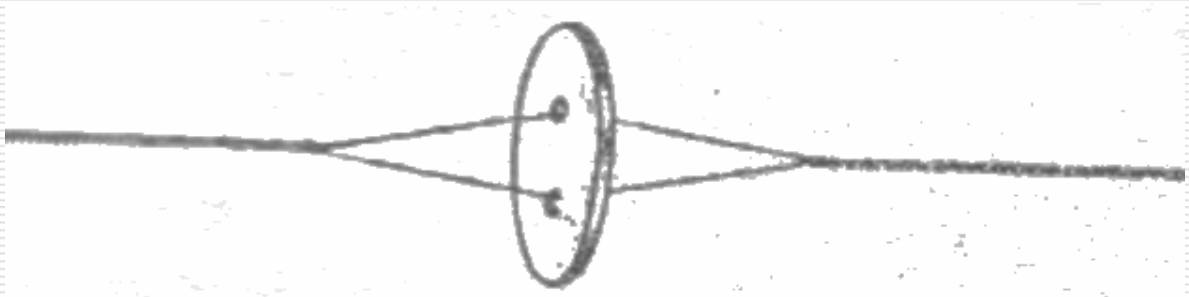
Problem 4.

Spring Thread

The Problem

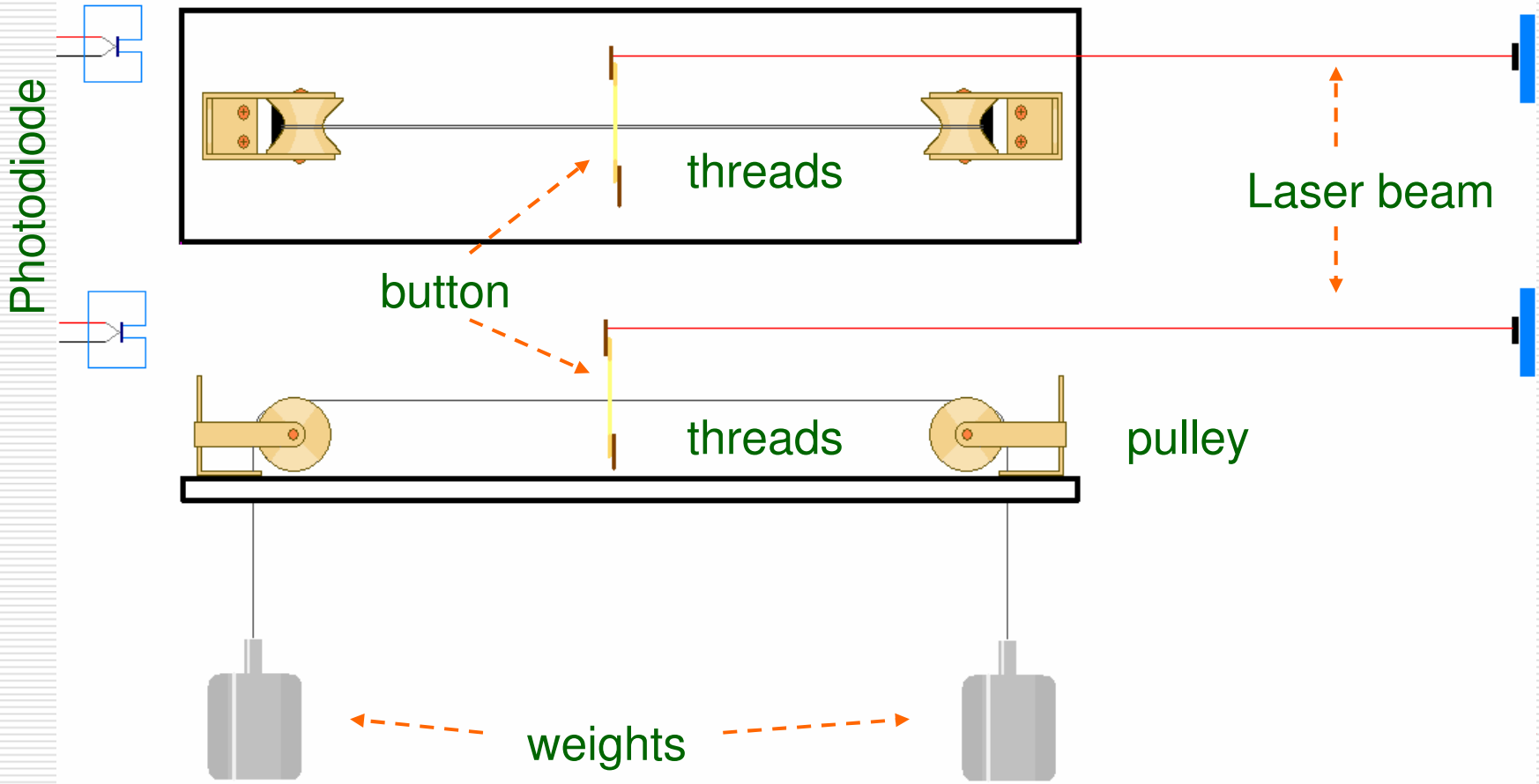
Pull a thread through the button holes as shown in the picture. The button can be put into rotating motion by pulling the thread. One can feel some elasticity of the thread. Explain the elastic properties of such a system.



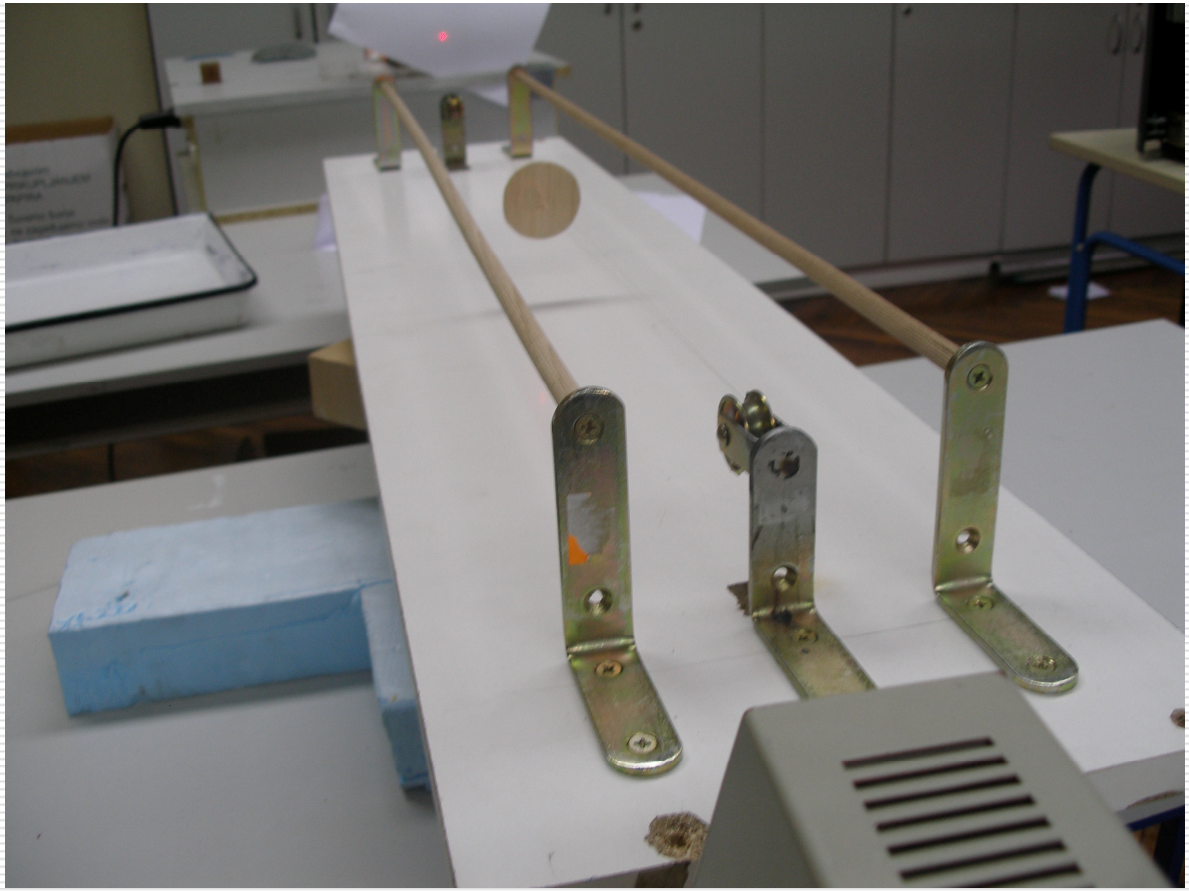
Overview

- Apparatus
- Measurements
- Simple theory
- Comparison
- Conclusion

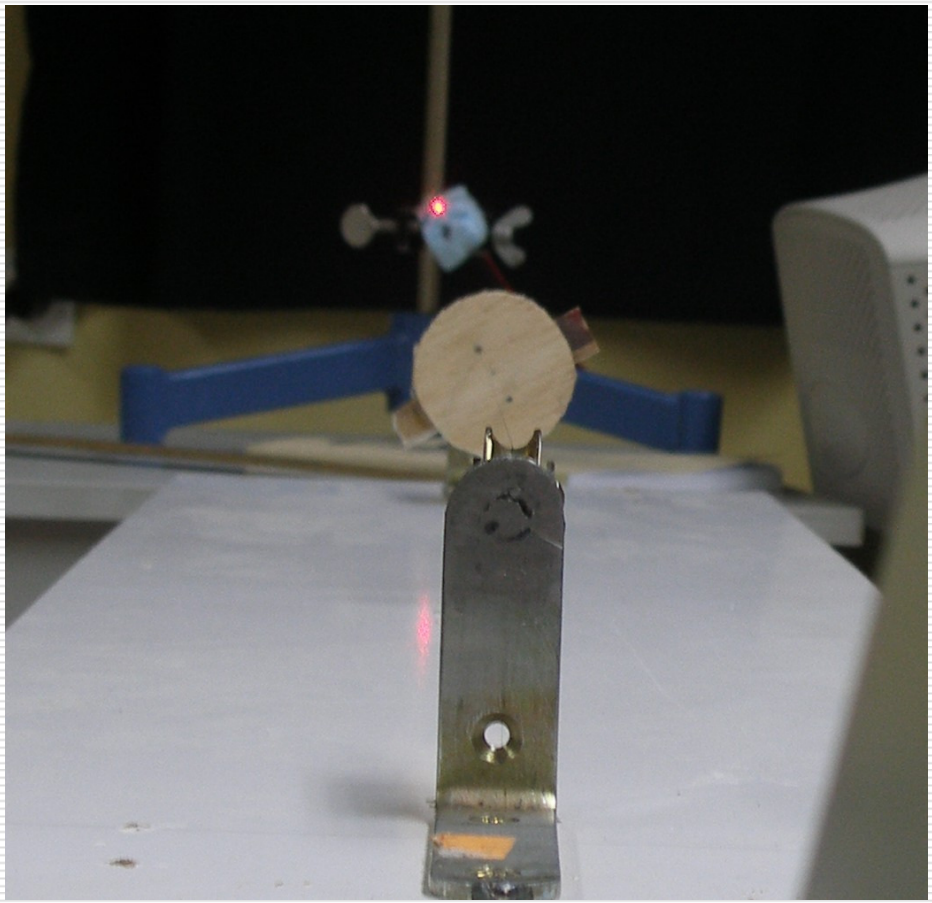
Setup


Photodiode
threads
Laser beam
button
threads
pulley
weights

Setup



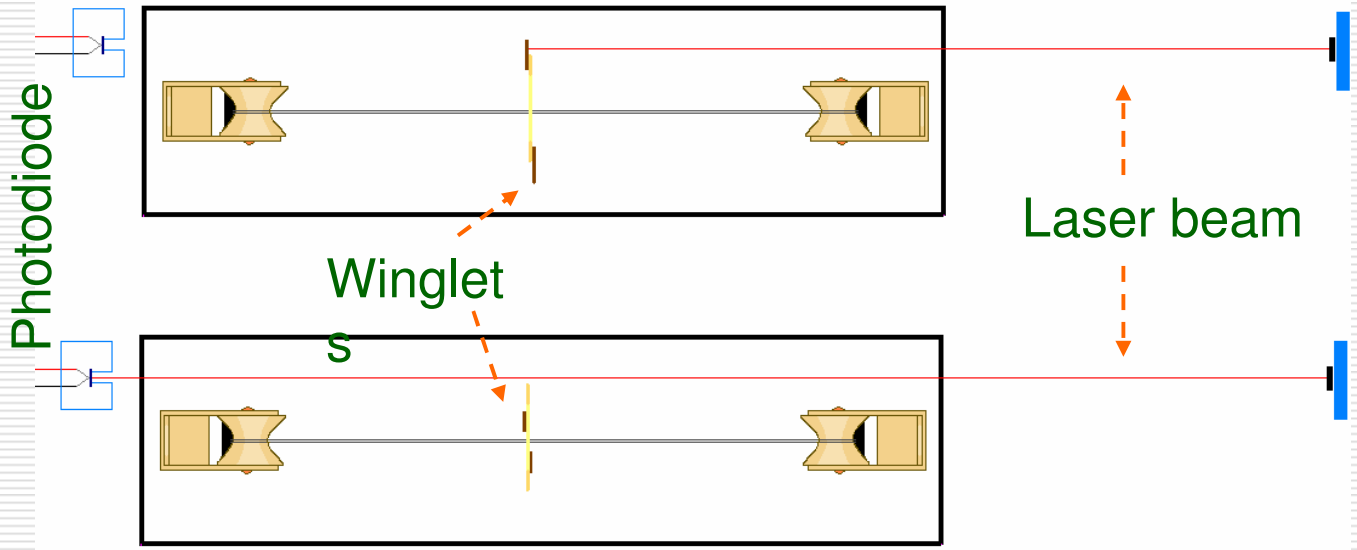
Setup



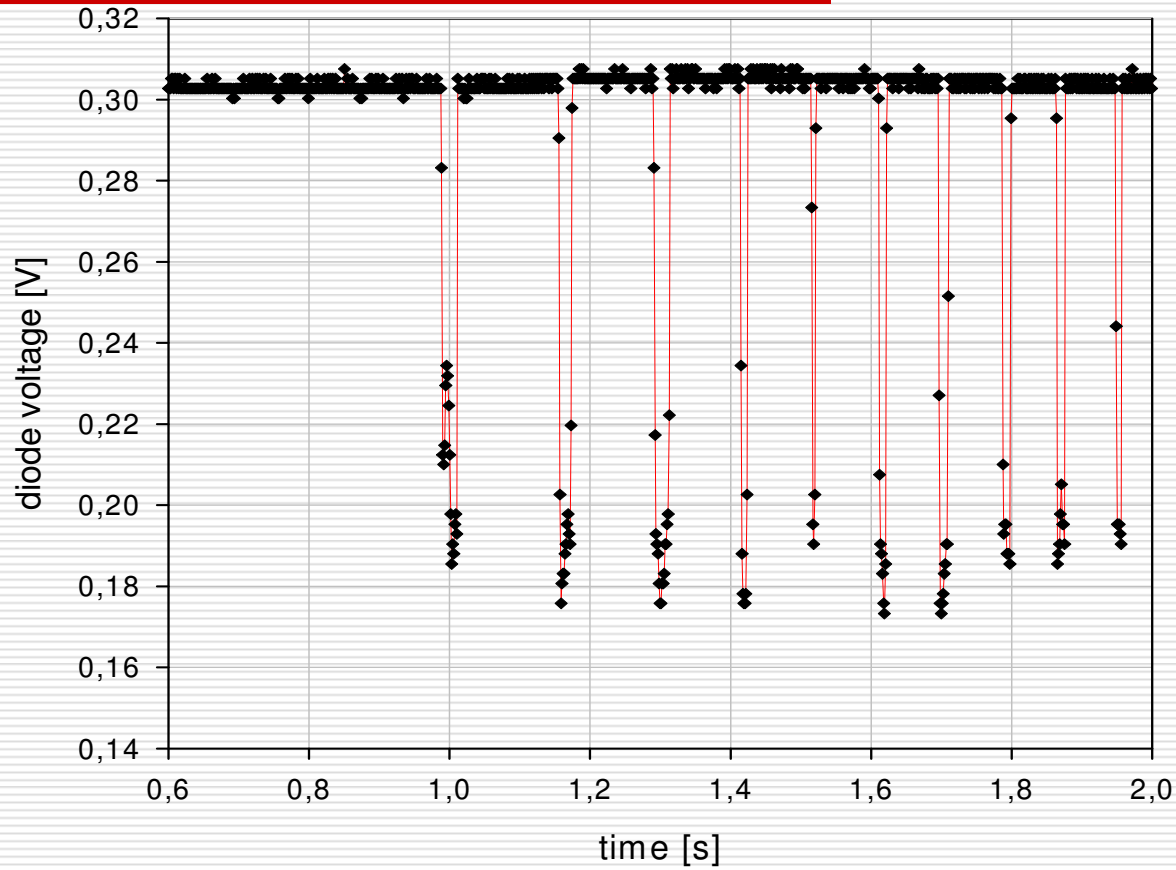


Setup

- Button angular velocity measurement – laser beam interrupt:



Setup



Signal from diode – digital reading by ADC

Measurements

- Measured:
  - Angular velocity in time
  - Period of button oscillation
- Parameters changed:
  - Button radius
  - Thread radius
  - Weight mass
  - Number of thread windings

Measurements

- Buttons used:

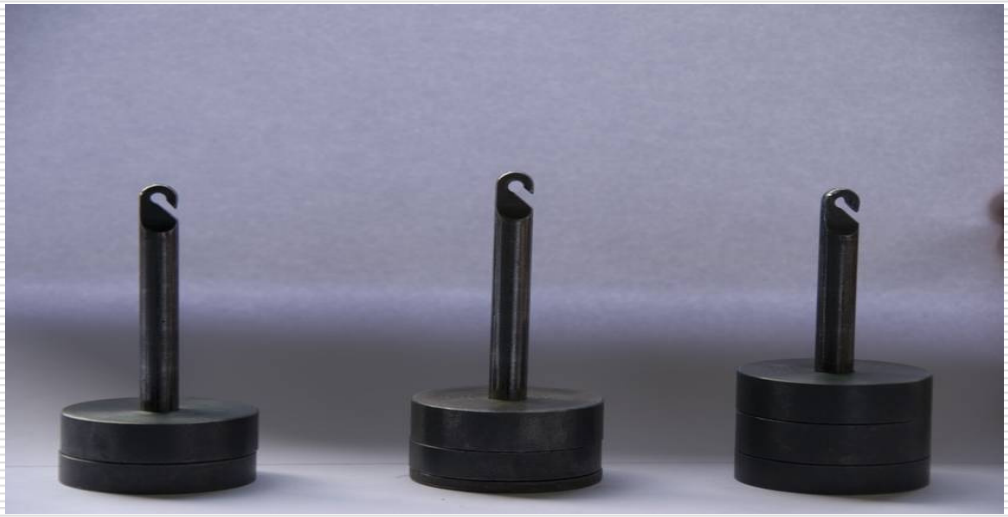


Radii:

1,5 cm
1,8 cm
2,5 cm
3,0 cm
3,4 cm
4,0 cm
4,5 cm

Measurements

- Weights: 200 – 450 g



- Threads:

Flax threads

Diameters: 0.1 – 0.6 mm

Explanation

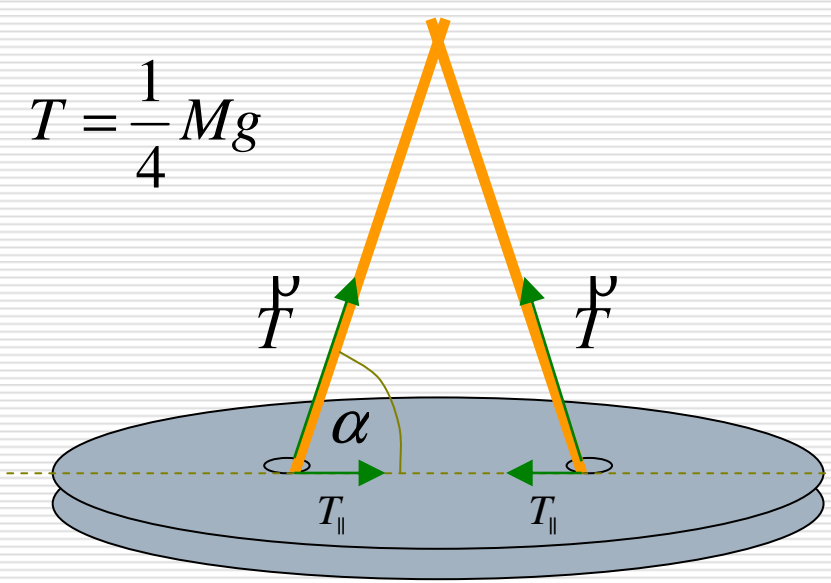
- Elastic properties – periodic motion of weights
- Analogy to spring coil
- ‘Elastic constant’ – due to spinning button
- Differences to spring:
    - Equilibrium position - lowest position
    - Much larger damping

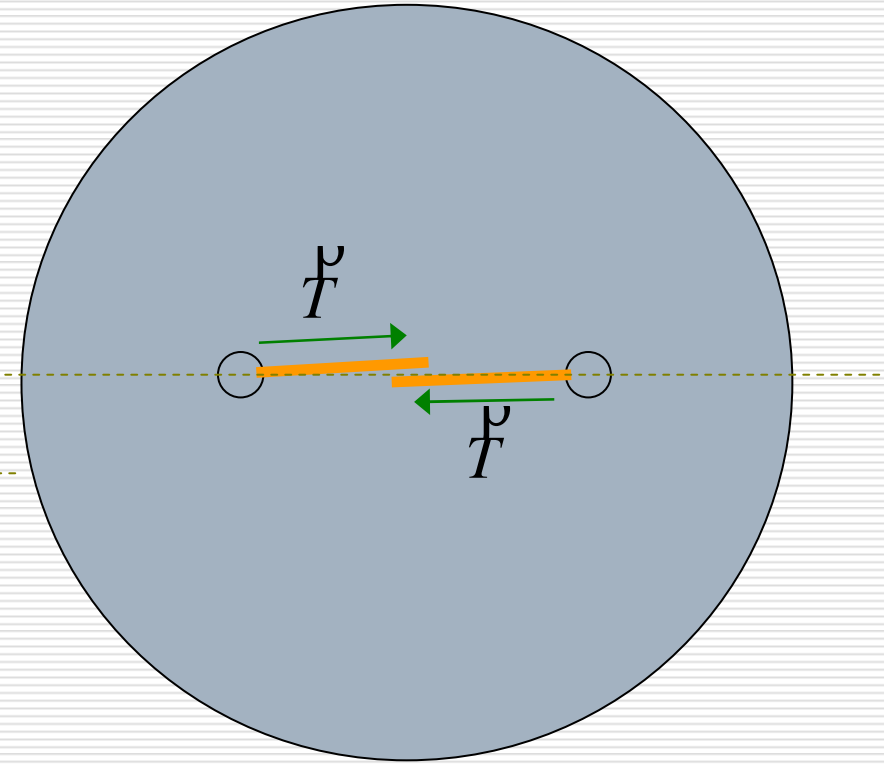
Explanation

- Theoretical goal: oscillation frequency in dependence on parameters
- Simple model – no damping!
- Overview:
    - Torque on button
    - Button equation of motion
    - Frequency calculation

Torque on button

- Torque caused by finite string radius:





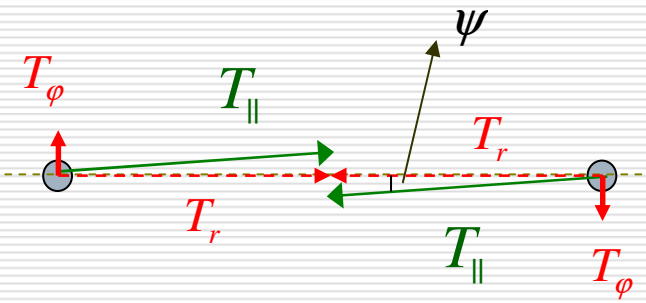
T – tension

α – angle between thread and button

M – weight mass

Torque on button

- Torque caused by finite string radius:



- The component T_φ causes torque!
- For one string:

$$T_\varphi = T\cos\alpha\sin\psi$$

T – tension

ψ – angle between tension and axis

Torque on button

- For small thread radii – small angle:

$$T_\varphi \cong T\psi\cos\alpha = T\frac{2r}{D}\cos\alpha$$

T – tension

ψ – angle between tension and axis

α – angle between thread and axis

r – thread radius

D – distrance between holes

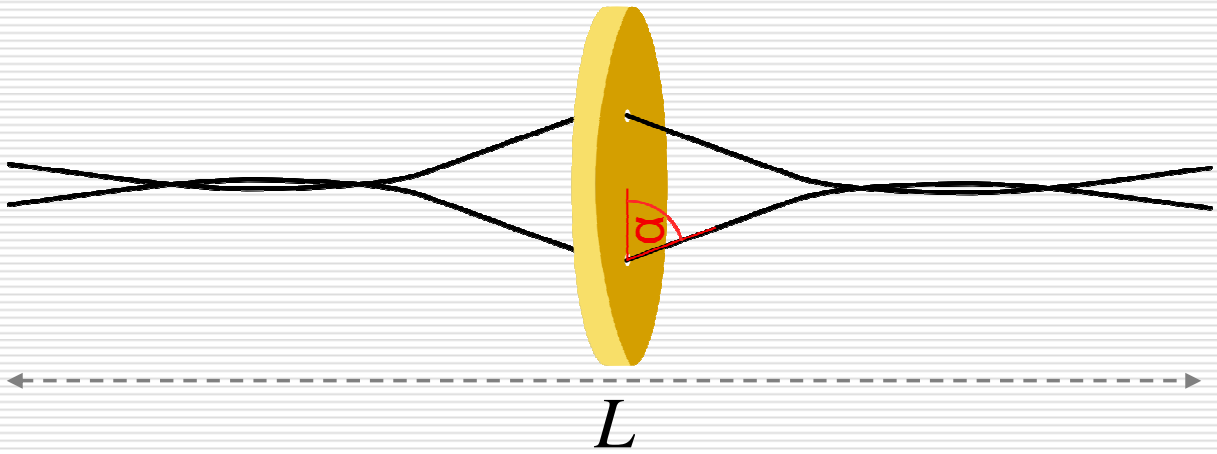
- Angle α depends on the number of turns of the thread

$\Rightarrow$ Torque depends on button turning angle

$\Rightarrow$ oscillations occur

Torque on button

- Calculation of α – thread shape
- Thread – double helix with N turns:



- From parametric helix equations:

Torque on button

$$L = L_0 \sqrt{1 - \left(\frac{r}{L_0}\right)^2 \phi^2}$$

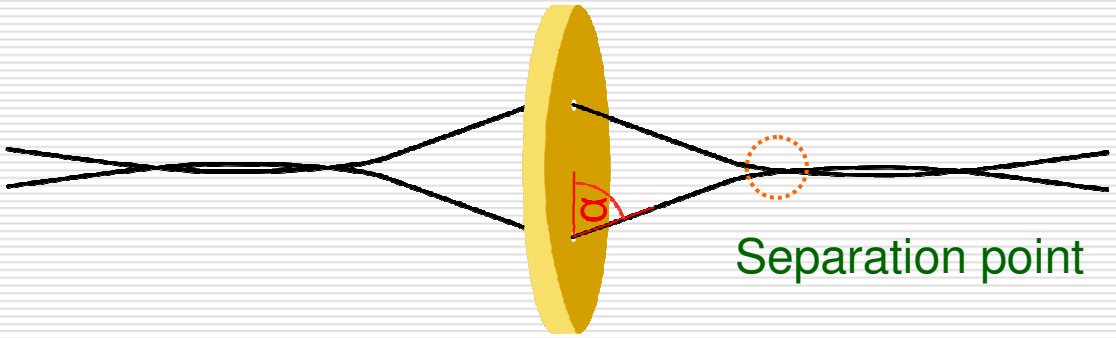
L – effective length

L_0 – thread length

ϕ – turning angle of button

r – thread radius

- Angle α can be found from tangent to helix in the separation point:



$$\Rightarrow \cos\alpha = \frac{r}{L_0}\phi$$

Button equation of motion

- Now that we know torque:

$$I\frac{d^2\phi}{dt^2} = -8\frac{Tr^2}{L_0}\phi - \tau_{fr}$$

I – button moment of inertia

L_0 – thread length

ϕ – turning angle of button

r – thread radius

T – tension

τ_{fr} – friction torque

- Neglecting frictional influence, frequency can be estimated:

$$\nu^2 \approx 2\frac{Tr^2}{\pi^2 IL_0} = \frac{Mgr^2}{4\pi^2 IL_0}$$

Frequency

- Button is a disc:

$$I \approx \frac{1}{2}\rho\pi HR^4$$

I – button moment of inertia

ρ – button density

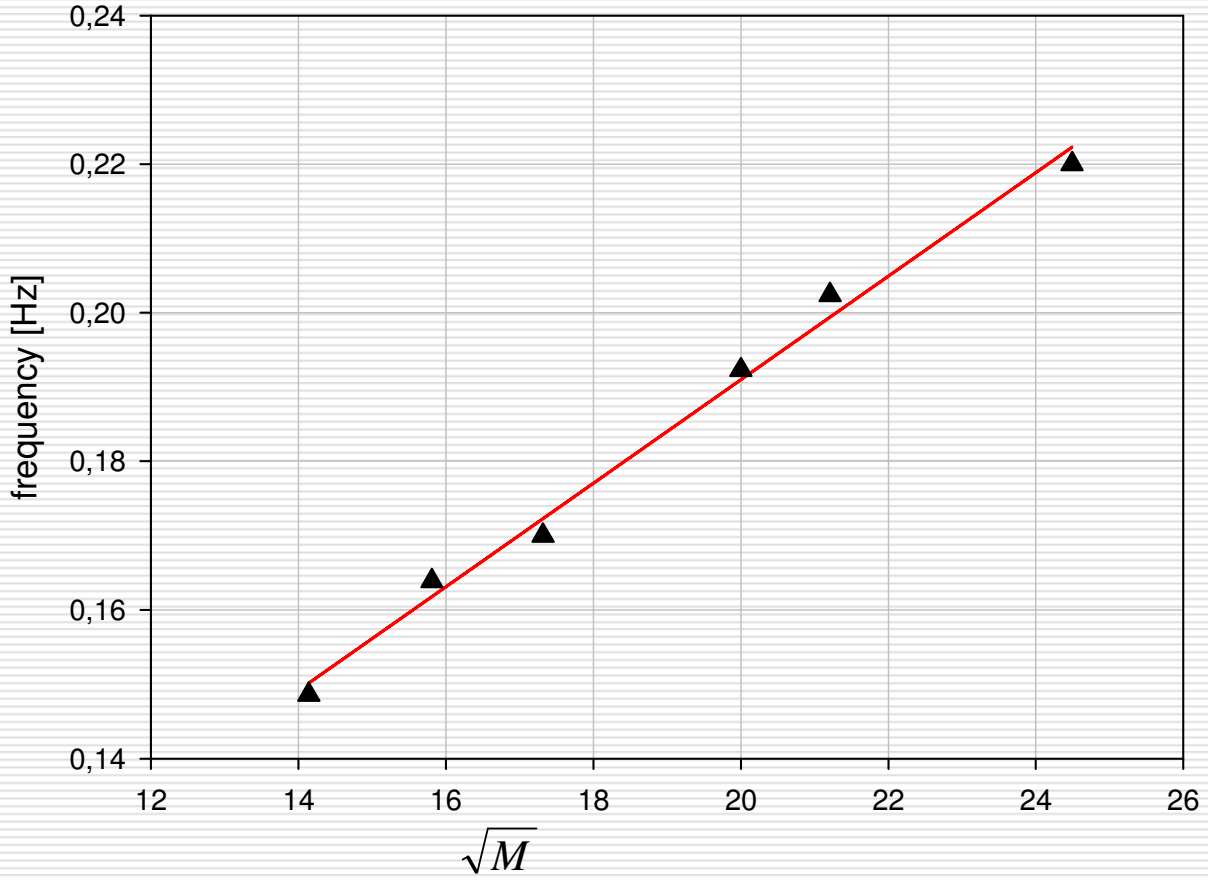
H – button thickness

R – button radius

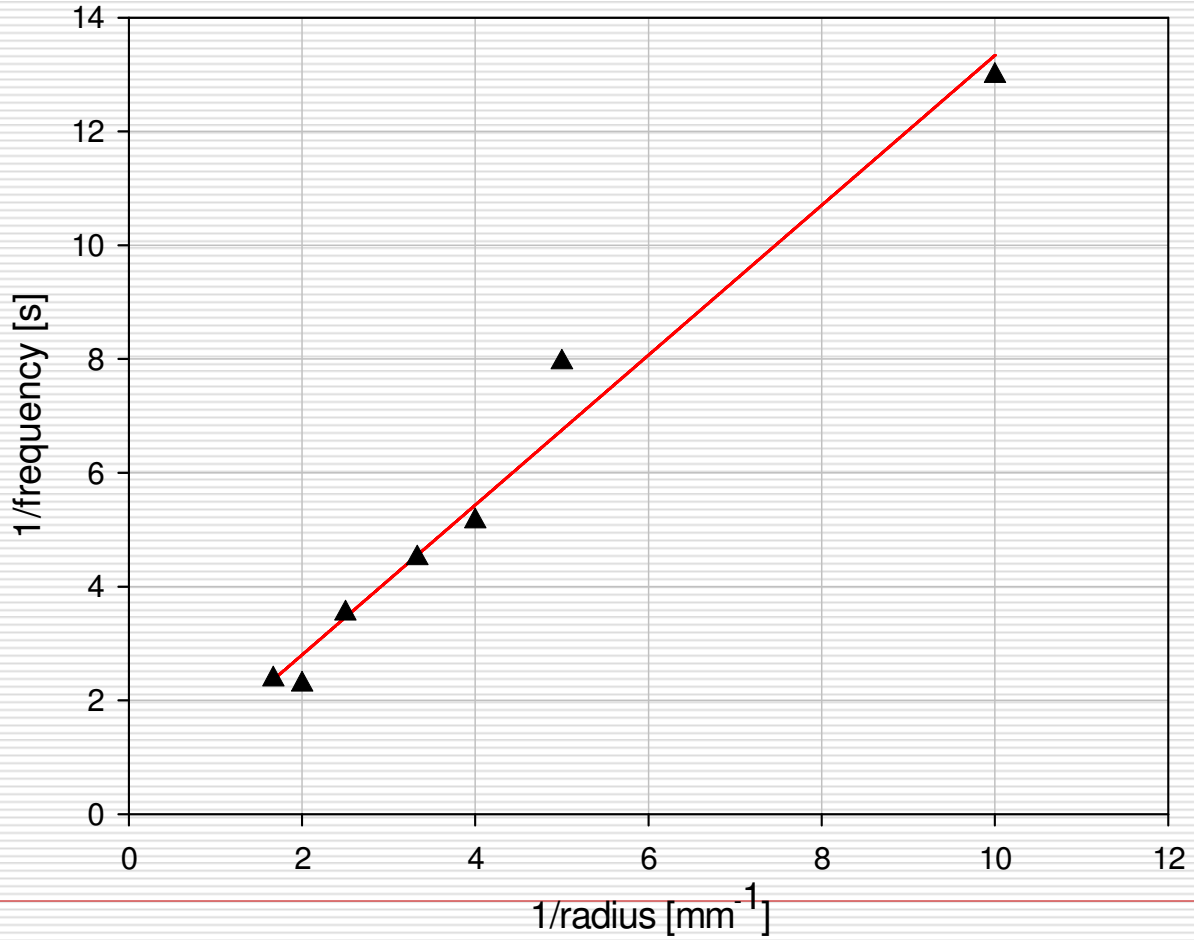
$$\Rightarrow \nu^2 \approx \frac{Mgr^2}{4\rho\pi^3 HR^4 L_0}$$

- Linear in M, r^2, $1/R^4$ – experimental check!

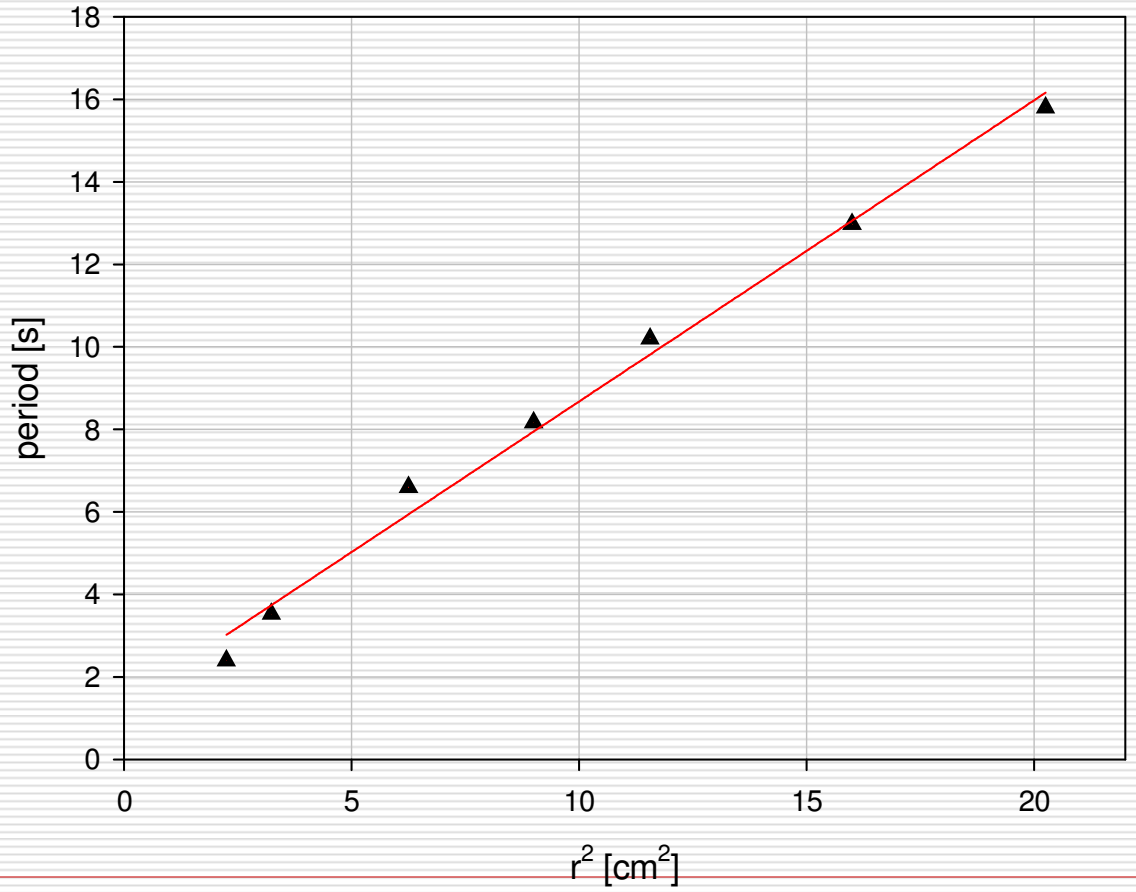
Frequency - mass



Frequency – thread radius



Frequency – button radius



Frequency – Number of thread windings

