Problem 11.

Water ski

The problem

What is the minimum speed needed to pull an object attached to a rope over a water surface so that is does not sink? Investigate the relevant parameters experimentally and theoretically.

Outline

- Scaling
- Experimental
- Basic explanation
- Measurements and results
- Conclusion

Scaling

- Our ski lenght scale: **10 cm**
- Our velocity scale: **1 m/s**
- Real skis length scale: **1 m**
- Real skis velocity scale: **10 m/s**
- Hydrodynamic conditions diagnostic:

  Reynolds number

- Reynolds number = inertia / viscosity

$$\mathrm{Re} = \frac{D\rho v}{\eta}$$

D – length scale
ρ – fluid density
v – velocity scale
η – fluid viscosity

Scaling

- Reynolds number ratio for our model:

$$\frac{\mathrm{Re}_{ski}}{\mathrm{Re}_{mdl}} \sim 100$$

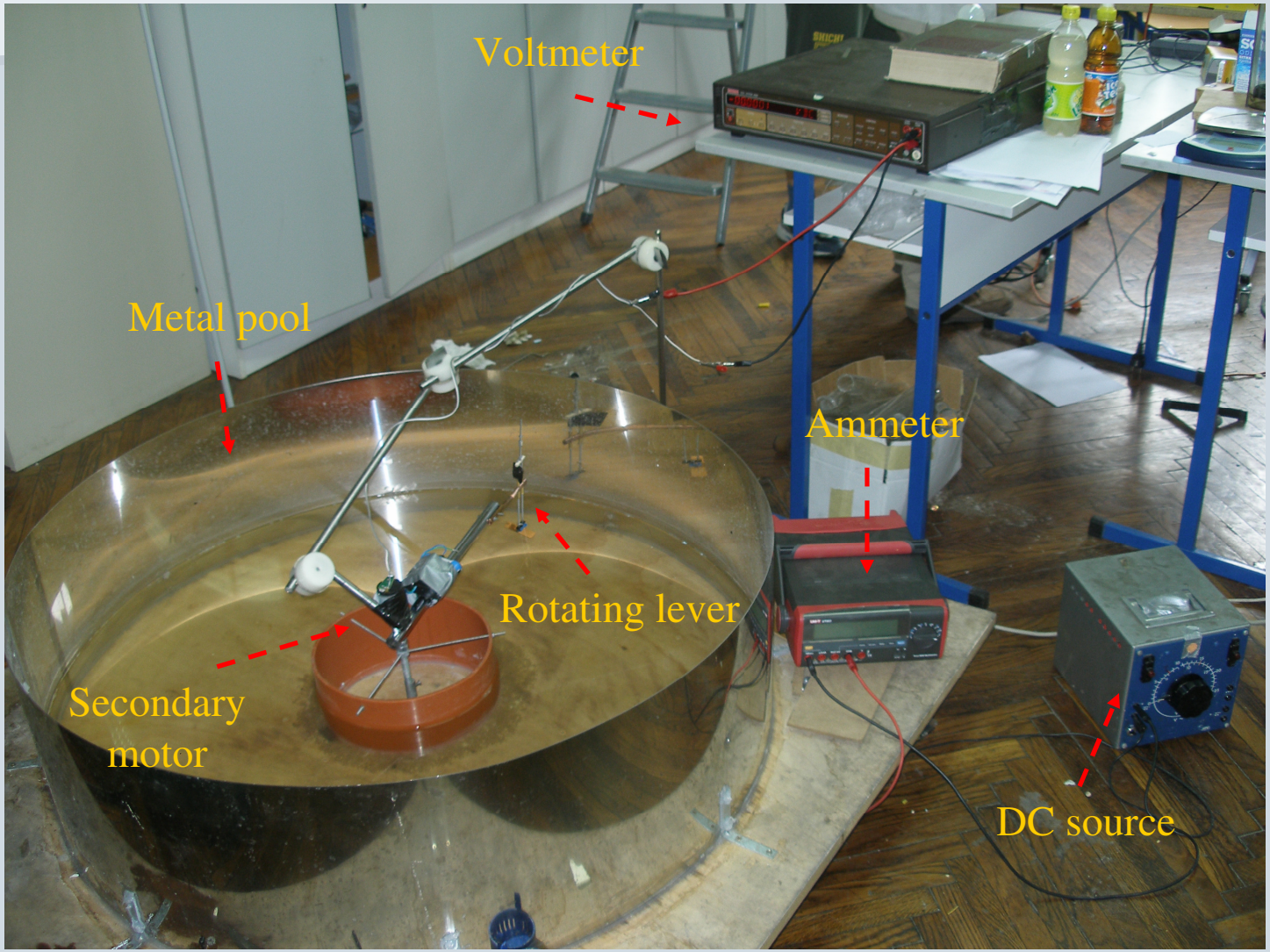
Re_{ski} – Reynolds number for real ski
Re_{mdl} – Reynolds number in our model

- **Our model doesn't match real ski conditions!**
- Only qualitative similarity
- Problem doesn't require quantitative matching!

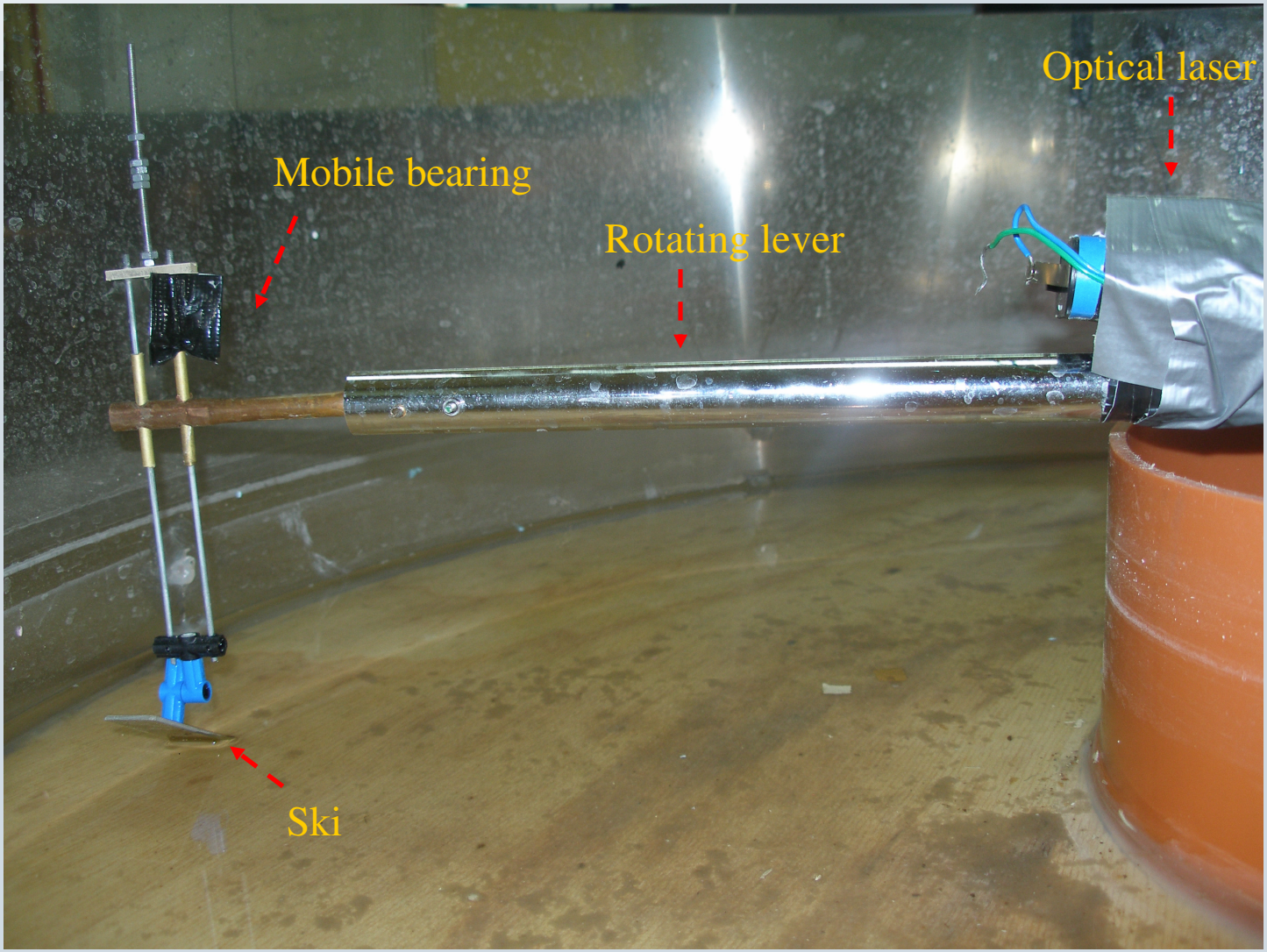
Experimental

- Metal pool with rotating lever
- Ski rotating on mobile bearing
- Lift force measurement – weights
- Drag force measurements – current on main motor
- Optical ski height detection
- Velocity measurement – inductive
- Fast photography setup

Experimental

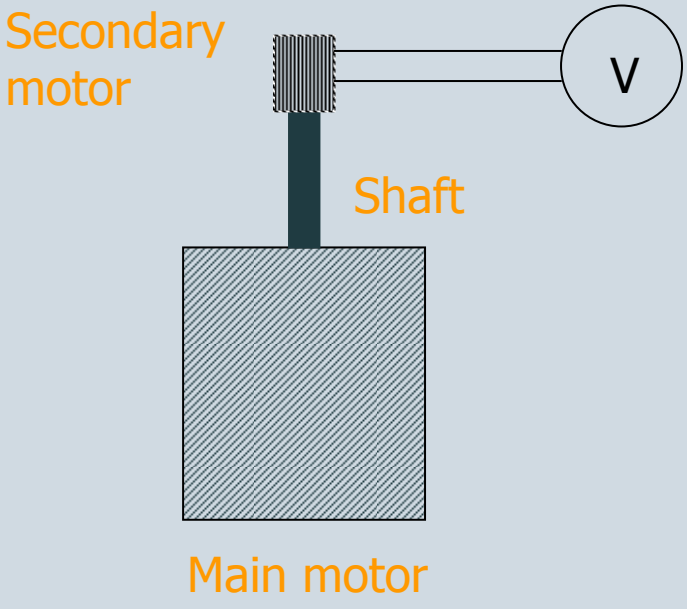


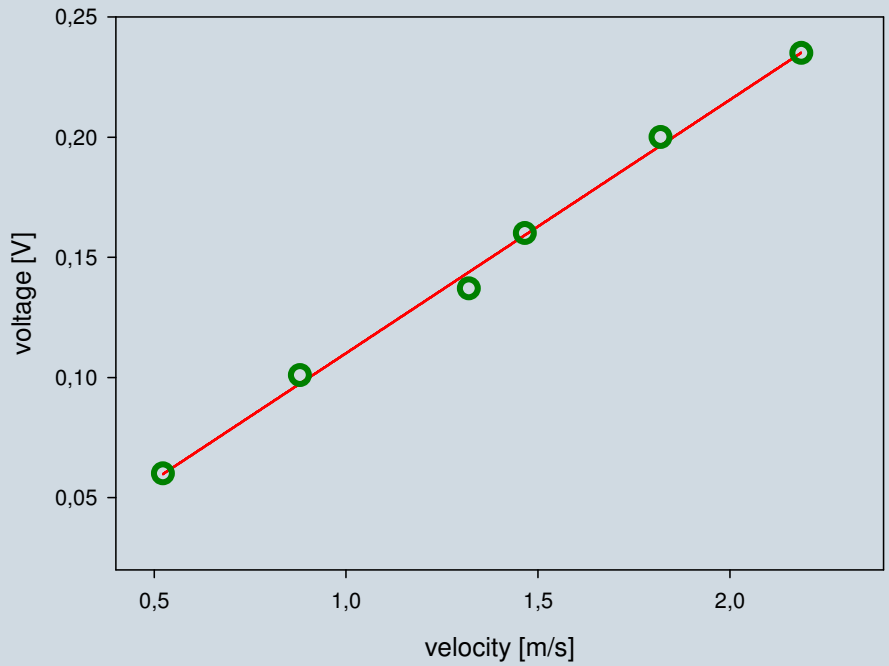
Experimental



Experimental

- Velocity range: 0.5 to 3.0 m/s
- Measurement – voltage on secondary motor:





Experimental

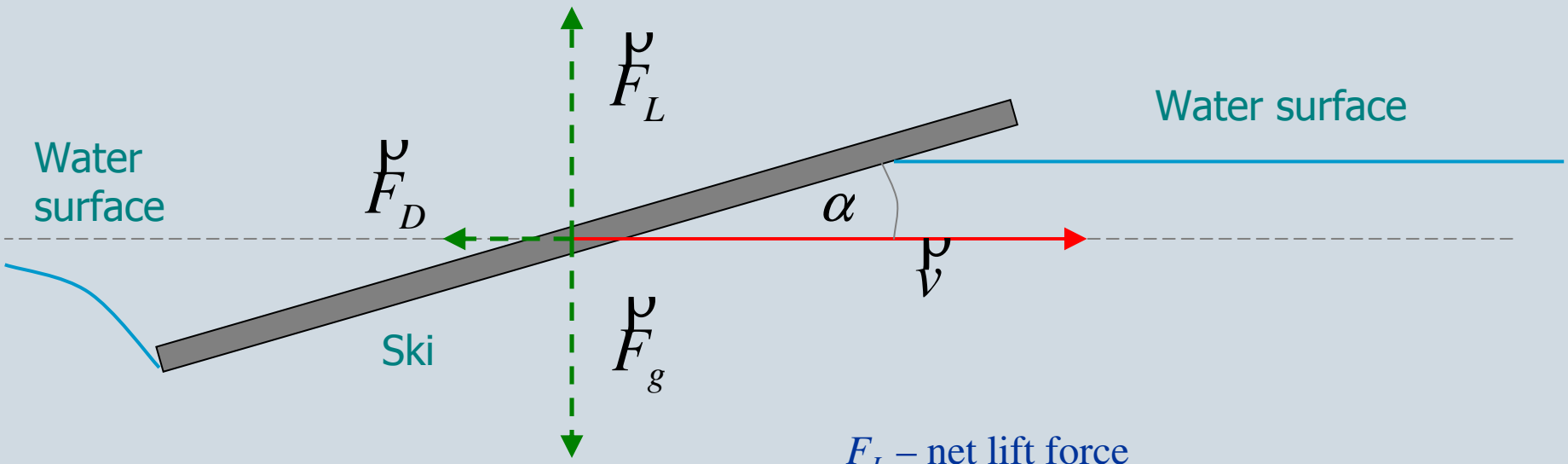
- Objects:
  - Skis of different size
  - (jos nesto)
- Parameters investigated for skis:
  - Velocity
  - Ski angle of attack
  - Ski surface area

Explanation

- Ski density > water density
- Buoyancy can't cause ski not to sink
- Ski in motion – dynamic forces
- Depend on:
  - Velocity
  - Attack angle
  - Shape and surface area
  - Water properties (viscosity, density)

Explanation

- Forces acting on ski:



F_L – net lift force
F_D – drag force
F_g – ski weight
v – ski velocity
α – angle of attack

Explanation

- The ski doesn't sink if

$$mg \leq F_L$$

$_{FL}$ – lift coefficient
m – net mass on ski
g – acceleration of gravity

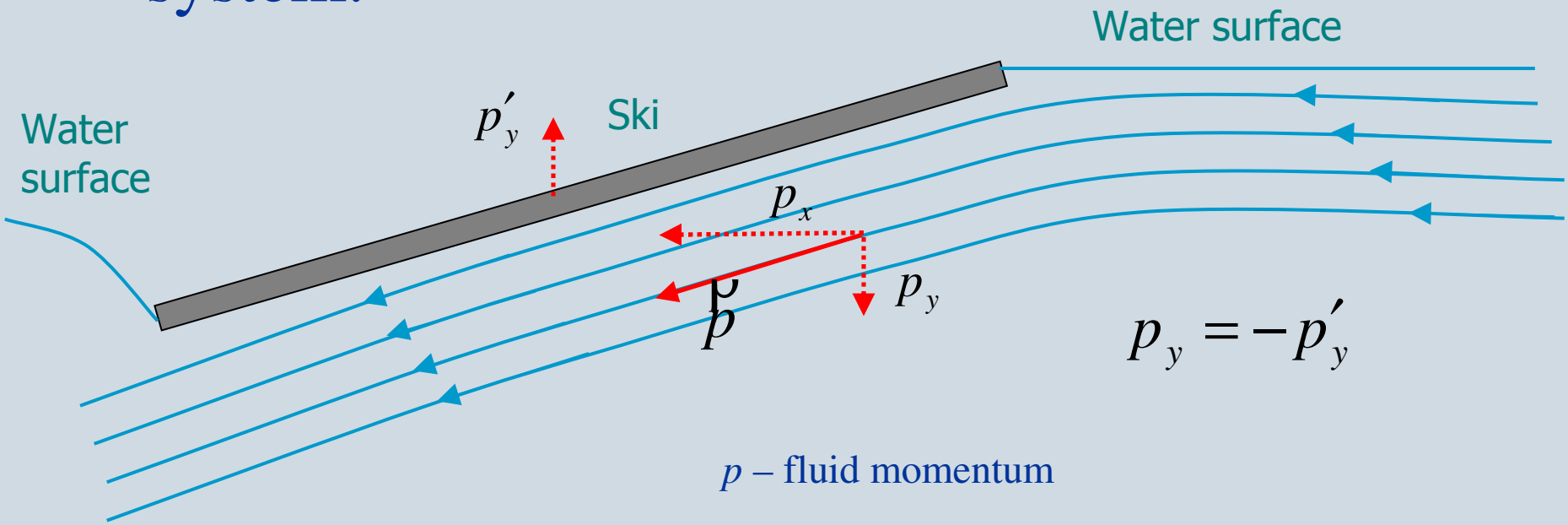
- The lift force depends on velocity

$\Rightarrow$ Critical velocity existence

- Obtaining critical velocity – lift force calculation:

Explanation

- Lift force – due to collisions between water and ski
- Momentum conservation in ski reference system:



Explanation

- Calculating:

$$\left|\frac{dp_y}{dt}\right| = \frac{1}{2} C'_L \rho S v^2 \sin^2 \alpha$$

C'_L – lift coefficient
ρ – fluid density
S – ski surface area
v – ski velocity
α – angle of attack

- Lift coefficient – correction for turbulent flows
- Depends on Reynolds number
- Must be measured!

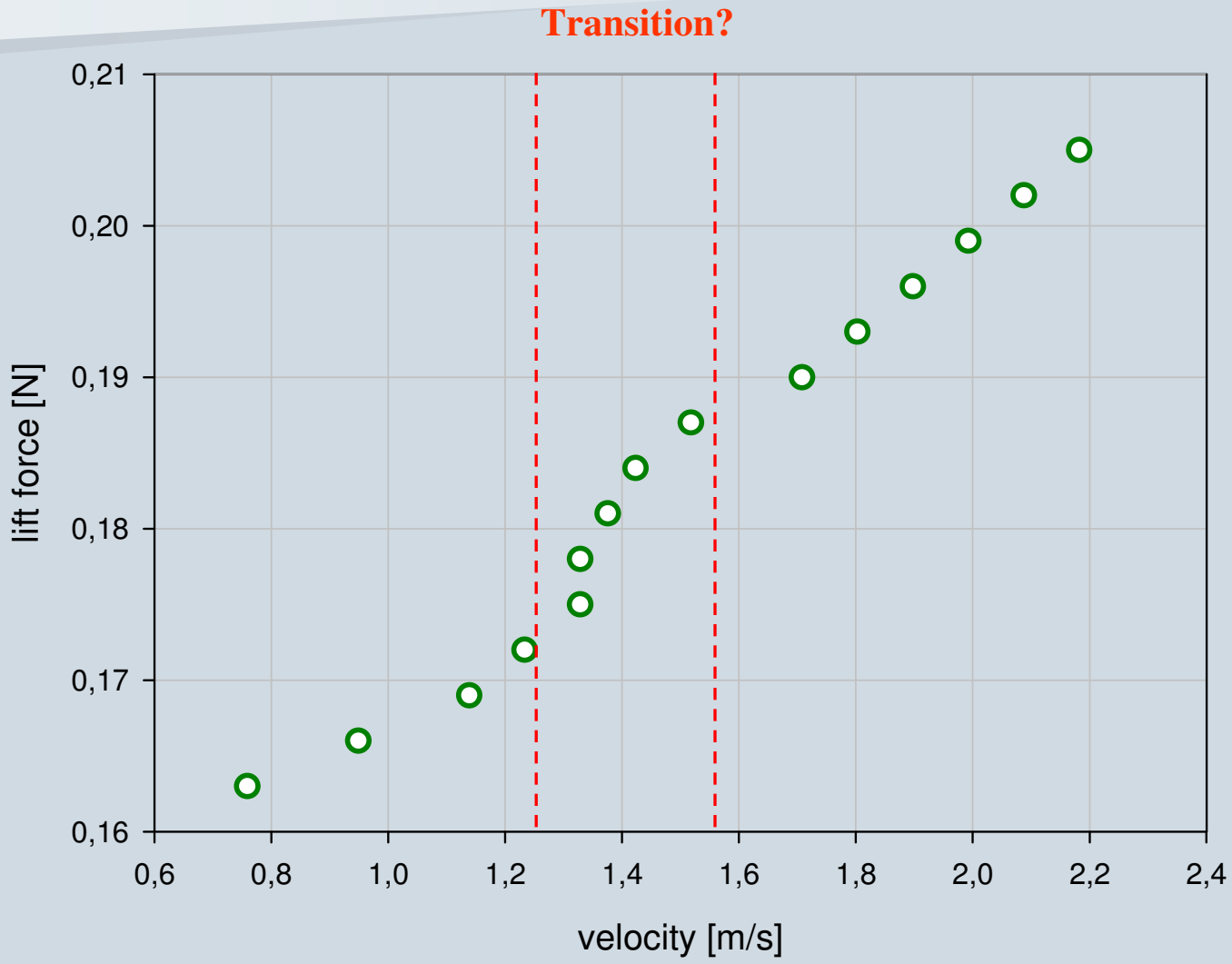
Explanation

- In our model apparatus:

$$10^4 < \mathrm{Re} < 5 \cdot 10^4$$

- High reynolds number regime
- Small turbulence
- Measurements give dependence on parameters:

Measurements and results



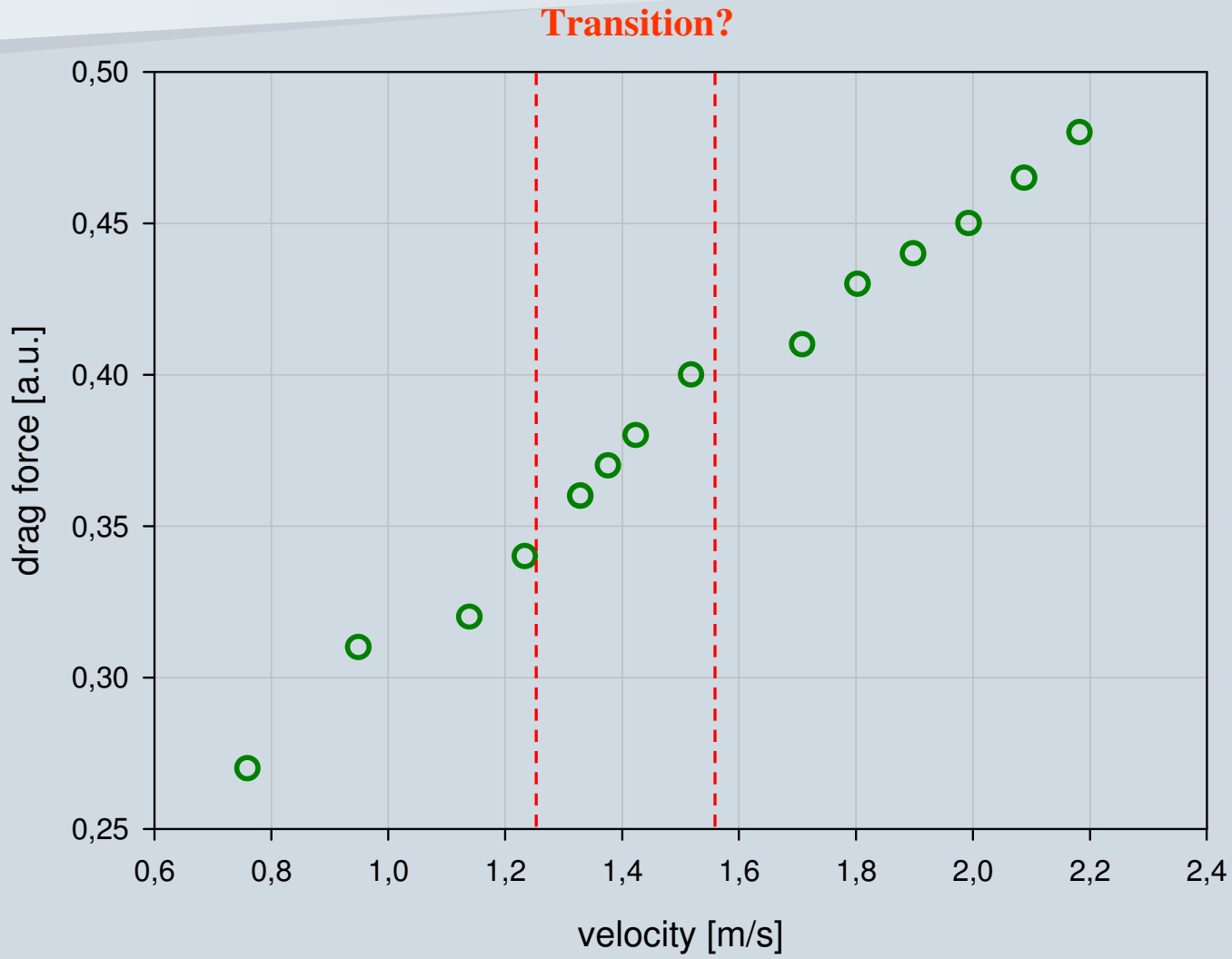
Ski surface area:

$10.8\,cm^2$

Angle of attack:

20°

Measurements and results



Ski surface area:

$10.8\ cm^2$

Angle of attack:

20°

Measurements and results



Velocity:

1.30 m/s

Turbulent wake

Measurements and results



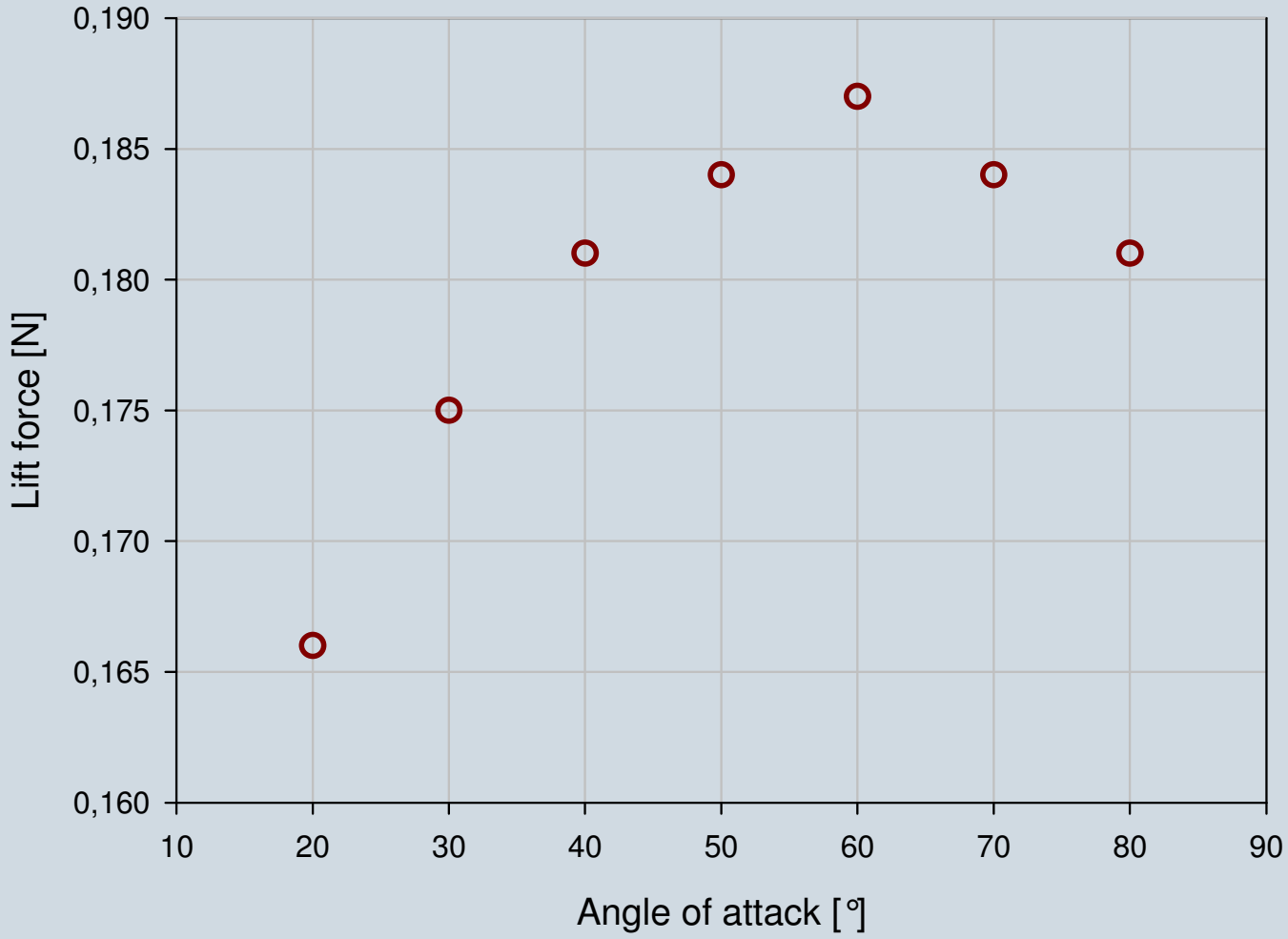
Velocity:

1.60 m/s

Smooth wake

⇒ **Transition exists!**

Measurements and results



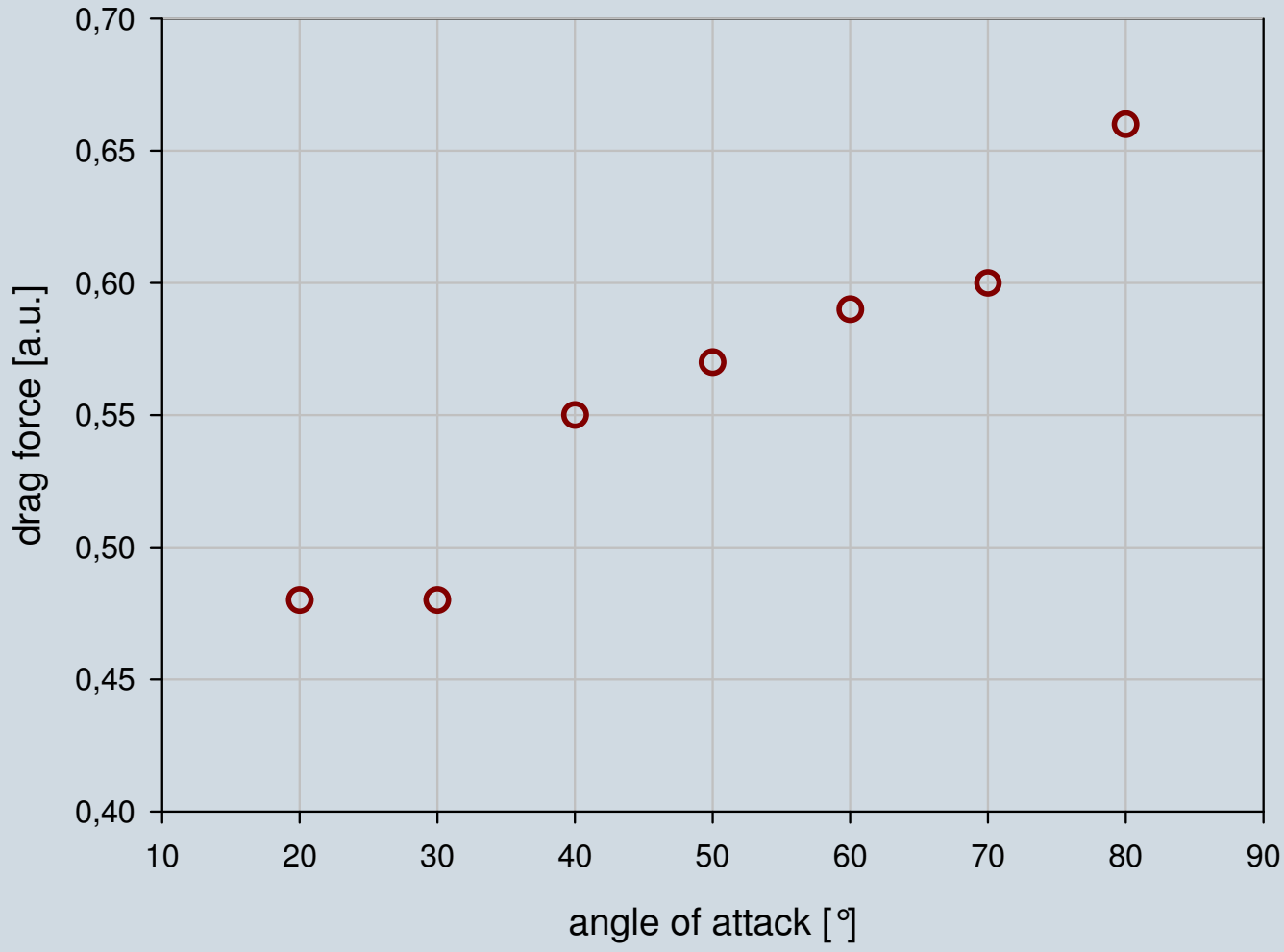
Ski surface area:

$10.8\ cm^2$

Velocity:

1.52 m/s

Measurements and results



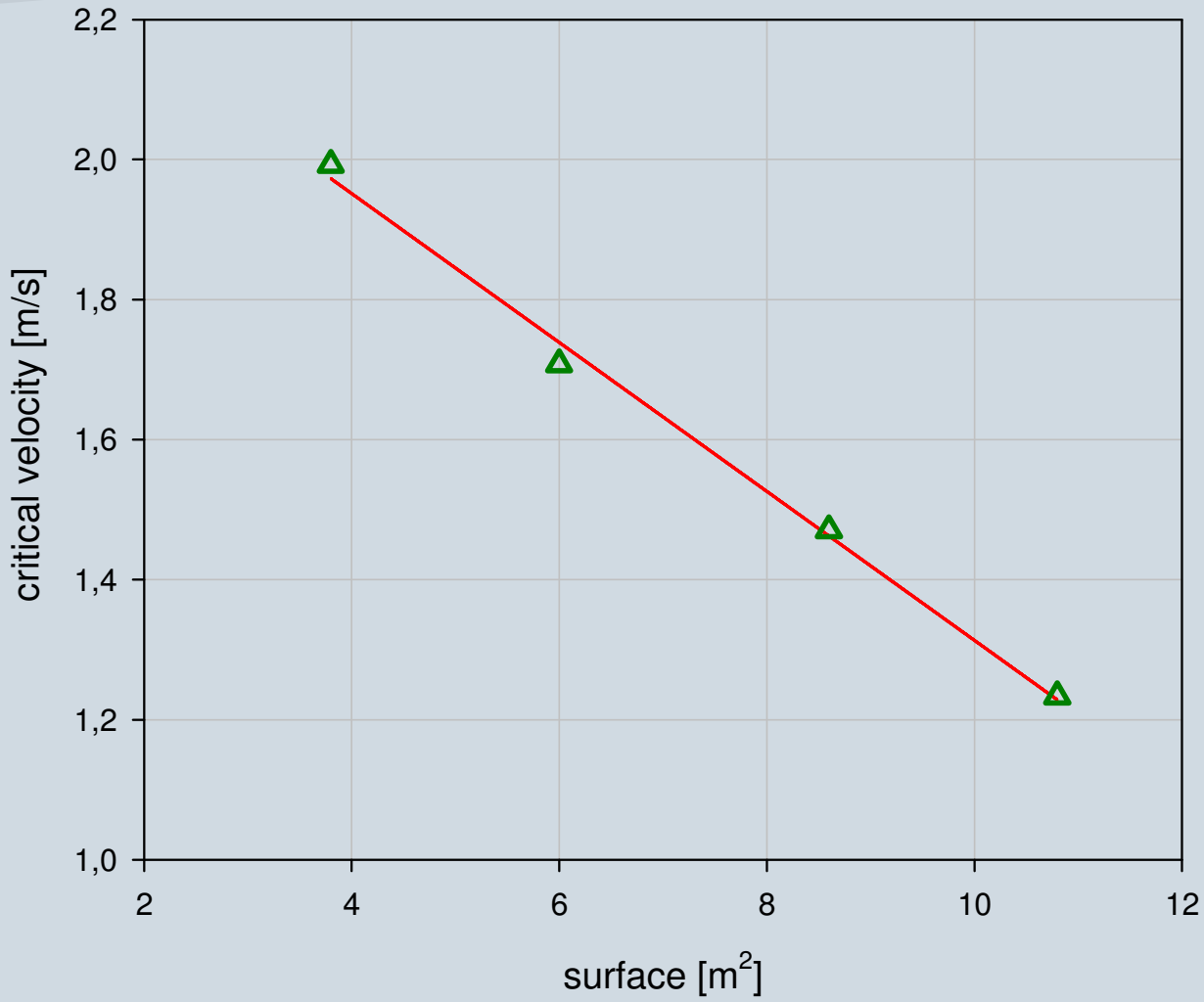
Ski surface area:

$10.8\ cm^2$

Velocity:

1.52 m/s

Measurements and results



Mass of ski + weights

16.9 g

Angle of attack:

20°

As predicted!

Measurements and results

- Ski doesn’t sink due to hydrodynamic lift
- Lift calculated for idealized case
- Measurement made for real case
- Qualitatively, critical velocity exists
- Parameters:
  - Mass
  - Ski angle of attack
  - Ski surface area