Problem 11.

Singing tube

The problem

A tube open at both ends is mounted vertically. Use a flame to generate sound from the tube. Investigate the phenomenon.

Experimental approach

- achieving effect
- experimental set-up
- measurements:
  - sound intensity vs. place of heating and heating power
  - sound intensity in time for different temperatures
  - harmonics vs. place of heating and power
  - air temperature distribution in tube
  - air flow through the tube vs. heating power

Achieving effect

- Two methods:
  - Flame heated metal mesh - demonstation
  - Electric heater in tube - measurements
- Flame heated mesh – tube sings when flame removed
- Heater in tube – continuous sound
- Vertical (or tilted) position neccesary – no sound when tube is horizontal

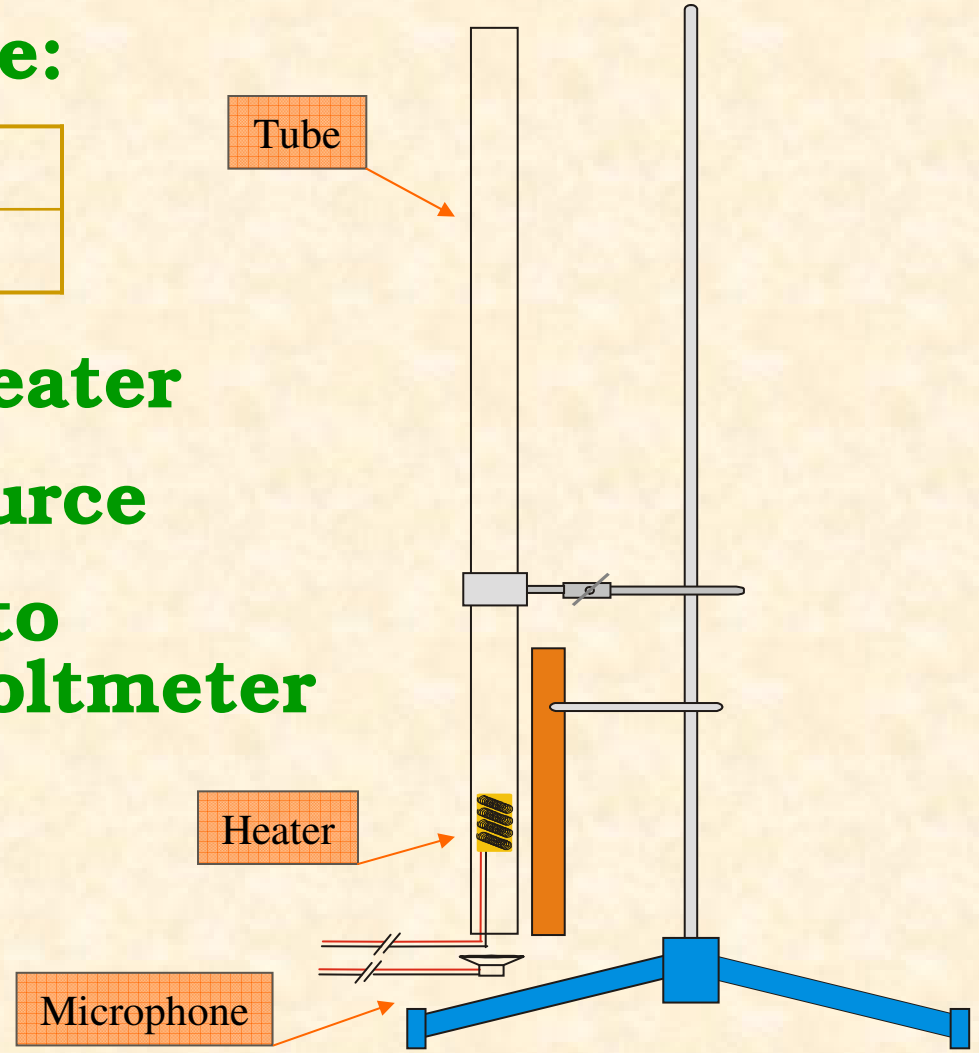
Achieving effect

Apparatus - basic

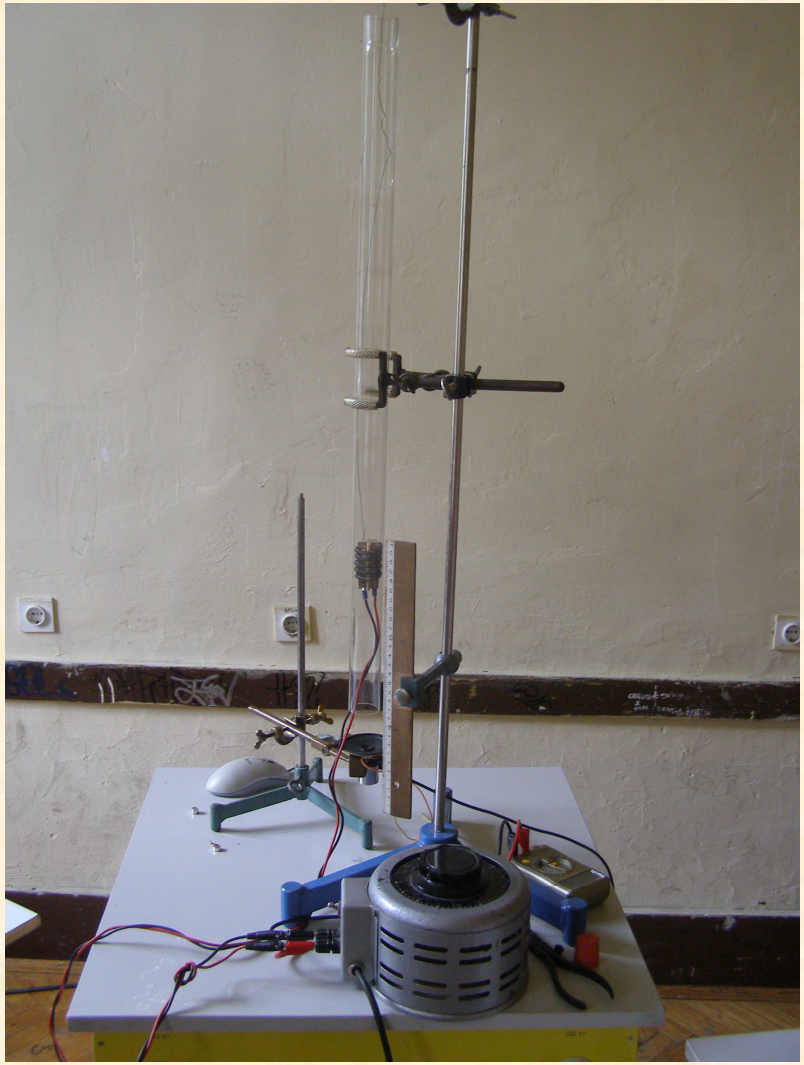
- **Vertical pyrex tube:**

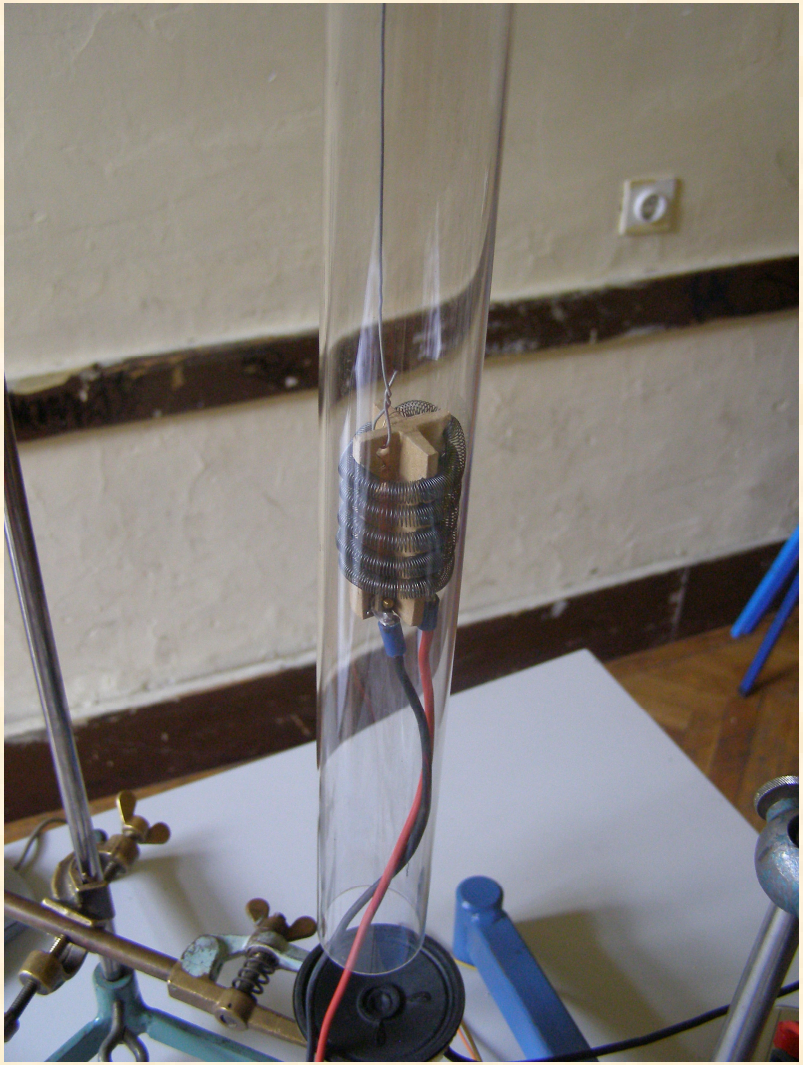
Tube length	80 cm
Tube diameter	3.6 cm

- **Resistance wire heater**
- **Variable power source**
- **Sound detector – to computer or AC voltmeter**



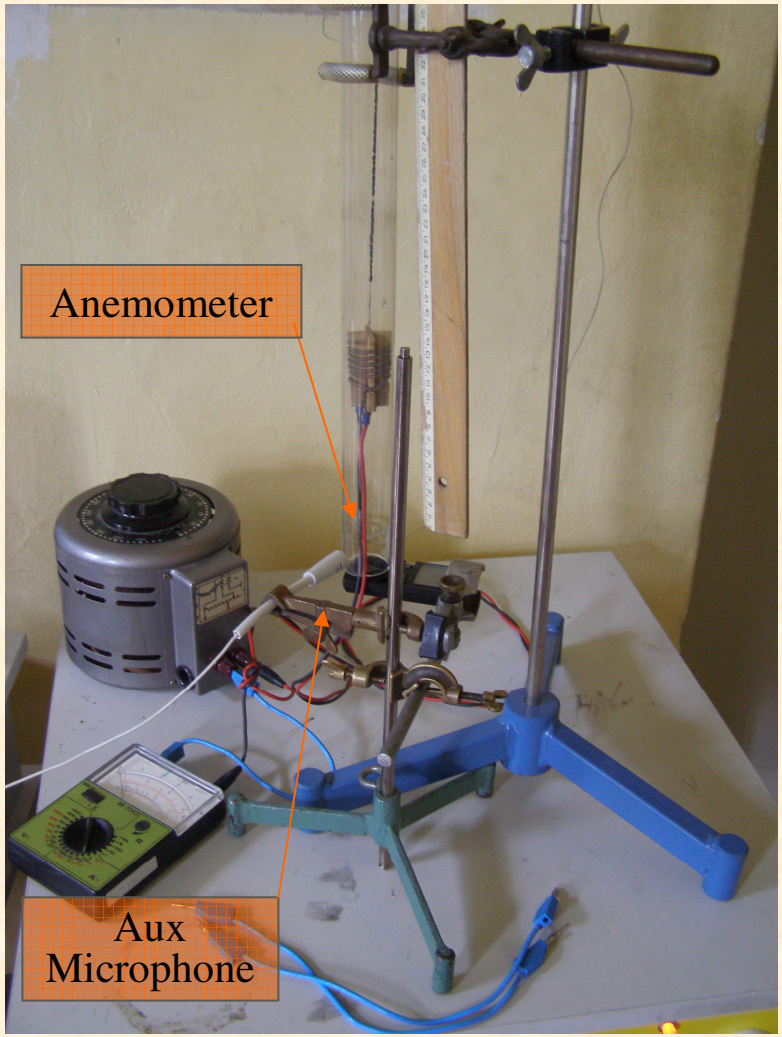
Apparatus - basic





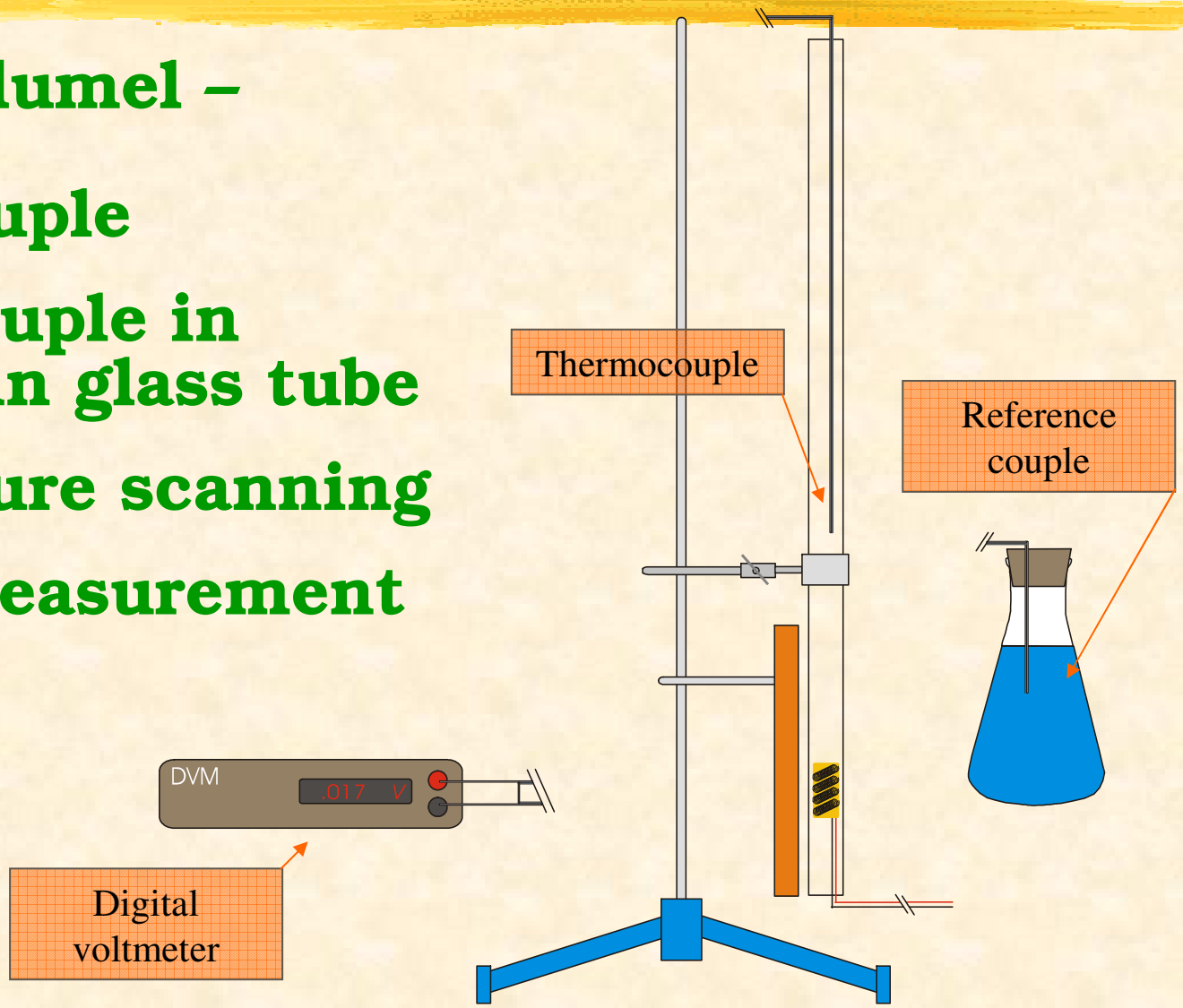
Apparatus - air movement

- ″Chimney″ effect
- Precision anemometer (0.1 m/s)
- Anemometer placed at tube ends
- Power variation – velocity change

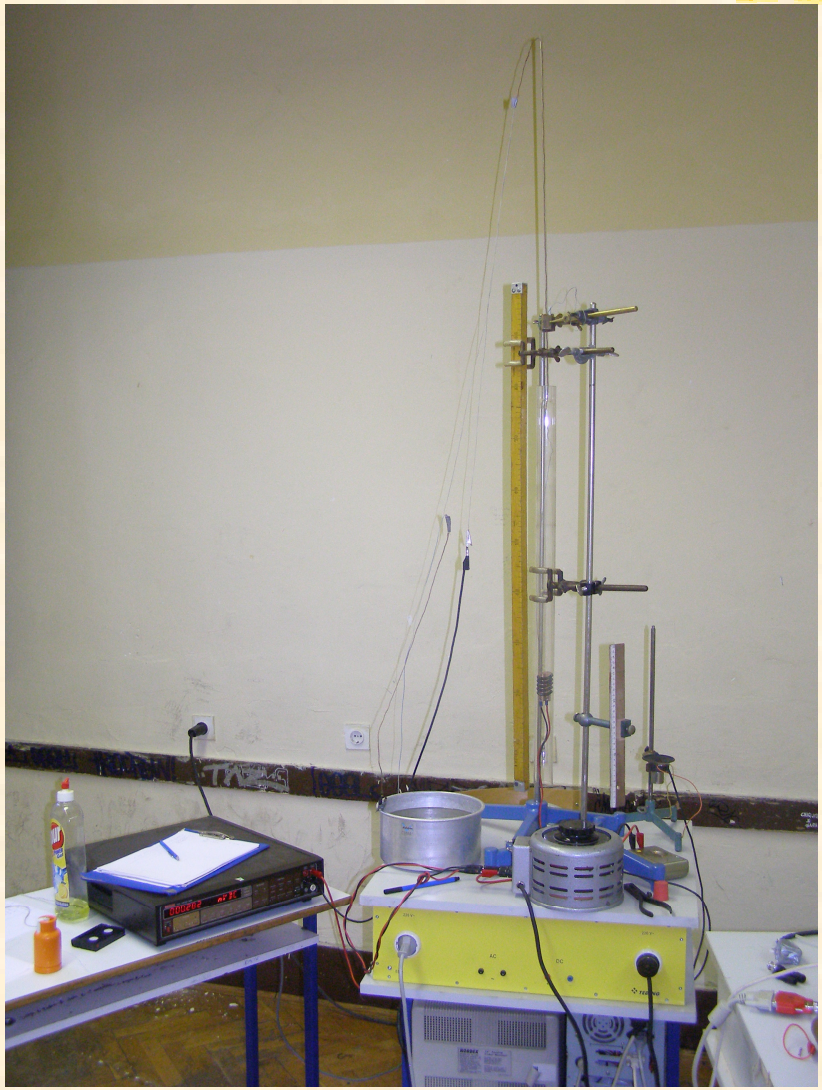


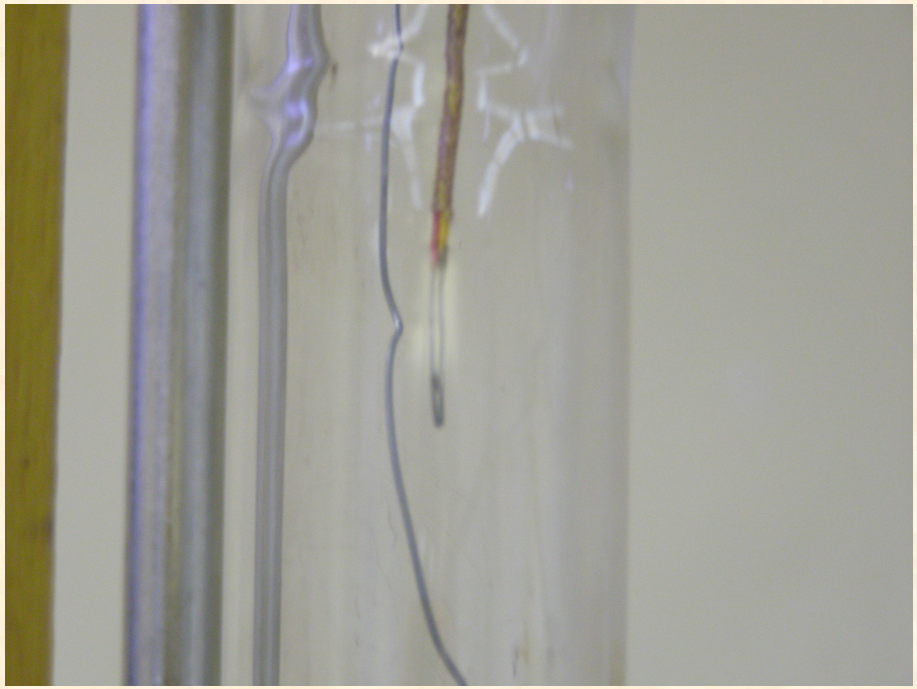
Apparatus - temperature

- Type K (Alumel – chromel) thermocouple
- Thermocouple in mobile thin glass tube
- Temperature scanning
- Voltage measurement - DVM



Apparatus - temperature





Theoretical approach

- Two stages:
  - Sound generation
  - Sound amplification and sustaining
- Basic mechanisms:
  - Oscillating heat exchange
  - Chimney effect
- Pitch – always ground mode of tube
- Spectrum – harmonics intensities dependent on power & heater position

Sound generation

- Boundary conditions at tube ends – constant pressure (atmospheric)
- Heater gives initial expansion push to air in tube
- Due to boundary conditions at tube ends – impulse reflection
- Standing wave formation
- Sound dies away quickly if not sustained by heating:

Sound sustaining

- **Flow in tube – two components:**

$$v_n = v_c + \tilde{v}$$

Net air velocity

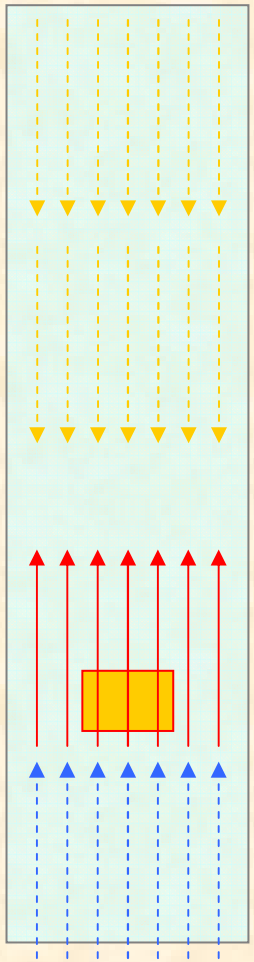
Constant velocity due to chimney effect

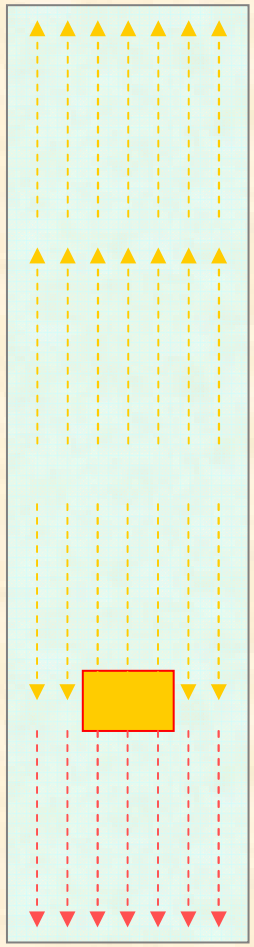
Time – changing sound air movement

⇒ **At some positions – variable heat transfer!**

- **Heater in lower half of tube:**
  - **Compression cycle – incoming cool air heated – larger heat transfer**
  - **Expansion cycle – heats already hot air – smaller heat transfer**

Sound sustaining cont.





$$dQ \sim (T_H - T_a)dt$$

Q – heat transferred

T_H – heater temperature

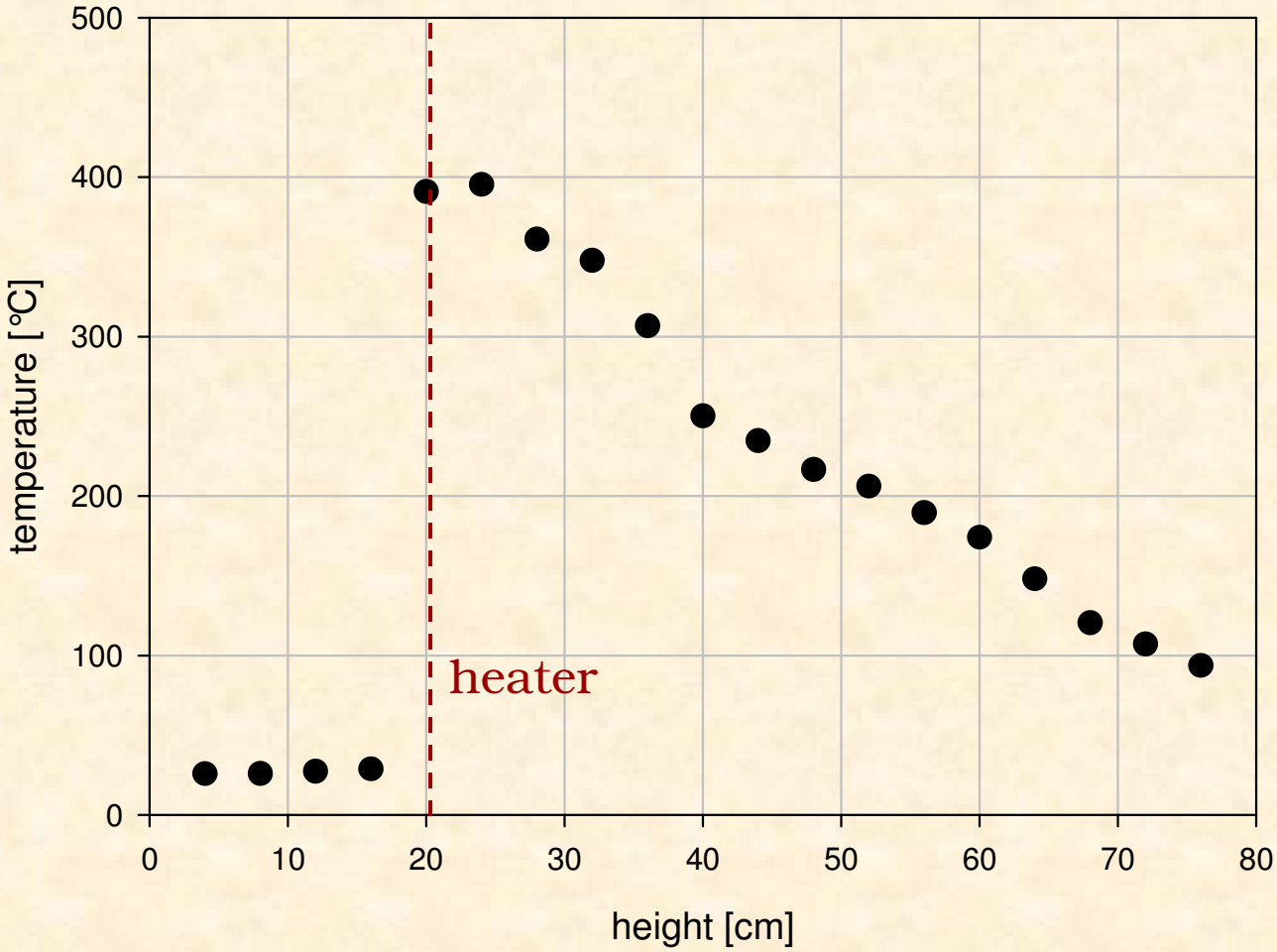
T_a – air temperature at heater location

t - time

- **In compression cycle – heat gives air push – amplitude increase**
- **In expansion cycle – no push**
- **In sum – amplification!**

Sound sustaining cont.

- **Real temperature distribution in tube:**



Sound sustaining cont.

⇒ **Two heat transfer components:**

$$Q = \overline{Q}\left[1 + \Gamma q(t)\right]$$

Q – steady heat transfer
Γ – constant
q – variable heat transfer

- **Criterion for amplification - convolution integral – measures phase relationship:**

$$I = \frac{1}{\tau}\int_{-\frac{\tau}{2}}^{+\frac{\tau}{2}} q(t)\tilde{v}\,dt$$

τ – oscillation period
t – time
v - variable velocity component

$I > 0$ - **amplification**

$I < 0$ - **damping**

Sound sustaining cont.

⇒ **Average heater power inversely proportional to *I***

- **In simplest approximation – heat transfer follows velocity:**

$$q(t) \sim \tilde{v}$$

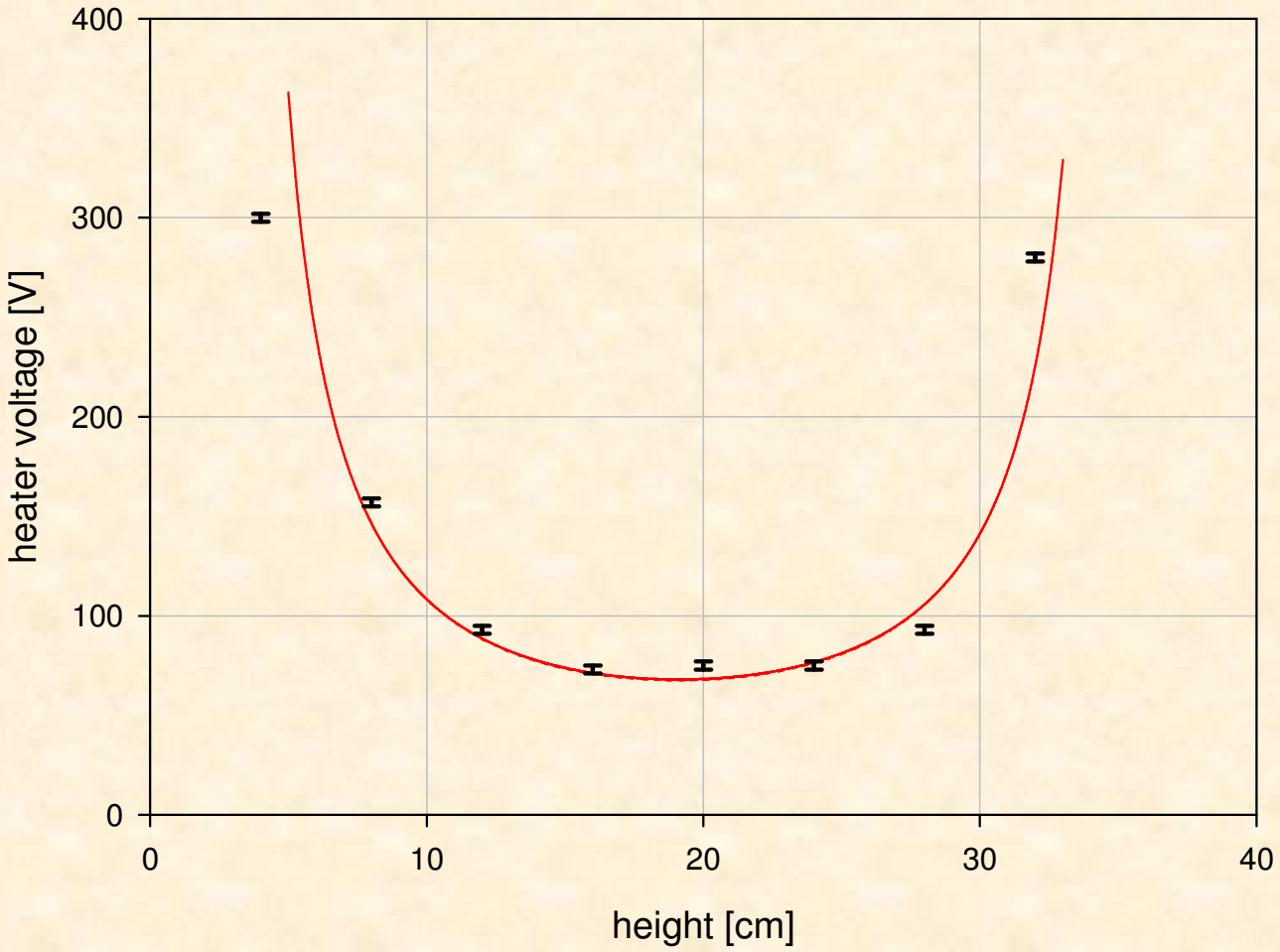
- **Ground mode generally loudest:**

$$\tilde{v} \approx A_0 \cos\frac{2\pi}{\lambda_0} z \cos\omega_0 t$$

A_0 – ground mode amplitude
λ_0 – ground mode wavelength
z – vertical coordinate
ω_0 – ground mode frequency

Sound sustaining cont.

⇒ Power neccesary for sound to start:



$$P = \frac{\Lambda}{I}$$

$$\Lambda =$$

Sound amplitude

- Amplitude – from energy conservation
- Heater power goes into:
  - Sound energy
  - Air heating and rising
  - Radiation
  - Acoustic losses…
- Acoustic amplitude dependent:
  - Sound energy
  - Acoustic losses

Sound amplitude cont.

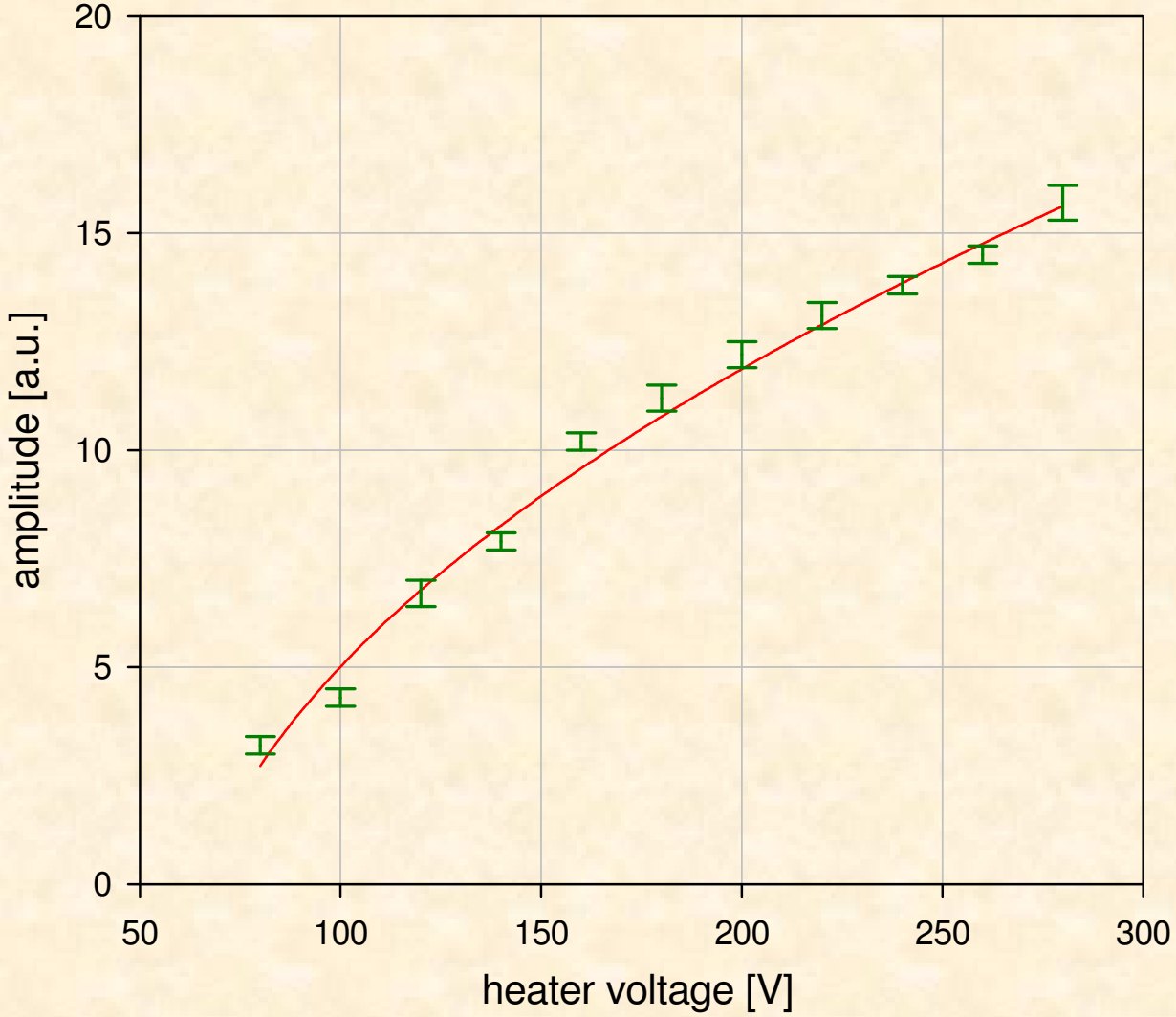
- **Energy conservation:**

$$P_H = \frac{1}{2} Sc\rho\omega^2 A^2 + P_{a.l.} + \Delta P$$

P_H – heater power
S – tube cross section
c – sound velocity
ρ – air density
ω – sound frequency
A – sound amplitude
$P_{a.l.}$ – acoustic losses

- **Acoustic losses:**
  - **End effects**
  - **Viscous losses**
- **Simplest approximation – linear in air velocity**

Sound amplitude cont.



$$A = \Xi + \sqrt{P_{eff} + \Xi^2}$$

Ξ – constant

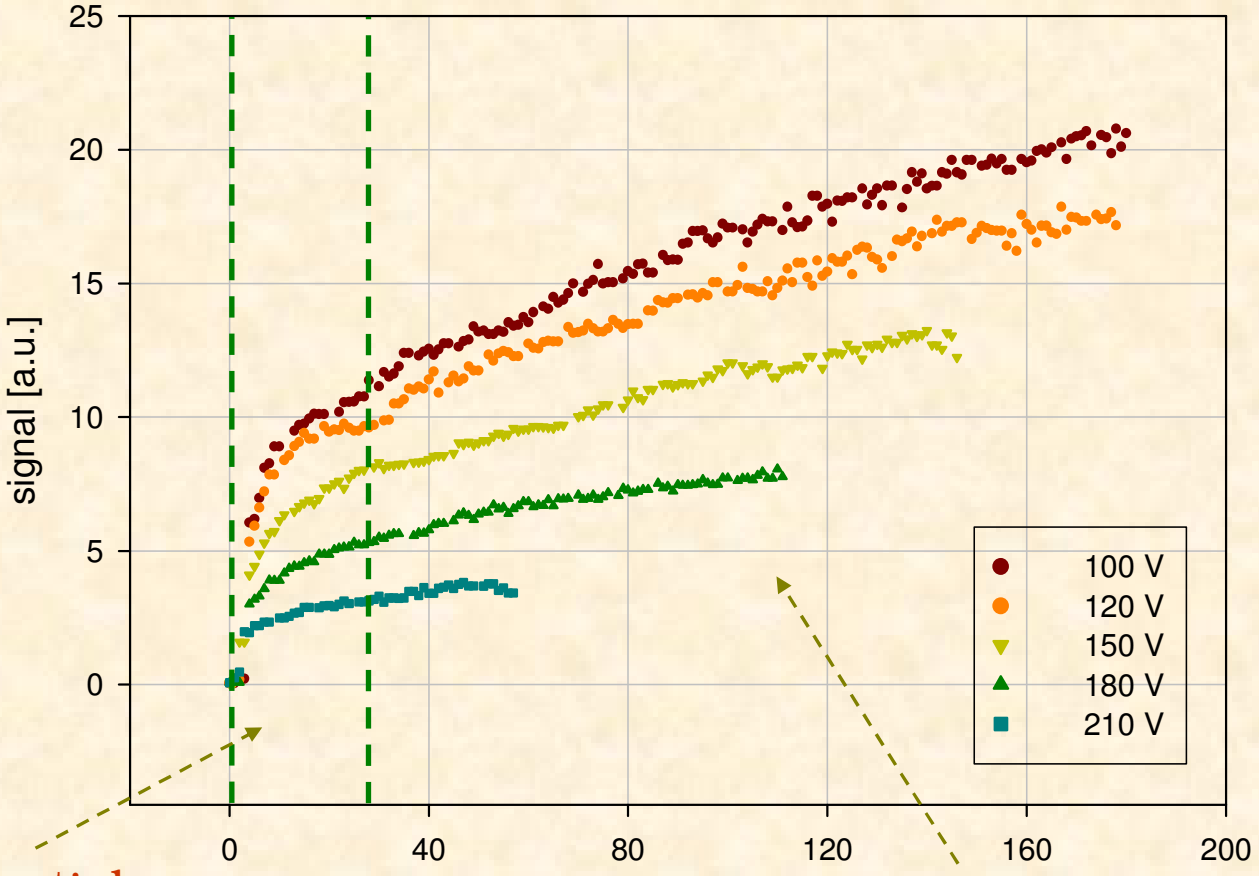
P_{eff} – power spent on wave motion

Sound amplitude cont.

- Amplitude in time – characteristic growth
- Medium – term (~100 s):
- Rise to maximum
    - Resonance with losses
    - Heater temperature growth
- Long – term (~1000 s and more):
    - Tube walls heat conduction – air below heater warmed!
    - Amplitude decrease

Sound amplitude cont.

- **Medium – term behaviour:**

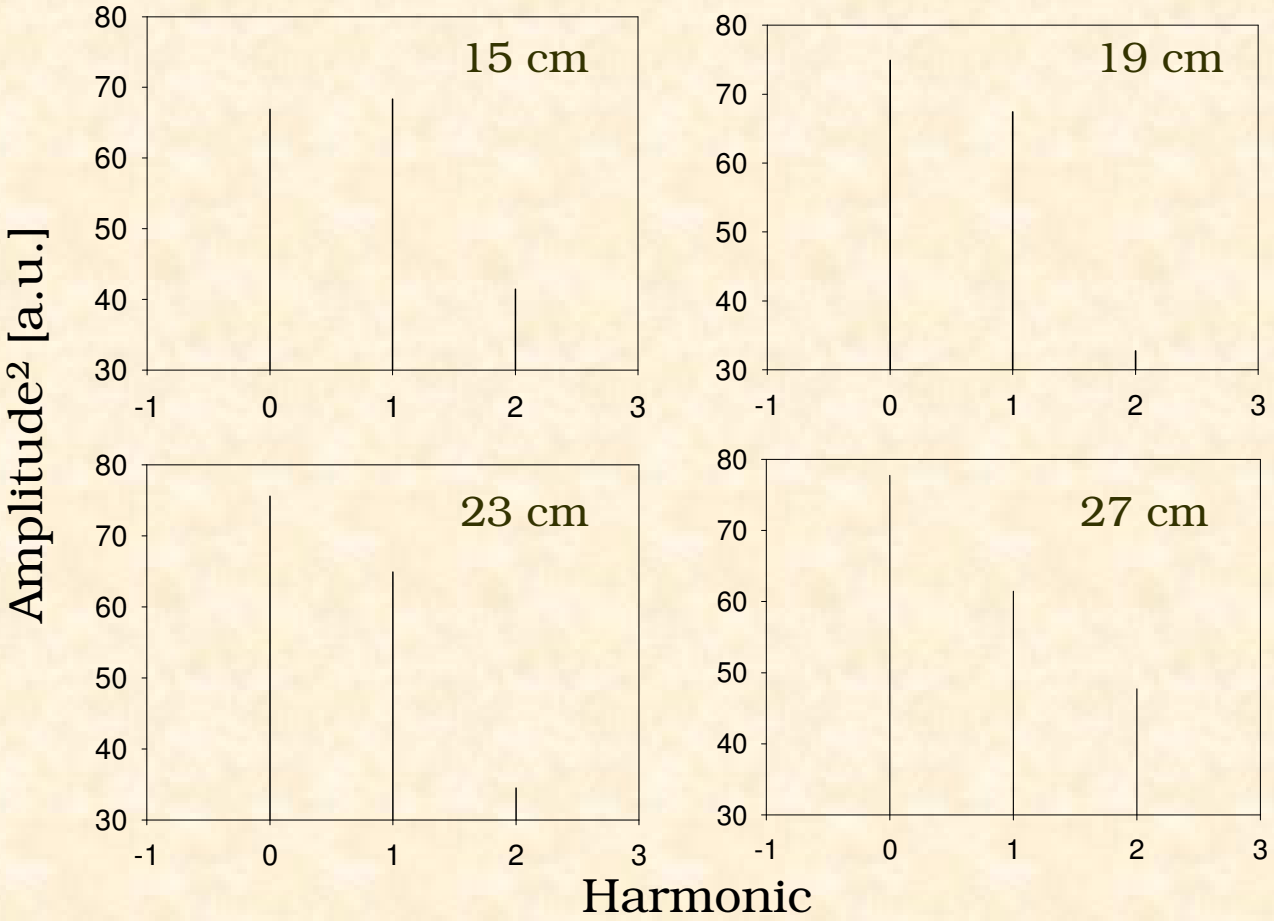


Exponential growth

Growth due to slow heater temperature increase

Spectral characteristics

- **Only first few harmonics present – acoustic losses grow steep with frequency**



Spectral characteristics

⇒ Ground mode and first harmonic dominate as supposed

⇒ Only at small heights – first harmonic stronger

- At some heater location/power – only first harmonic?

  never obtained in our experiments