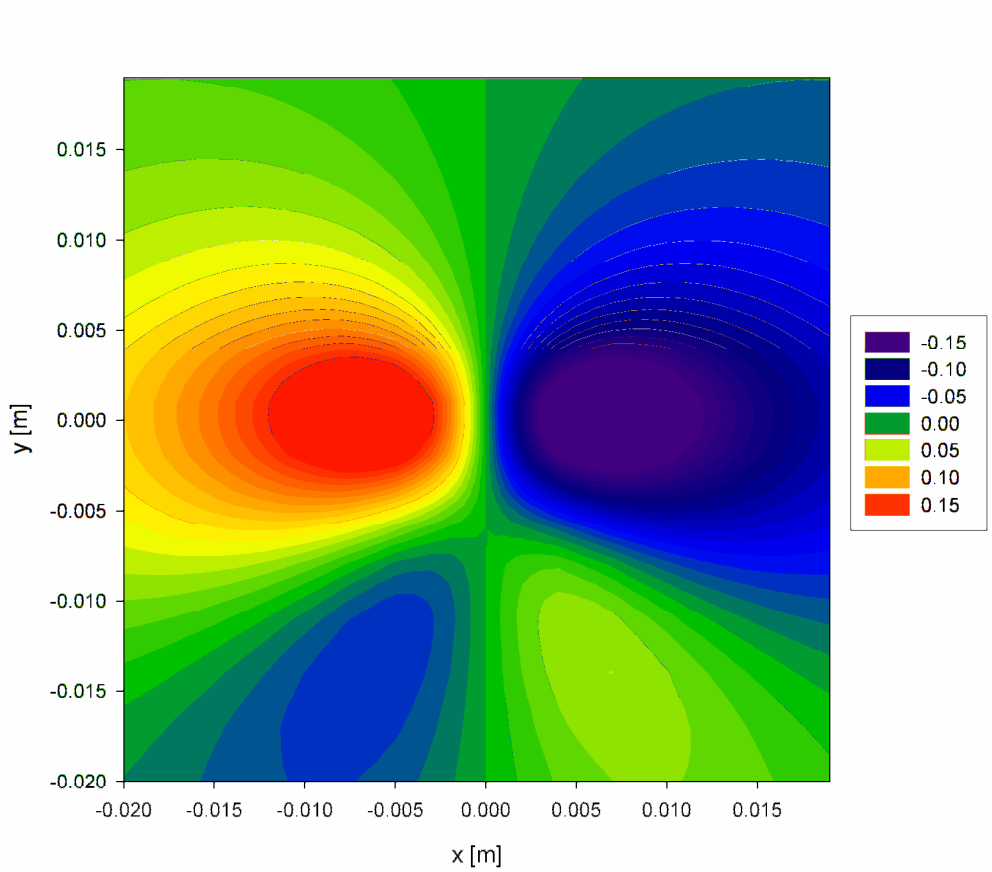
- From three measurements the coefficient Λ is obtained:

$\Lambda \cdot 10^9$	
Experiment	Theory
1.21 ± 0.02	1.19(3)
53.2 ± 0.2	52.8(4)
2.97 ± 0.02	2.95(2)

- Agreement is very good – justification of linearization!

2. Boundary effects

- Close to edge – nonsymmetric induced currents:

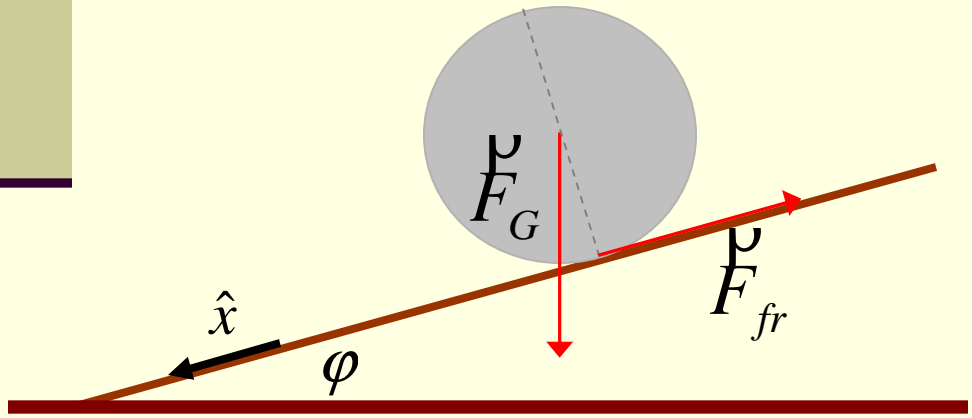


2. Boundary effects cont.

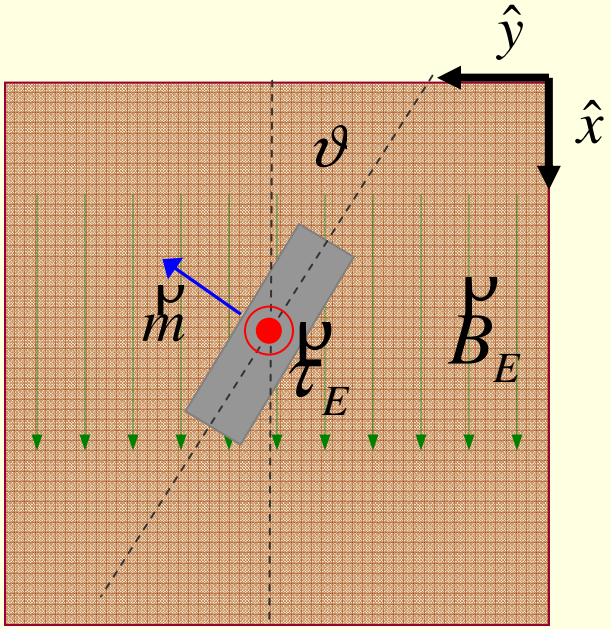
- Repulsive force occurs
- Magnet follows a quasiperiodical trajectory
- Exact modeling very difficult

Theory

- Acting on the magnet rolling motion:
  - Gravity
  - Earth field torque
  - Friction



From the side



From above

x,y – unit vectors
m – magnetic moment

Theory cont.

⇒ Trajectory equation:

$$(x - x_0)^2 + (y - r)^2 = r^2$$

$$r = \frac{\pi}{6} \frac{g \sin\varphi}{mB_E} R^4 \rho D$$

x_0 – initial x – position of magnet

R – magnet radius

ρ – magnet density

D – magnet thickness

- Trajectory – portion of circle
- For different initial angles numerical solution neccesary

Theory cont.

- Linear acceleration while rolling:

$$a = -\frac{2}{3} g \sin\varphi \sin\vartheta$$

g – acceleration of gravity

φ – plate inclination

θ – angle between magnetic moment and Earth field vector

- Special case: magnetic moment initially normal to Earth field – simple trajectory
- Magnetic field torque:

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}_E$$

$$I\ddot{\vartheta} = -mB_E \sin\vartheta$$

m – magnetic moment vector

$\mathbf{B}_E$ – Earth field vector

I – moment of inertia of magnet

Results and comparation

Theory cont.

- Magnet ↔ an array of infinitely thin dipoles
- Force on one dipole:

$$d\mathbf{F} = \left|\nabla\left(d\mathbf{m}\cdot\mathbf{B}_{in}\right)\right|_{(x_m,y_m,z_m)}$$

B_{in} – induced field

x_m, y_m, z_m – dipole coordinates

⇒ Force on magnet in our geometry:

$$\mathbf{F} = R^2\left|M\right|\int_{-\frac{D}{2}}^{+\frac{D}{2}}\left|\nabla B_{iny}\right|_{(0,\varepsilon,R)}d\varepsilon$$

R – magnet radius

D – magnet thickness

B_{iny} – y - component of induced field

ε - parameter