Problem 3.

Water Jets

The Problem

What can be observed when two water jets collide at different angles?

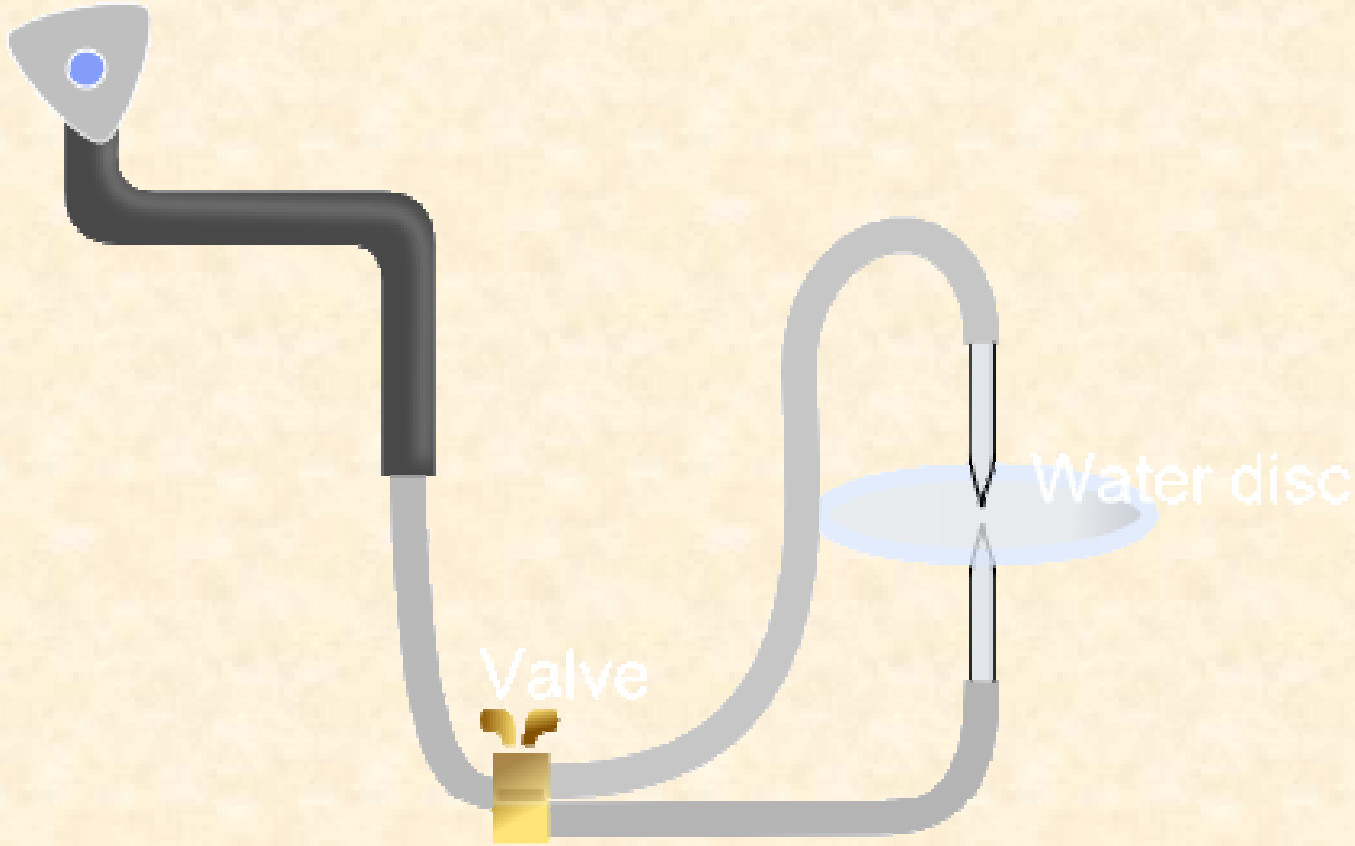
Overview

- Experimental apparatus
- Results
- Simple explanation
- Mathematical solution
- Comparison

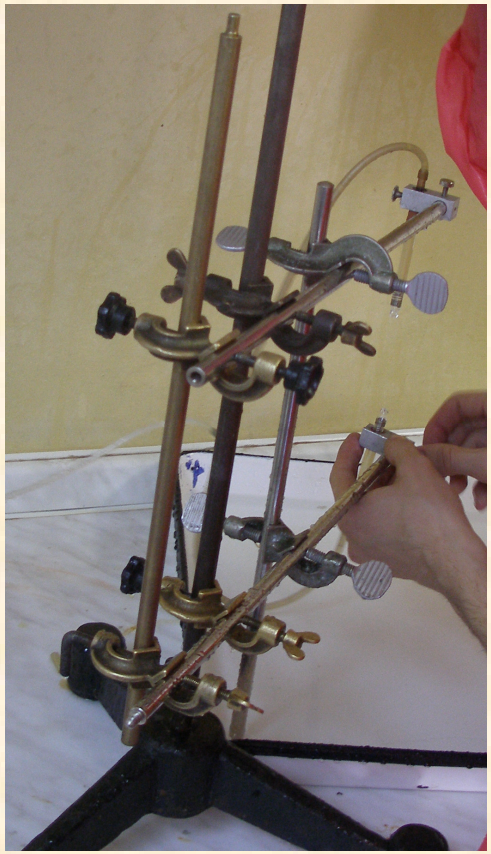
Experimental

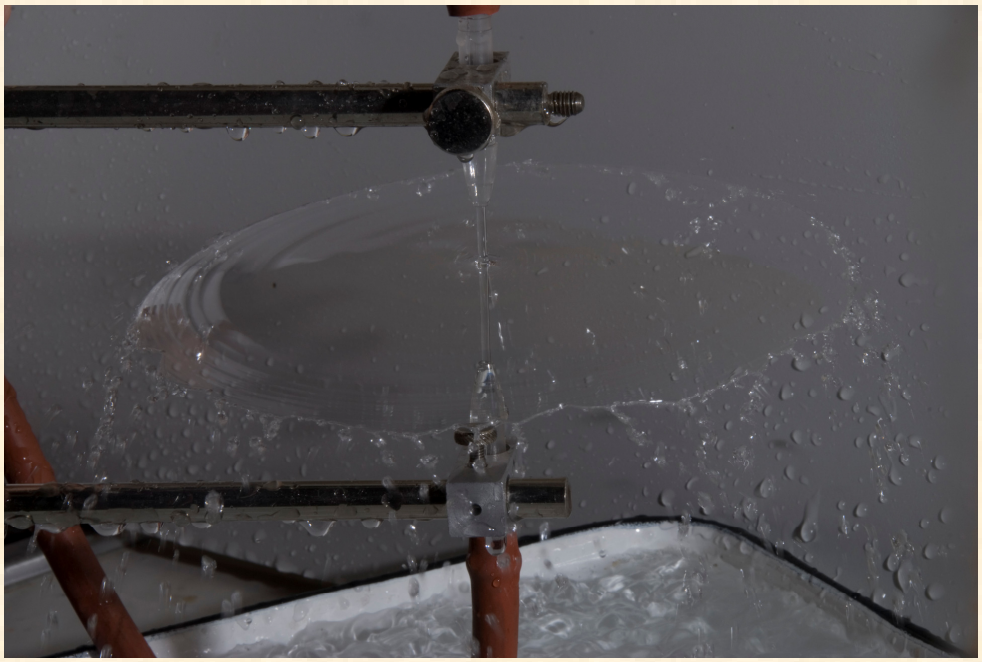
- Two equal jets – fine-tuning valves
- Nozzles with anti – turbulence system
- Jet angle control
- Sheet thickness measurement
- High-power flash photograpy

Apparatus



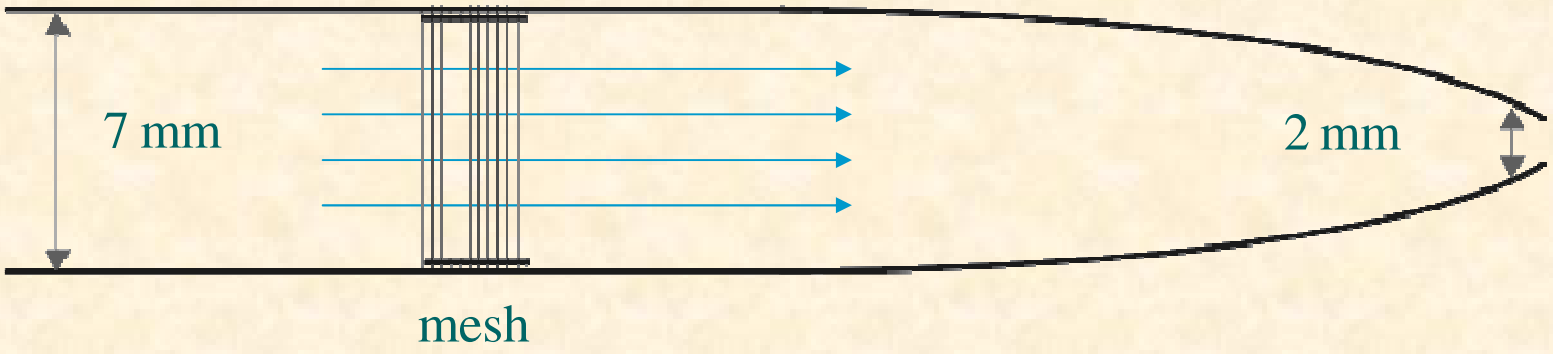
Apparatus cont.





Apparatus cont.

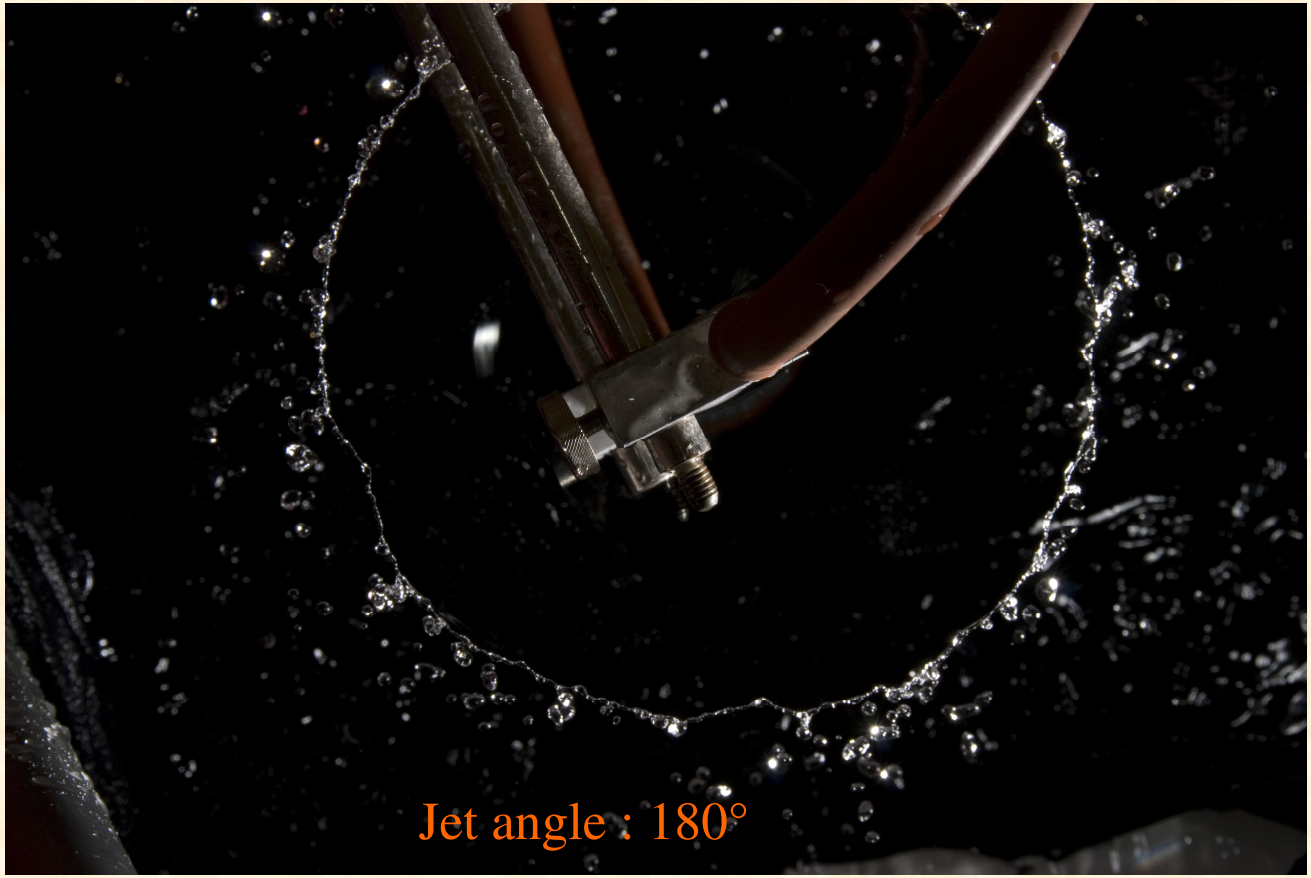
- For smooth jets – turbulence removal
- Streamlined nozzles with mesh:



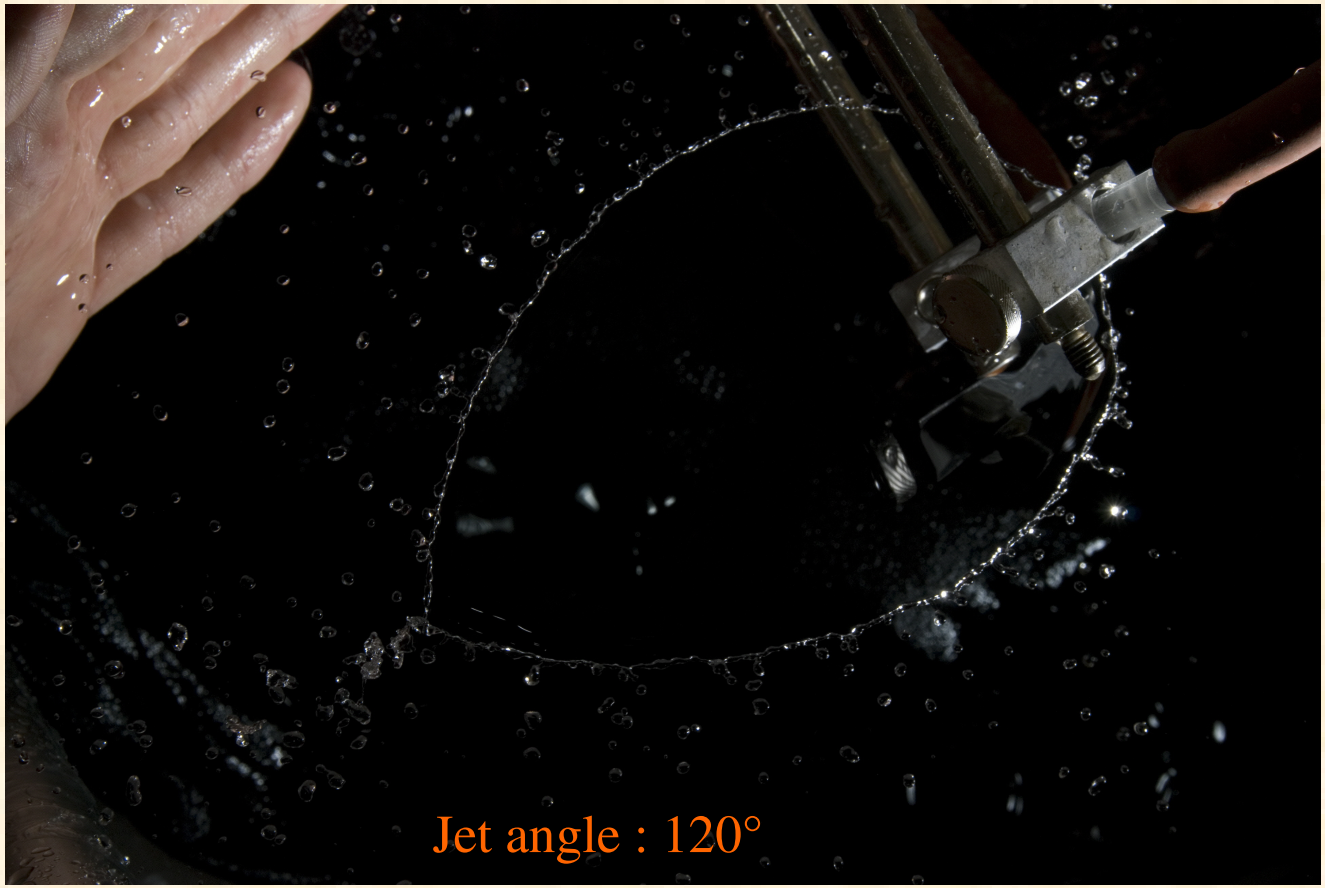
Apparatus cont.

- **Sheet thickness measurement:**

Results



Results



Jet angle : 120°

Results

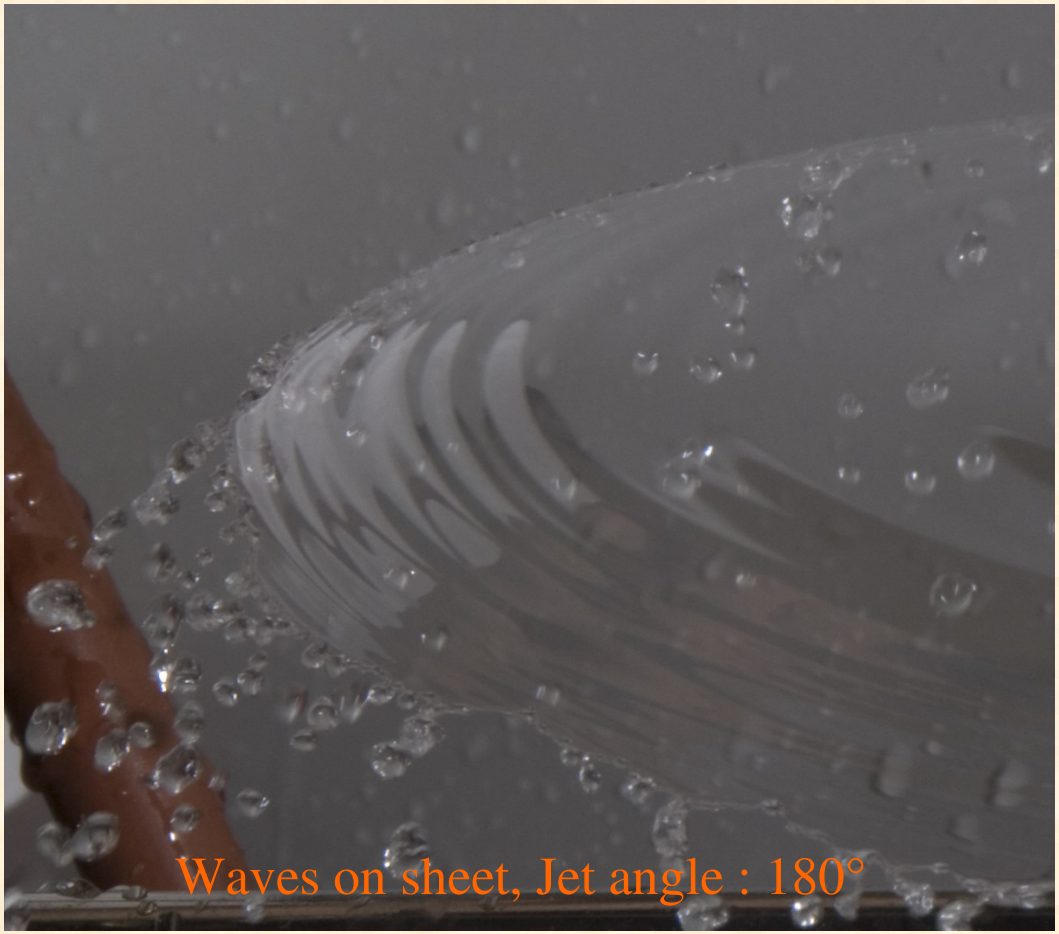


Results



Flag-like instability, Jet angle : 90°

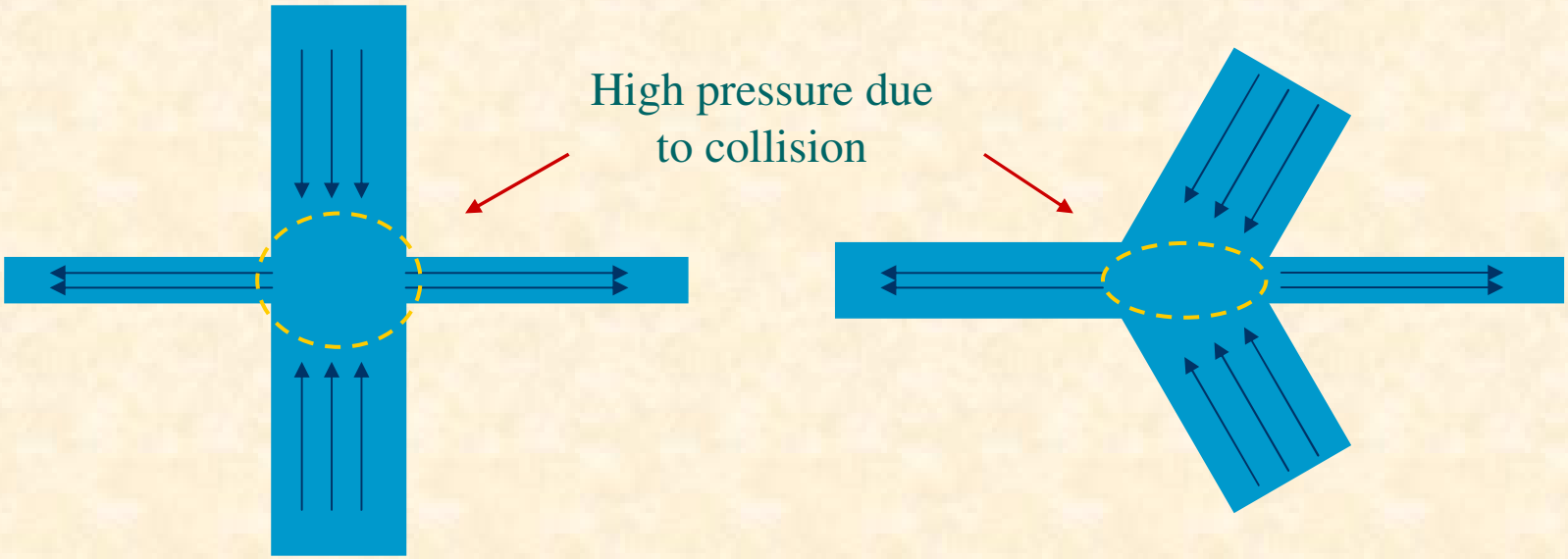
Results



Waves on sheet, Jet angle : 180°

Basic explanation

- In collision zone – elastic fluid collision
- Resultant pressure gives fluid velocity in horizontal plane:



Diagnostic Numbers

- **Reynolds number – ratio of inertia to viscosity:**

$$\mathrm{Re} = \frac{D\rho v}{\eta}$$

D – pipe diameter
ρ – water density
v – water velocity
η – water viscosity

- **In our jet configuration:** $\mathrm{Re} \sim 10^3$

⇒ **possible turbulence**

⇒ **Mesh and profile neccesary**

Diagnostic Numbers

- **Bond number – ratio of gravity to curvature:**

$$Bo = \frac{D}{\sqrt{\sigma / \rho g}}$$

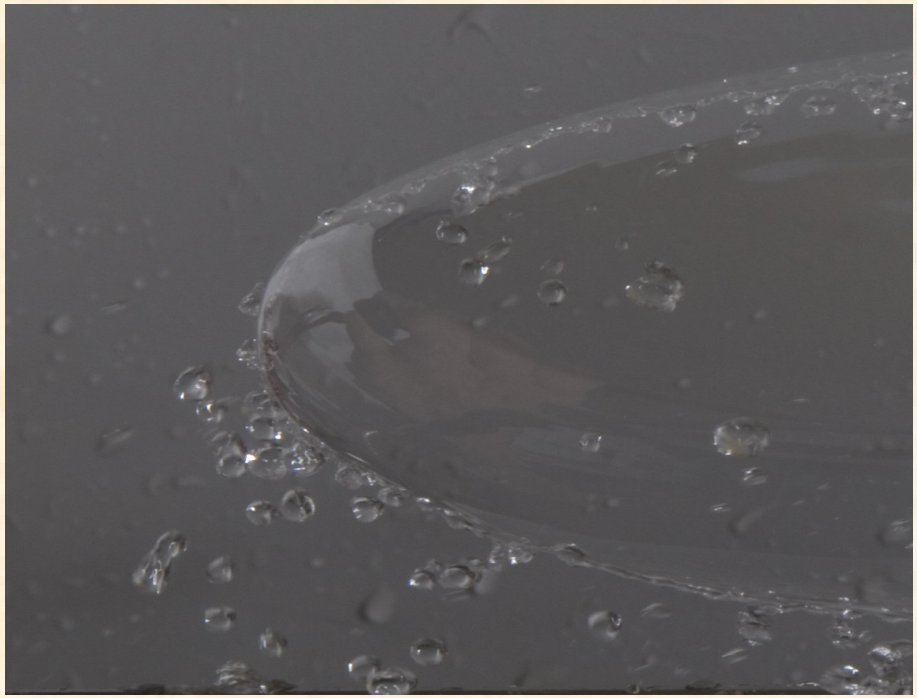
D – pipe diameter
ρ – water density
g – acceleration of gravity
σ – water surface tension

- **Measures influence of gravity**
- **In our jet conditions:** $Bo < 1$

  $\Rightarrow$ **gravity influence small**

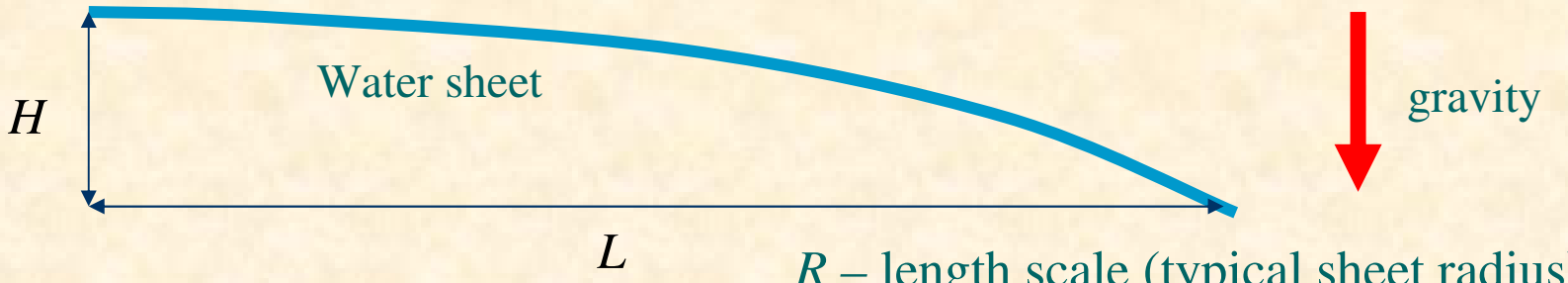
Diagnostic Numbers

- In sheet – gravity bends the rim for smaller velocities:



Diagnostic Numbers

- **Importance of gravity measure:**



$$Gr = \frac{L}{H} = \frac{2v}{g\tau} = \frac{2v^2}{gR}$$

R – length scale (typical sheet radius)
v – water velocity scale
g – acceleration of gravity
τ – time scale

- **In our conditions:** $Gr \sim 10$

$\Rightarrow$ **gravity unimportant**

Diagnostic Numbers

- **Weber number – ratio of inertia to surface energy:**

$$We = \frac{\rho h v^2}{2\sigma}$$

h – length scale (jet diameter, sheet thickness)

ρ – water density

v – velocity scale

σ – water surface tension

- **For jet:** $We \sim 100$

⇒ **Surface tension not important**

- **For sheet:** $We \sim 1$

⇒ **important and complicated effects!**

Sheet parameters

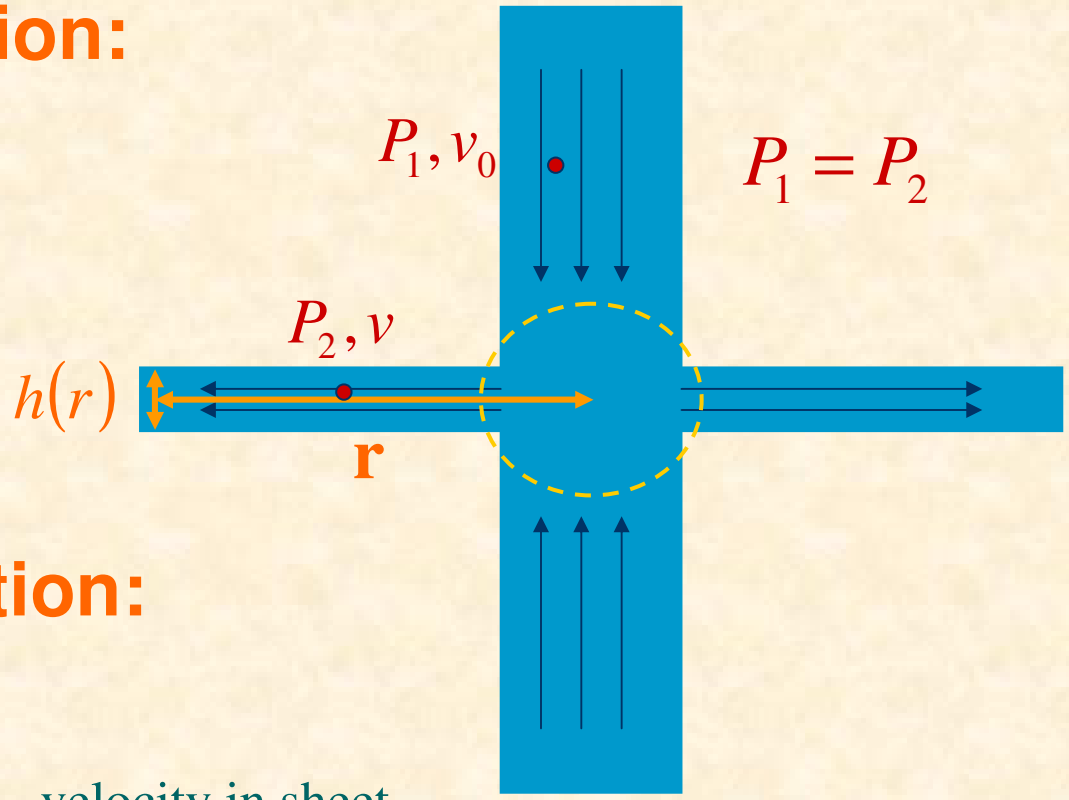
- Parameters: thickness and velocity distribution in sheet
- Subcases:
  - Frontal collision – simple
  - Jets under different angles
- Conservation laws:
  - Mass
  - Energy
  - Momentum

Frontal collision

- **Bernoulli equation:**

$$\frac{1}{2}\rho v_0^2 = \frac{1}{2}\rho v^2$$

v_0 – jet velocity
ρ – water density
v – velocity in sheet



- **Mass conservation:**

$$2r\pi h(r)v = Q$$

Q – volume flow rate
r – radial position
v – velocity in sheet
h – sheer thickness

Frontal collision

⇒ **Velocity in sheet constant**

$$v = v_0$$

v_0 – jet velocity
v – velocity in sheet

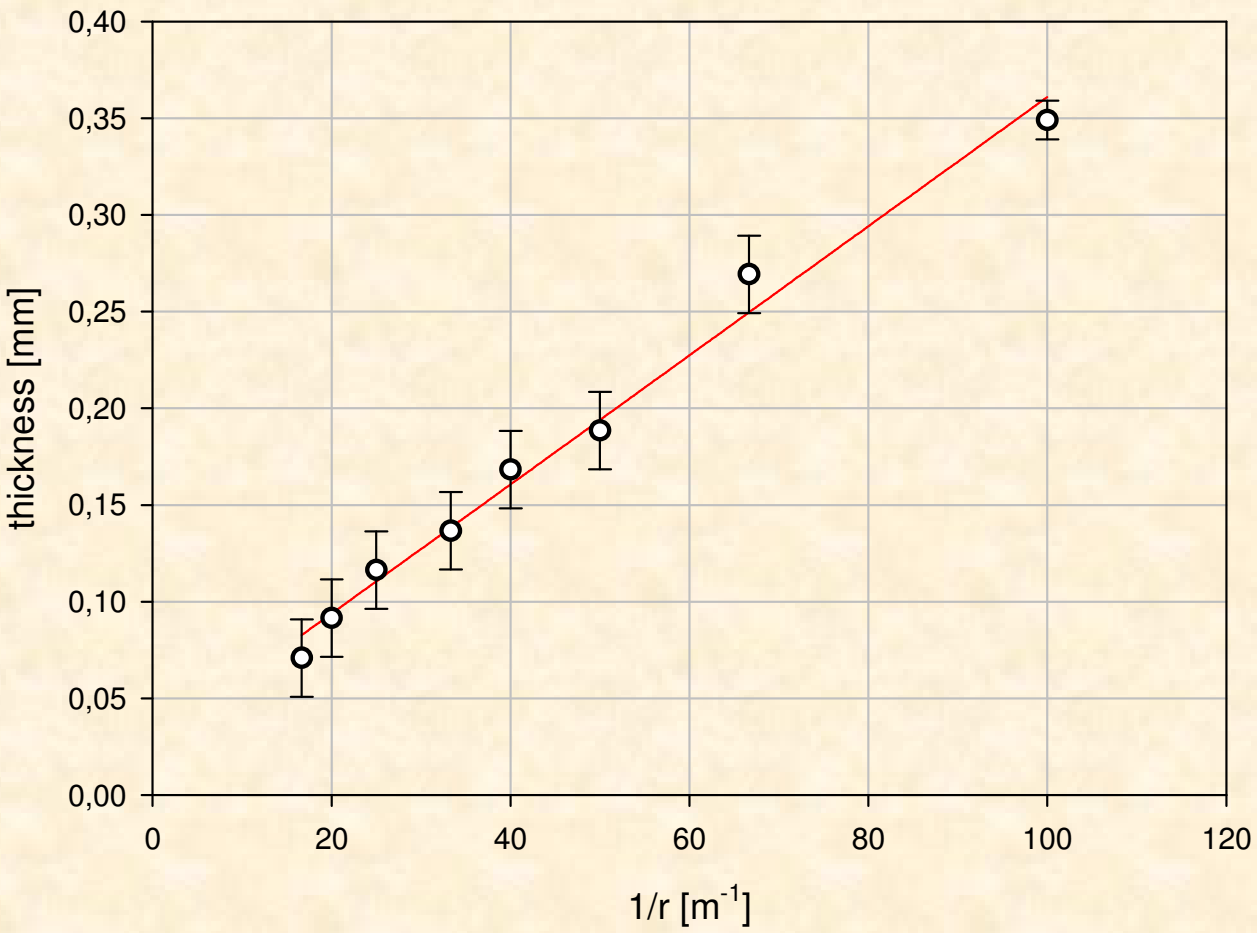
⇒ **sheet thickness radially symmetric:**

$$h(r) = \frac{Q}{2r\pi v_0}$$

Q – volume flow rate
r – radial position
v_0 – velocity

- **Thickness dependence on position experimentally tested**

Frontal collision



Volume flow rate:
11 ± 0.1 mL/s
Nozzle radius:
1.0 mm
Test tube diameter:
10.0 mm

Theoretical slope:
$5 \cdot 10^{-7}$ m^2
Experimental slope:
$(3.5 \pm 0.2) \cdot 10^{-7}$ m^2

Nonfrontal collision

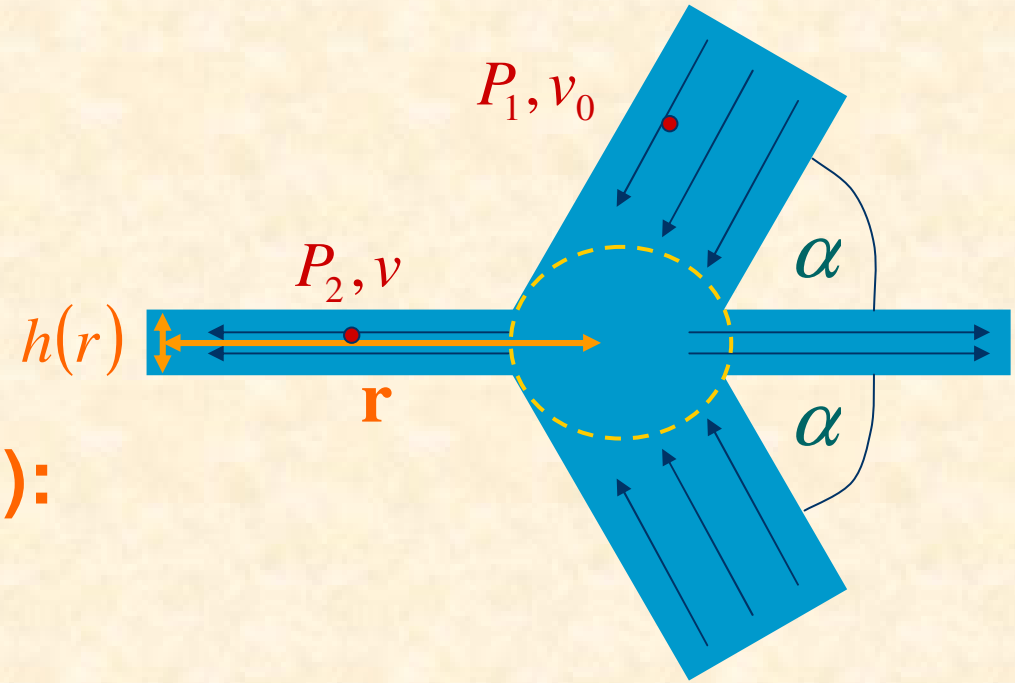
- **From Bernoulli equation:**

$$v = v_0$$

v_0 – jet velocity
v – velocity in sheet

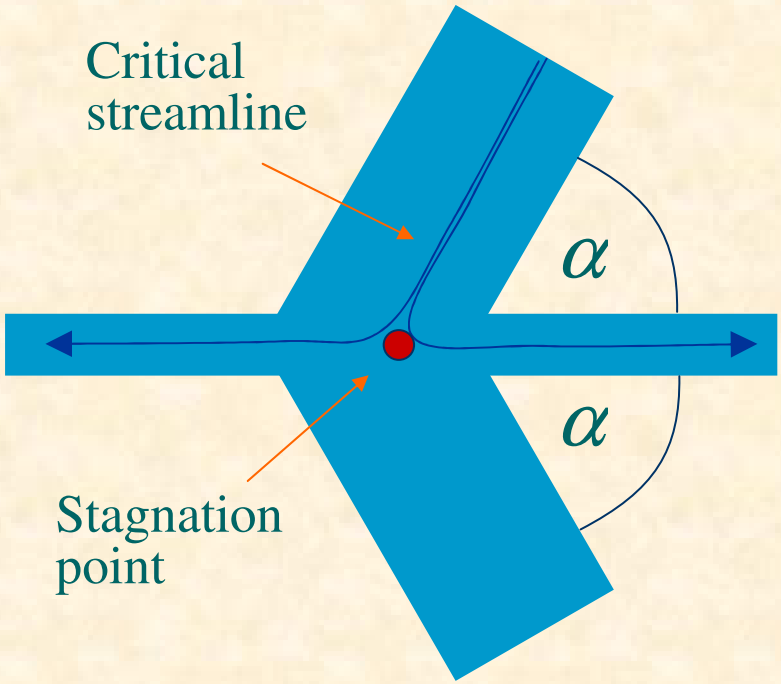
- **Momentum conservation (paralell direction):**

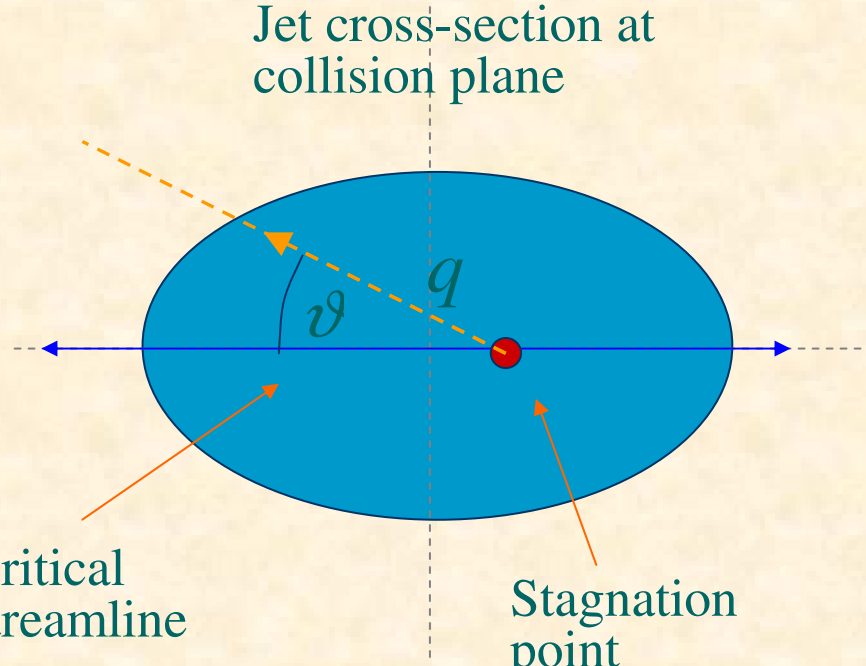
$$\int_0^{2\pi} v_0 h(\alpha, \vartheta) \cos\vartheta d\vartheta =$$



Nonfrontal collision

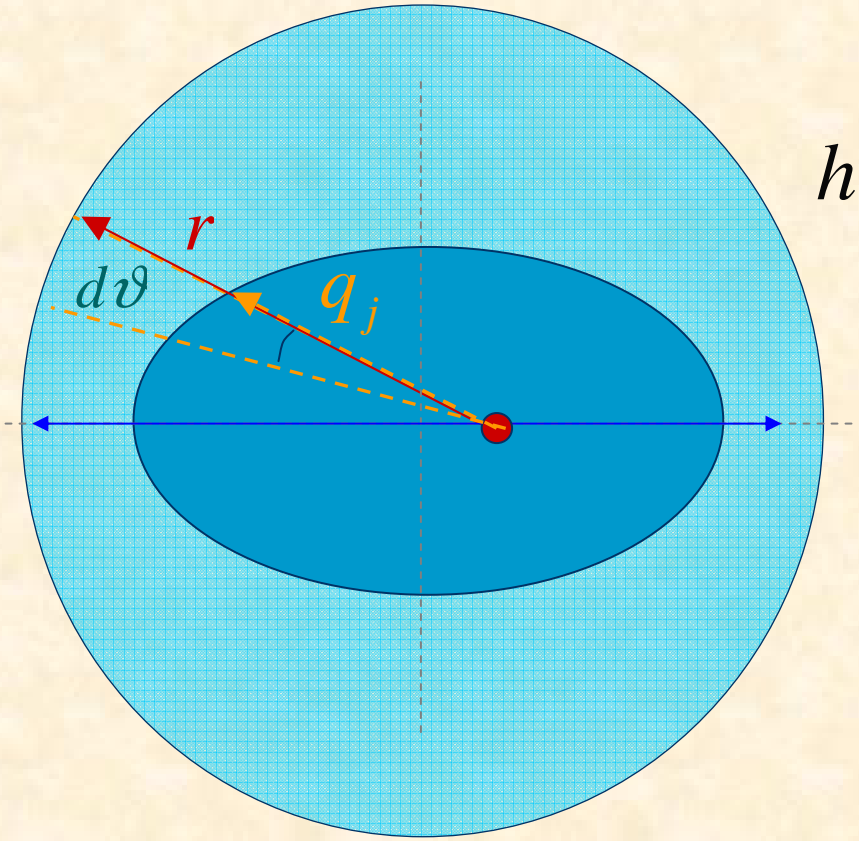
- **'Critical' streamline:**





Nonfrontal collision

- **Mass conservation:**



$$h(\vartheta,\alpha)rd\vartheta = 2\int_0^{q_j} \sin\alpha q dq d\vartheta$$

h – sheet thickness
r – radial position
θ – polar angle
α – jet inclination
q – coordinate in jet
q_j – jet boundary

Nonfrontal collision

- Height distribution follows from conservation laws
- Numerical integration
- Direct measurement difficult
- Confirmation – sheet shape, geometrical properties

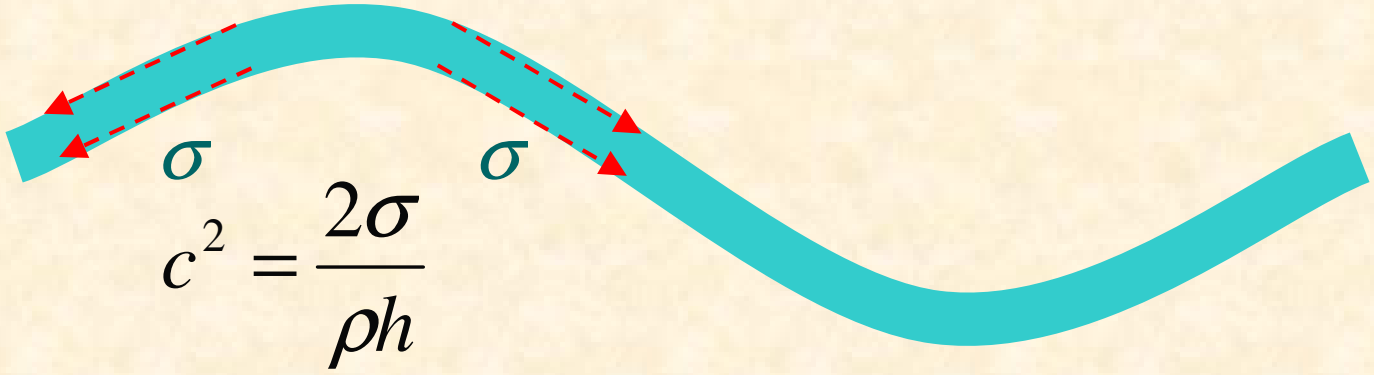
Sheet disintegration

- Disintegration into droplets – surface tension effect
- Droplets – lower surface energy
- Rayleigh - Plateau instability – surface tension waves amplification
- Disintegration front – standing wave
- Wave velocity:

Sheet disintegration

- **Surface – analogous to stretched string**
- **Tension:** 2σ
- **Mass per unit length:** ρh

h – sheet thickness
ρ – density
σ – surface tension
c – wave velocity



$$c^2 = \frac{2\sigma}{\rho h}$$

- **Velocity independent of wavelength!**

Sheet disintegration

- **Sheet boundary – stationary wave**
- **Lines of constant phase:**

$$\sin\psi = \frac{c}{v_0} = \frac{1}{\sqrt{We}}$$

ψ – angle between line and radiusvector

v_0 – fluid velocity

c – wave velocity

We – Weber number

- **Shape calculation possible!**
- **Sheet rim – wavefront must exist for all polar angles**

Sheet disintegration

- For frontal collision – circle
- Furthest steady wavefront:

$$We = 1$$

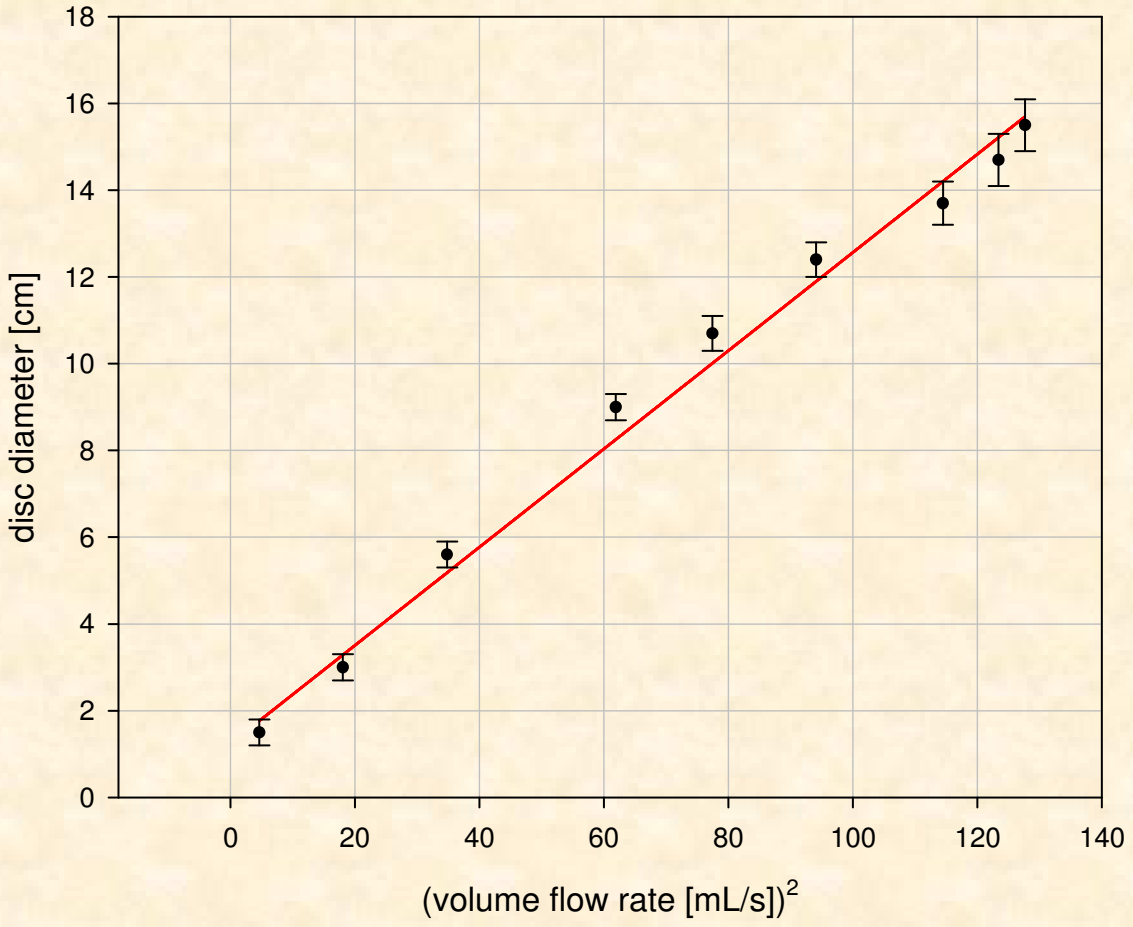
$\Rightarrow$ disc radius must follow

$$R = \frac{\rho}{4\pi\sigma S} Q^2$$

ρ – fluid density
σ – surface tension
S – jet surface area
Q – volume flow rate
We – Weber number

- In real conditions – critical We < 1 :

Sheet disintegration



Water surface tension:

0.073 N/m

Nozzle radius:

1.0 mm

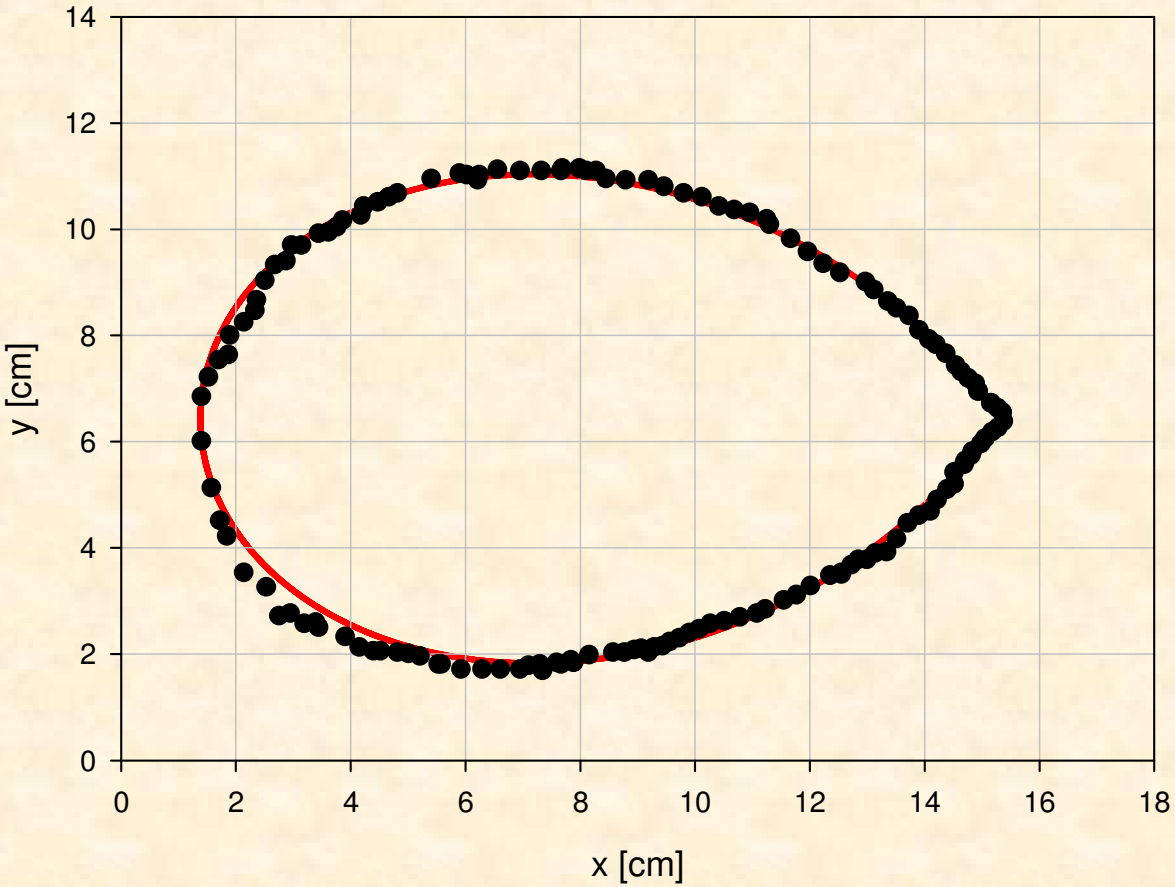
Measured critical We number:

0.83 ± 0.03

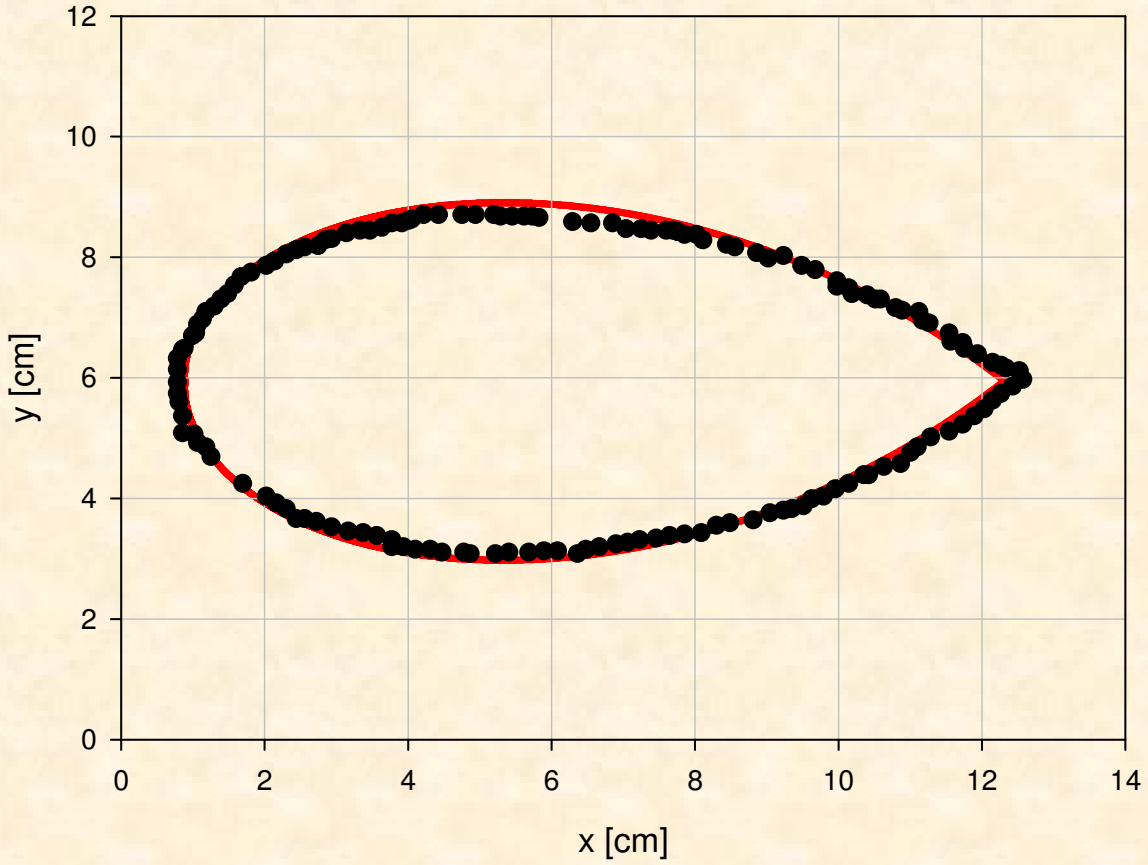
Sheet disintegration

- For nonfrontal collision – oblique shape
- Numerical calculation of critical wavefront neccesary
- Results:
    - Sheet shape
    - Ratio of large and small axis of the shape

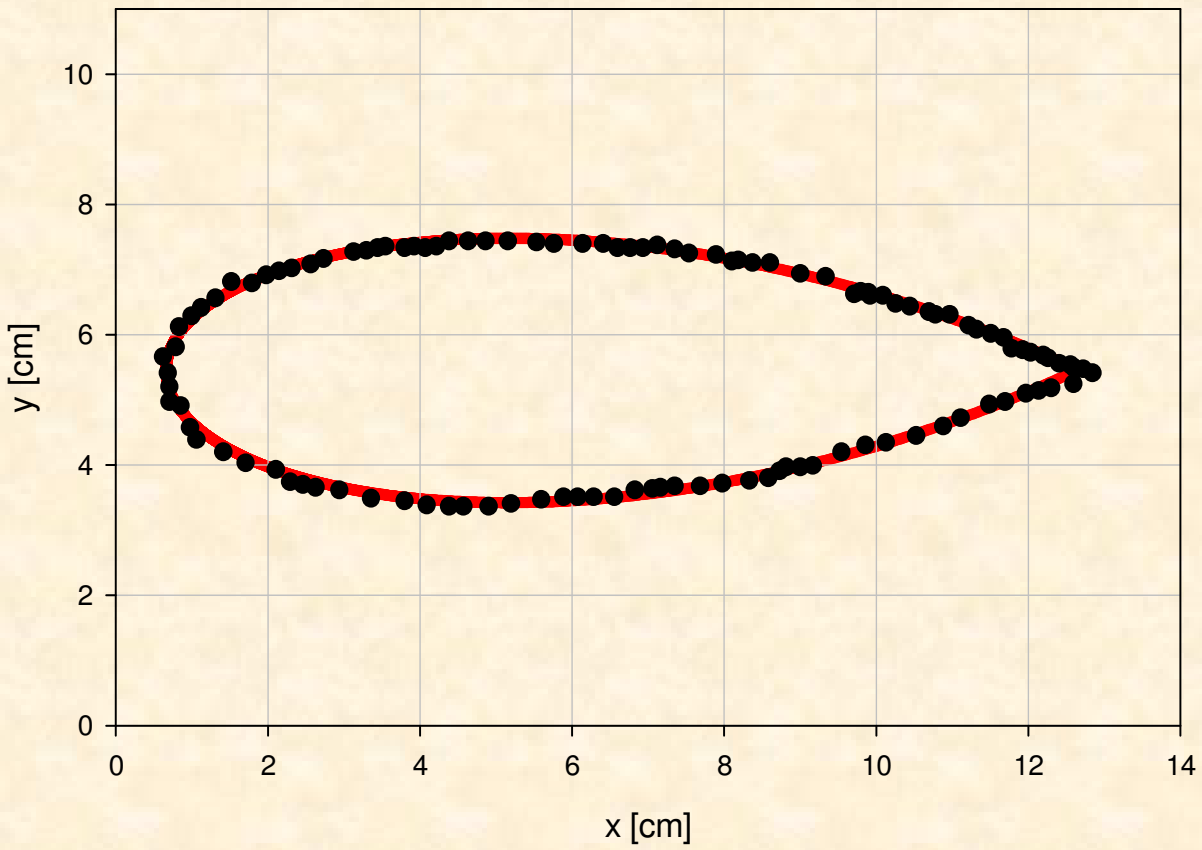
Sheet disintegration



Sheet disintegration



Sheet disintegration



Sheet disintegration

