8. Astroblaster

The Problem

When a large ball is dropped, with a smaller one stacked on top of it, onto a hard surface, the smaller ball will often rise much higher than it would if dropped onto the same surface by itself while the larger ball hardly bounces at all. Investigate this phenomenon and design a multiple-ball system, using up to 4 balls, that will reach the greatest elevation of the top ball.

The Toy

- Illustrates the laws of conservation of momentum and energy during the creation of a supernova



Contents

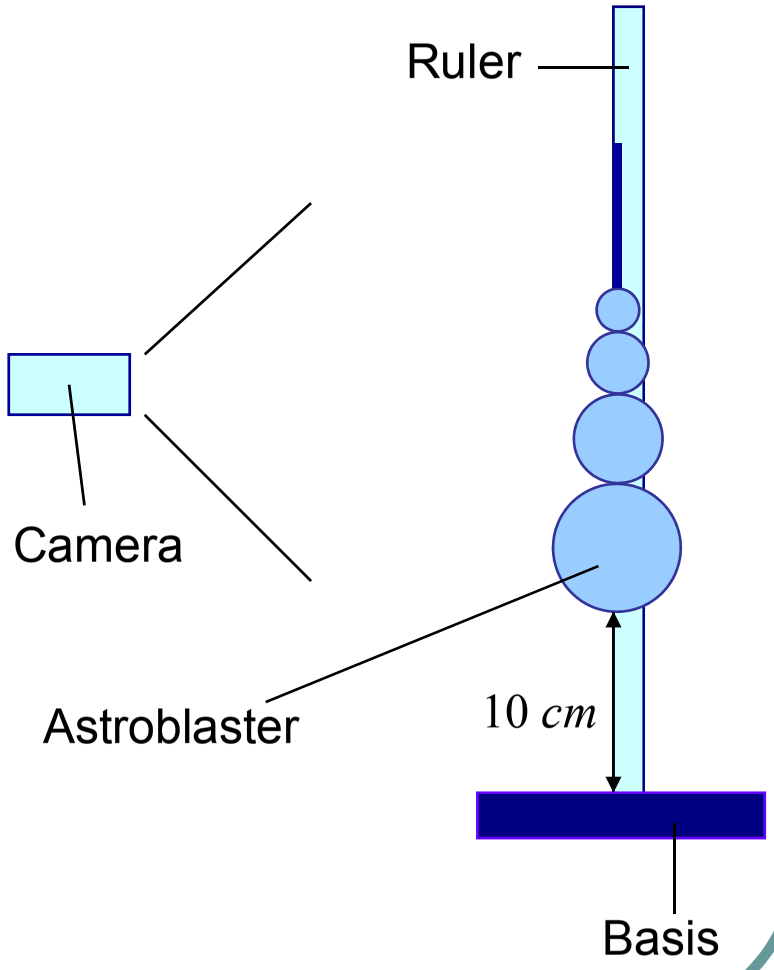
- Astroblaster – The Toy
- Apparatus
- Positon vs. Time Plots
- Theoretical Model With Basic Explanation
- Dependence of the Rebound Speed on Certain Masses
- Interpretation of the Results
- Achieving the Maximum rebound speed
- Conclusion

The Experiment

- Two apparatus were made
  - Apparatus 1
    - Studying the dependence of the rebound speed on particular masses
    - Maximization of the rebound speed
  - Apparatus 2
    - Achieving the maximum speed by minimizing the energy dissipation

Apparatus (1)

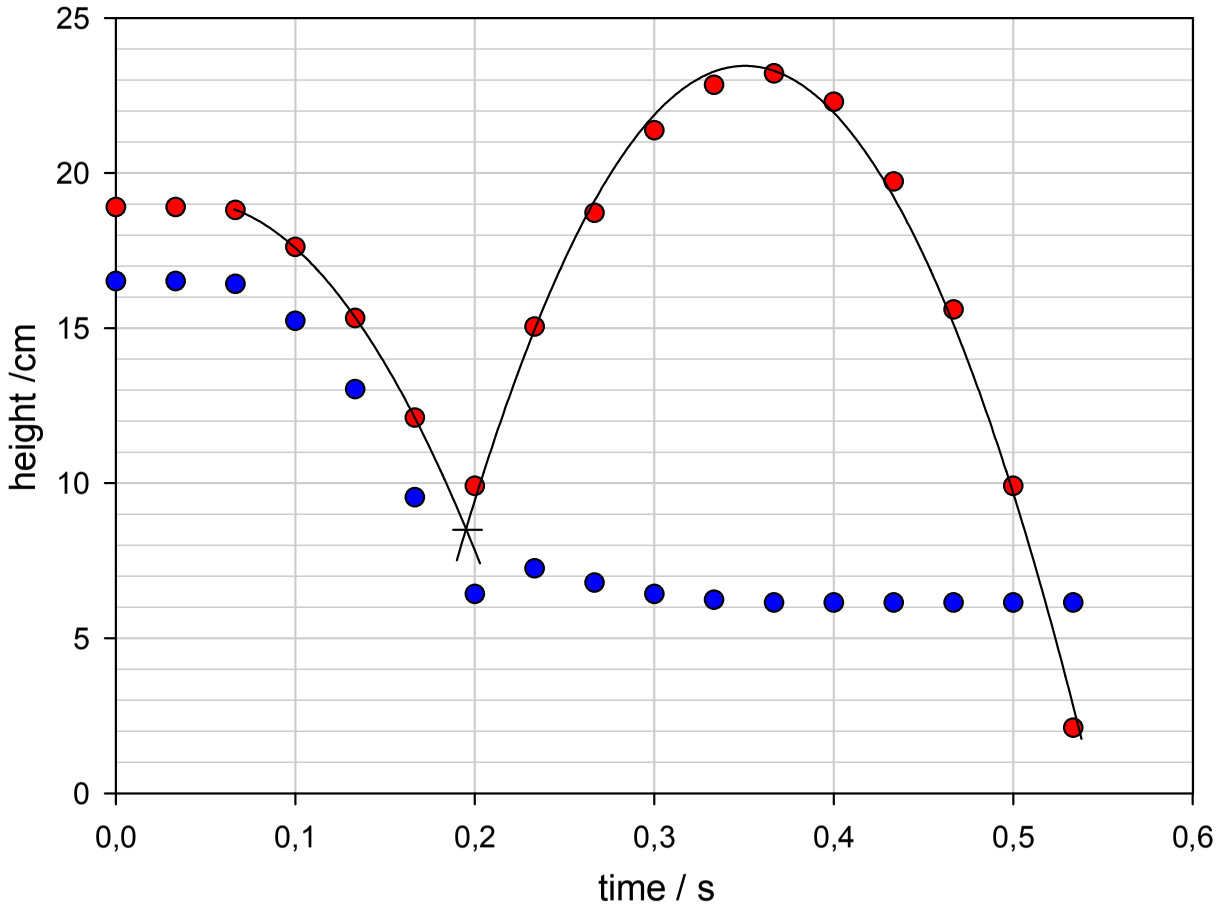
- Wooden balls (22 balls with 11 different masses)
  - The bottom ball having an iron rod fixed as a pillar for the other balls
- Steel basis
- Camera
- Ruller
  - Simple performance
  - 1320 combinations



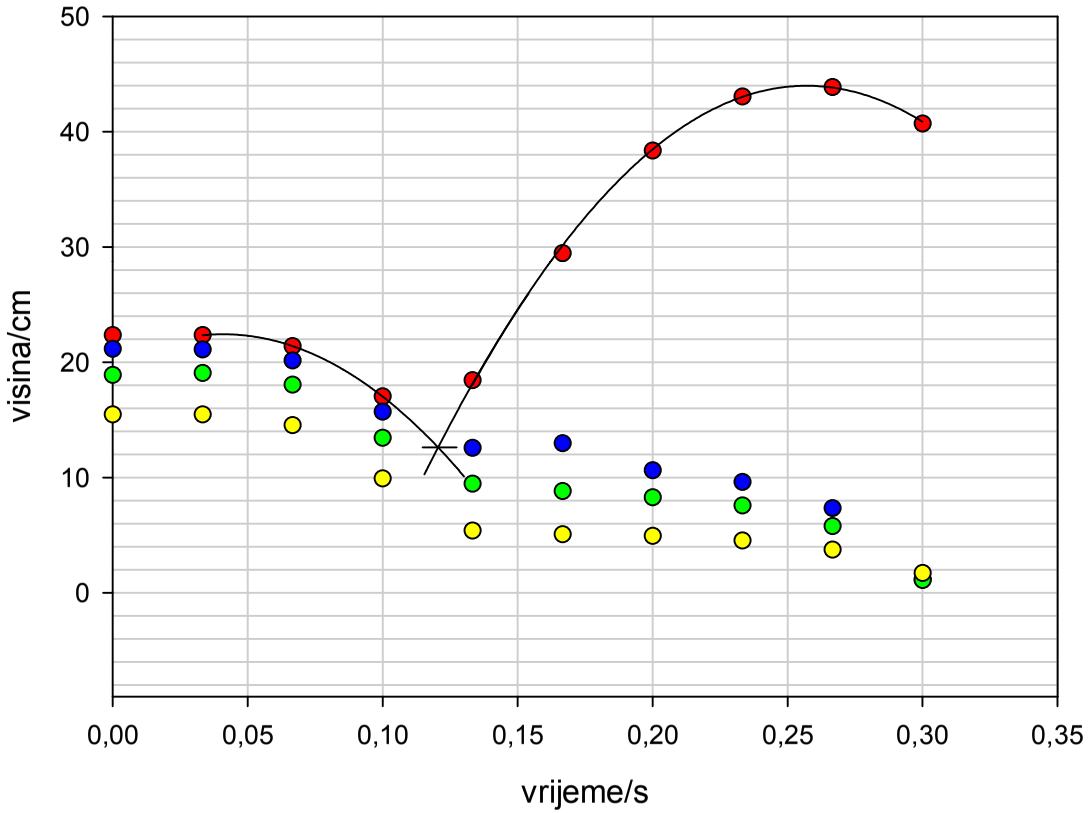
Measurements

- One measurement includes:
  - Dropping the Astroblaster from the height $h = 10cm$
  - Sequential photographing (30 photos per second)
  - Measuring the rebound speed of the top ball from two consecutive photographs
- Numerous measurements (8-10) were made for one combination of ratios
- An optimized astroblaster was made and recorded

Positon vs. Time Measurement



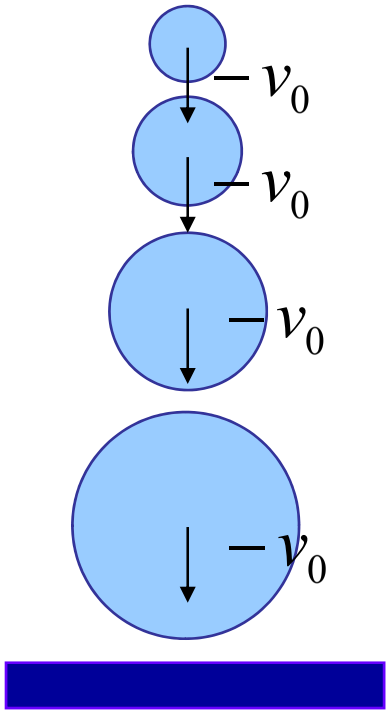
Positon vs. Time Measurement



- Vanishingly small collision time

Theoretical Model

- A small gap between two consecutive balls
- The collisions are
  - Instantaneous
  - Independent
  - Central
- The balls are numbered 1, 2, 3, 4 from the bottom.

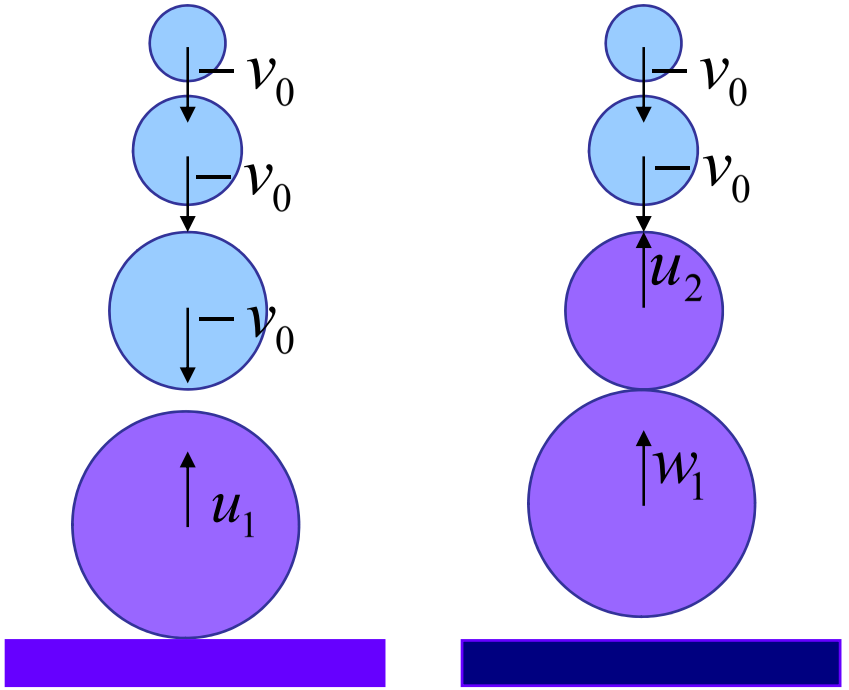


Right before collision 1

Basic Explanation of the Model

- A collision between 2 balls moving in opposite directions → the ball of smaller mass gains greater speed then the initial

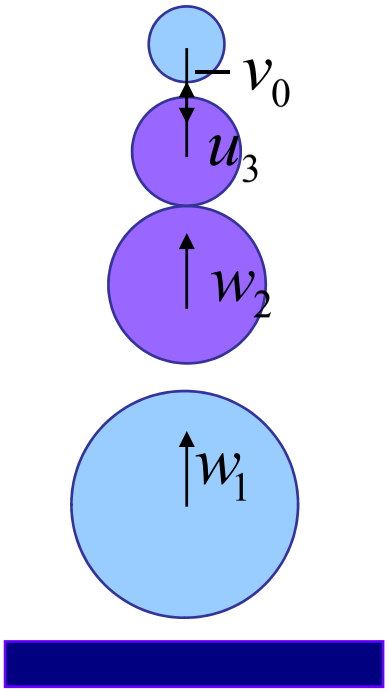
→ Collision 2 – ball 2 gains speed $u_2 > u_1$



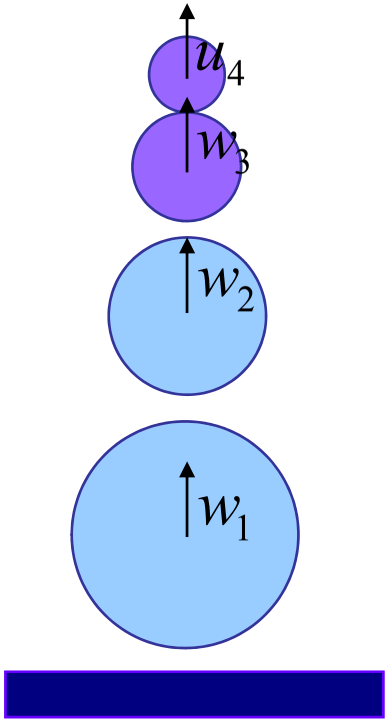
Right after collision 1

Right after collision 2

Basic Explanation of the Model



Right after collision 3



Right after collision 4

→ Collision 3 – ball 3 gains speed $u_3 > u_2$

→ Collision 4 – ball 4 gains speed $u_4 > u_3$

→ The rebound speed repeatedly increases

Calculus

- A collision among two balls in general form (u_i calculated from previous collisions):
  - Momentum conservation

$$m_i u_i - m_{i+1} v_0 = m_i w_i + m_{i+1} u_{i+1}$$

  - Energy conservation
    - λ=0,65 – restitution coefficient measured using one ball

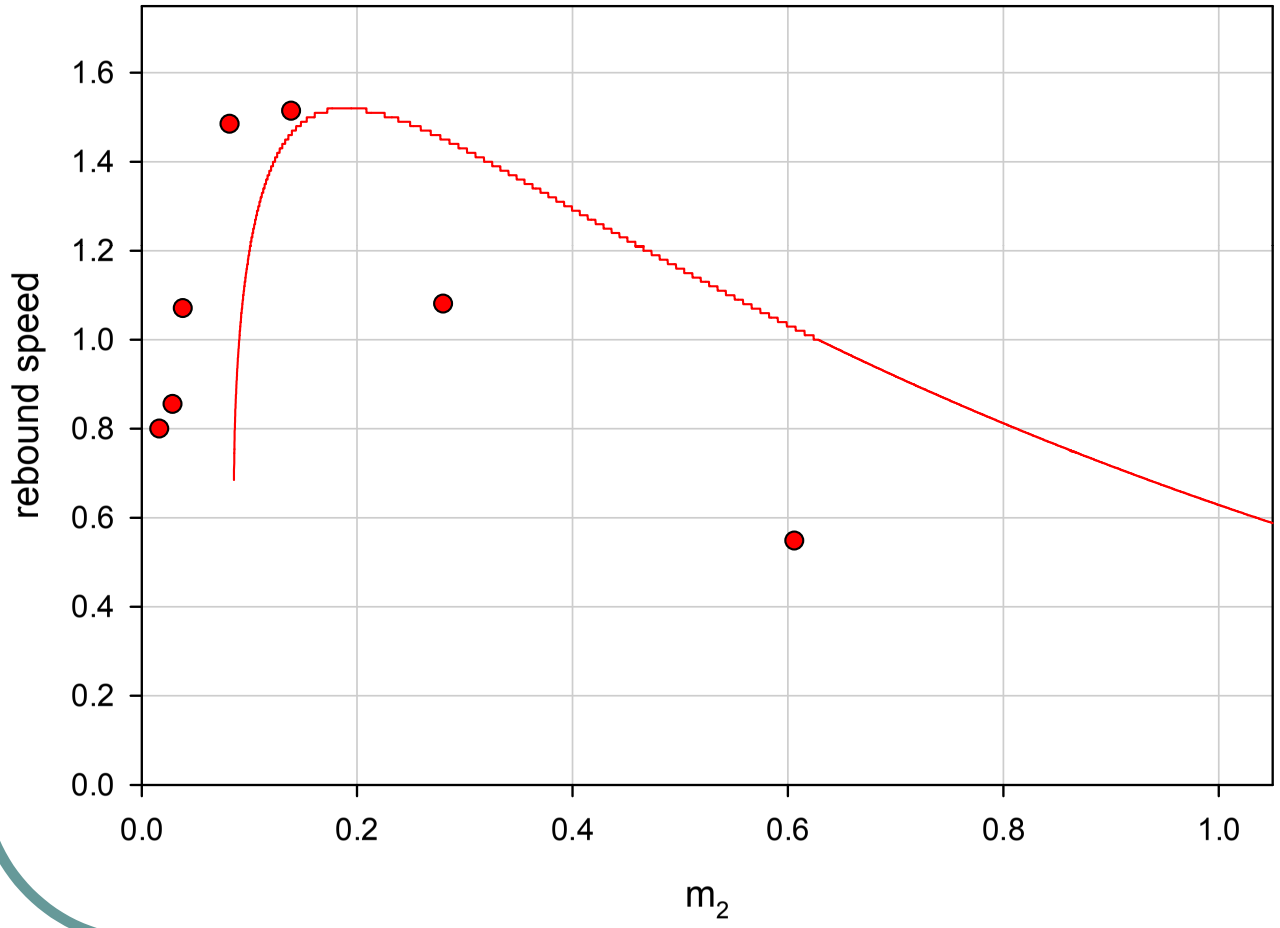
$$\lambda\left(m_i u_i^{\,2} + m_{i+1} v_0^{\,2}\right) = m_i w_i^{\,2} + m_{i+1}^{\,2} u_{i+1}$$

- Theoretical values obtained by a simulator

Measurements

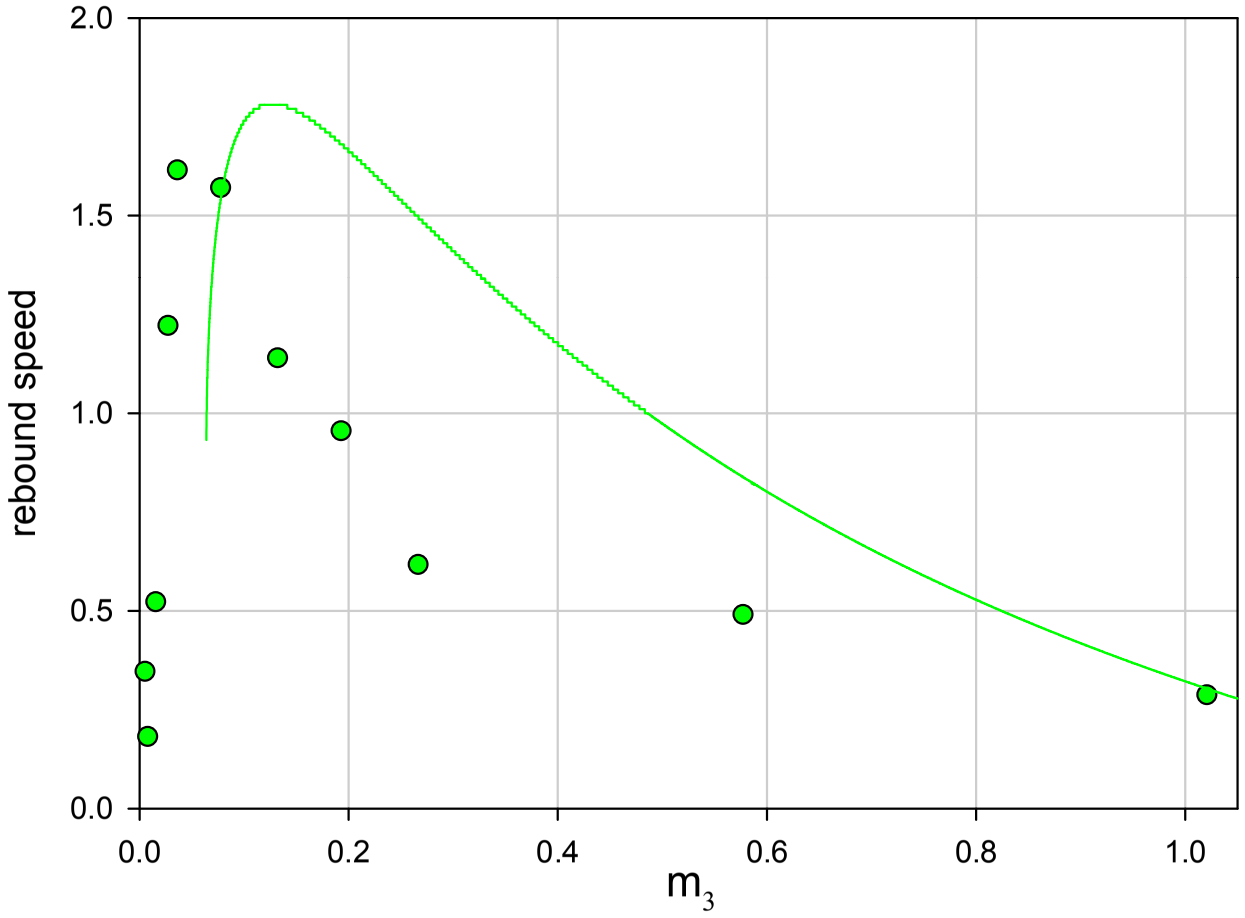
- 3 series of measurements for n=2, 3, and 4
- Dependence of the rebound speed (u_n) on each mass
  - $m_1=1$
  - $v_0=1$
- Theoretical values obtained using a simulator
- Position vs. time plots

Results (2 ball)



$m_1=1,0$

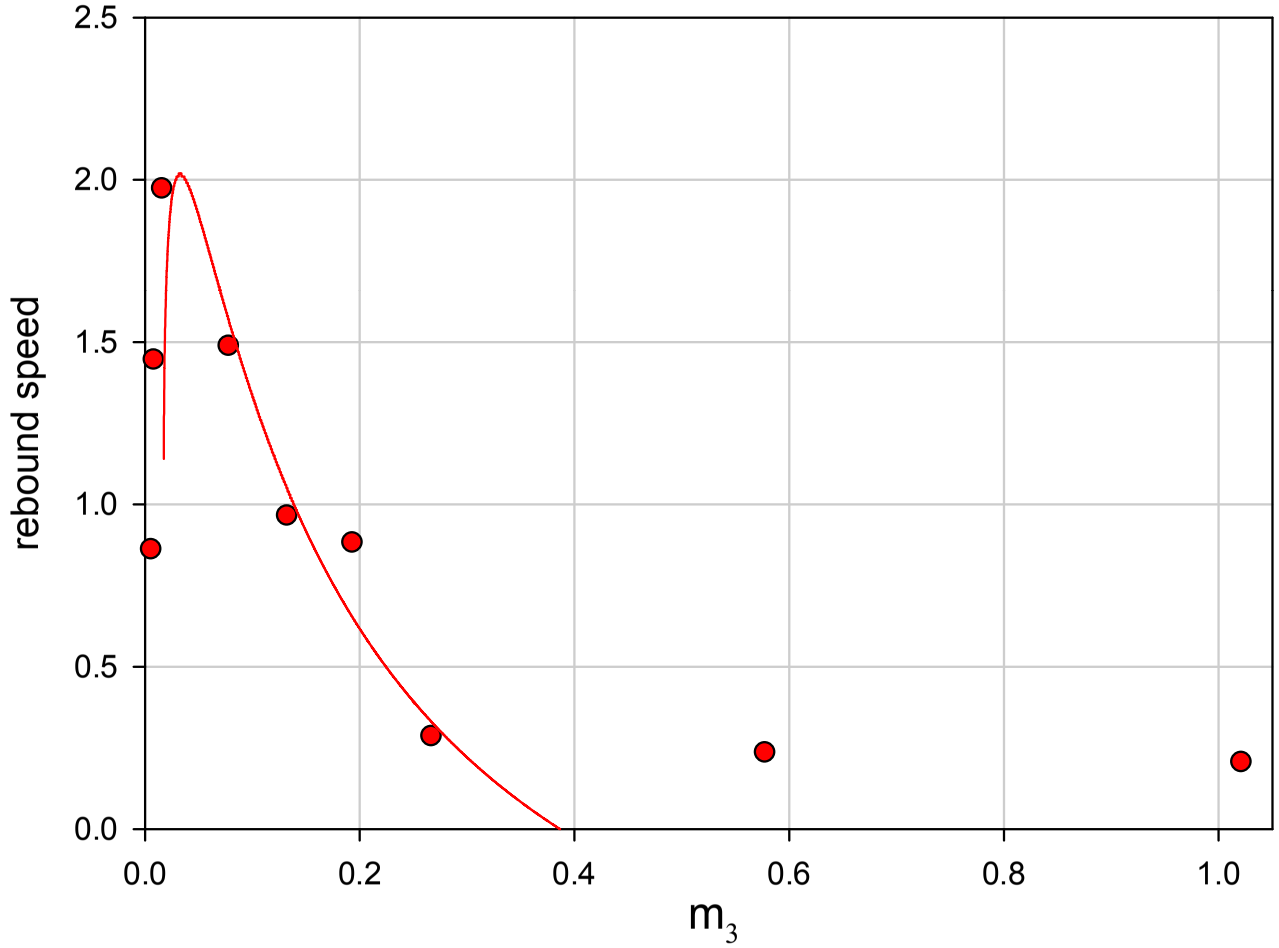
Results cont. (3 balls)



m_1=1,0

m_2=0,54

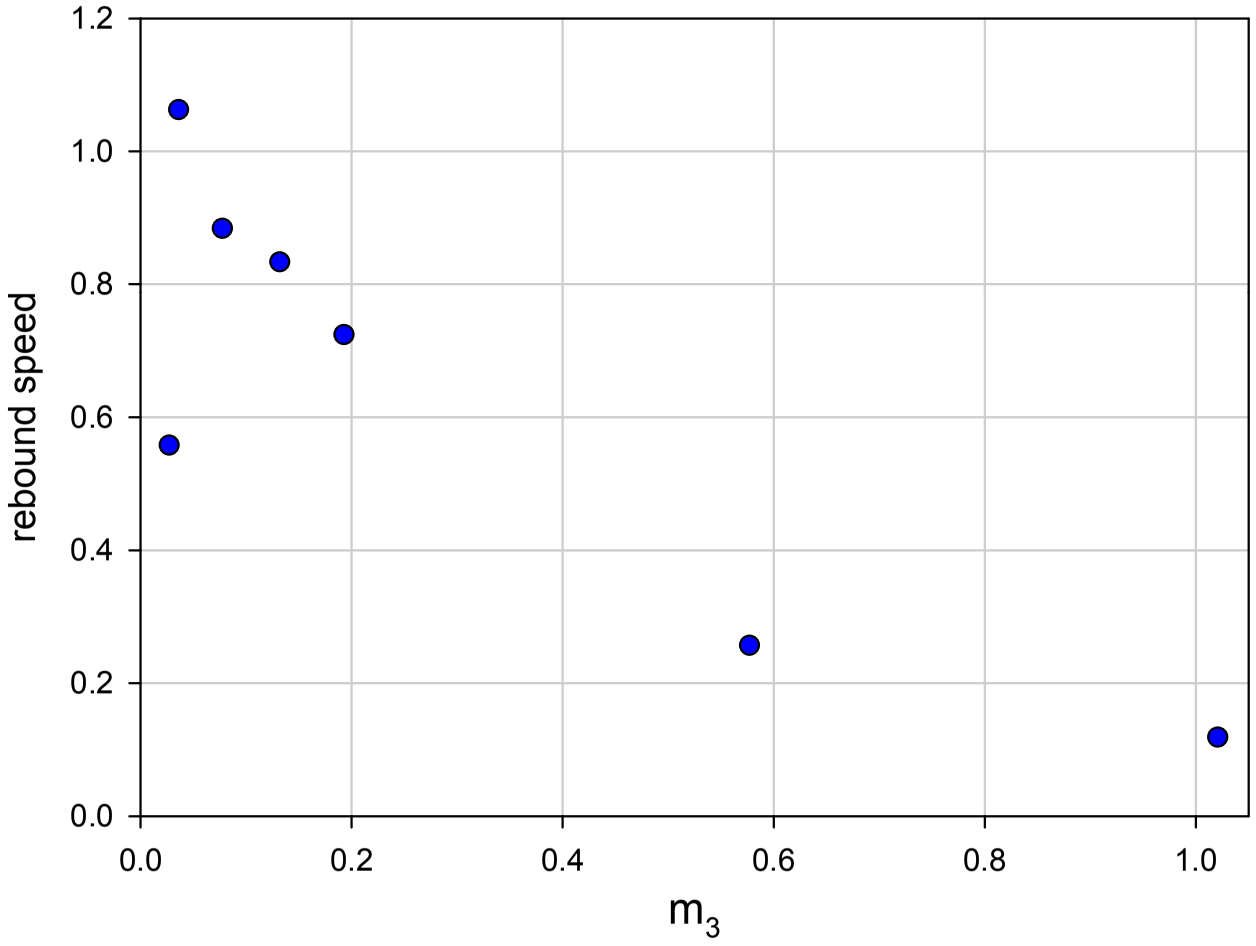
Results cont. (3 balls)



m_1=1,0

m_2=0,12

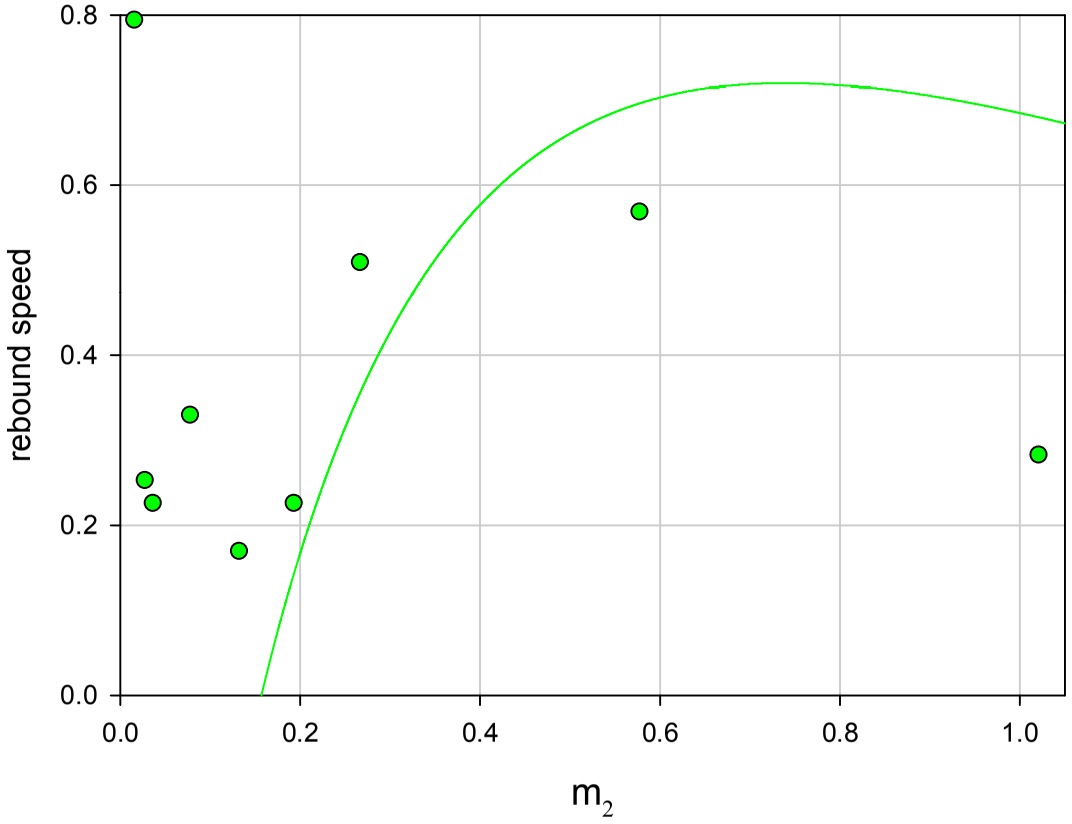
Results cont. (3 balls)



m_1=1,0

m_2=0,015

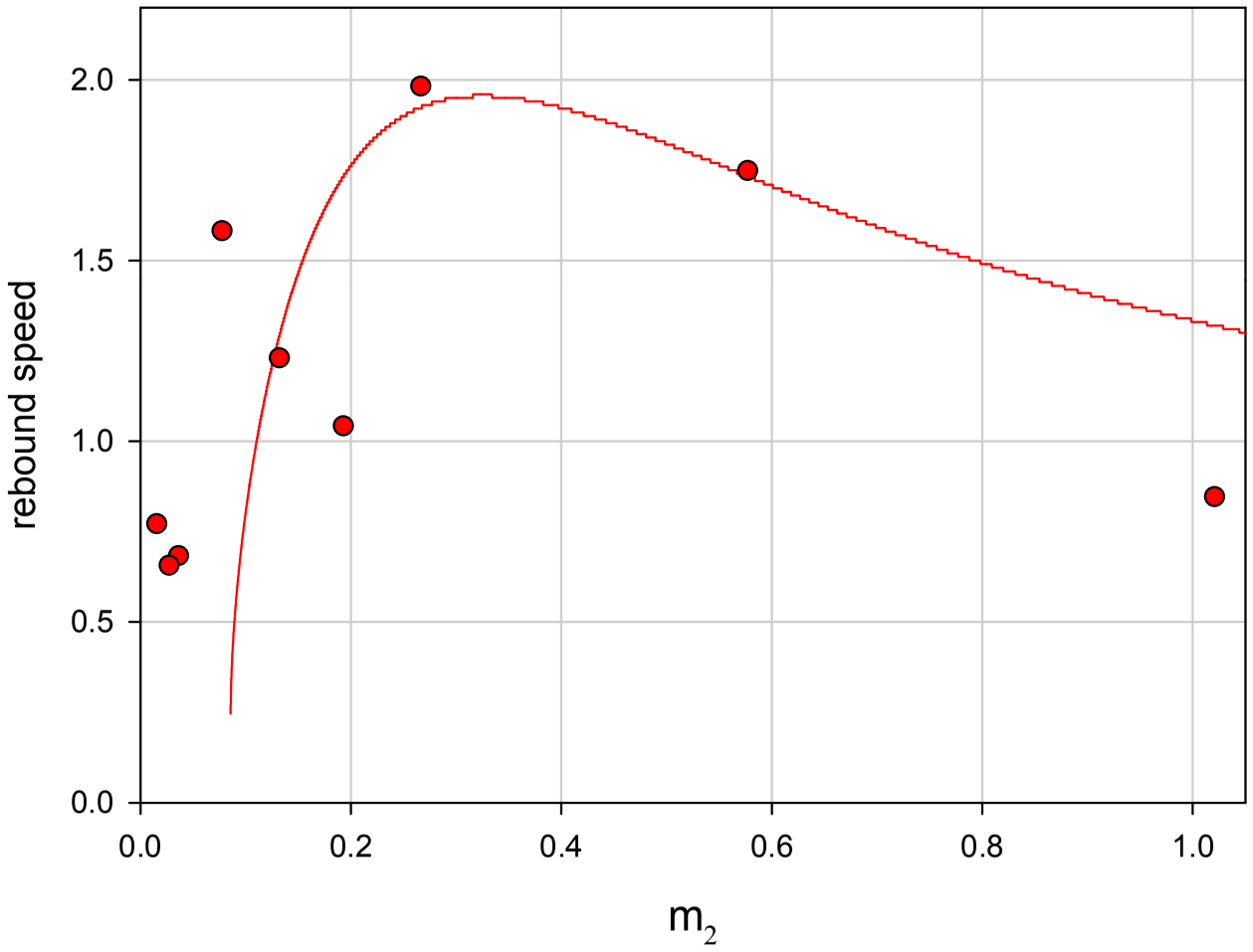
Results cont. (3 balls)



m_1=1,0

m_3=0,54

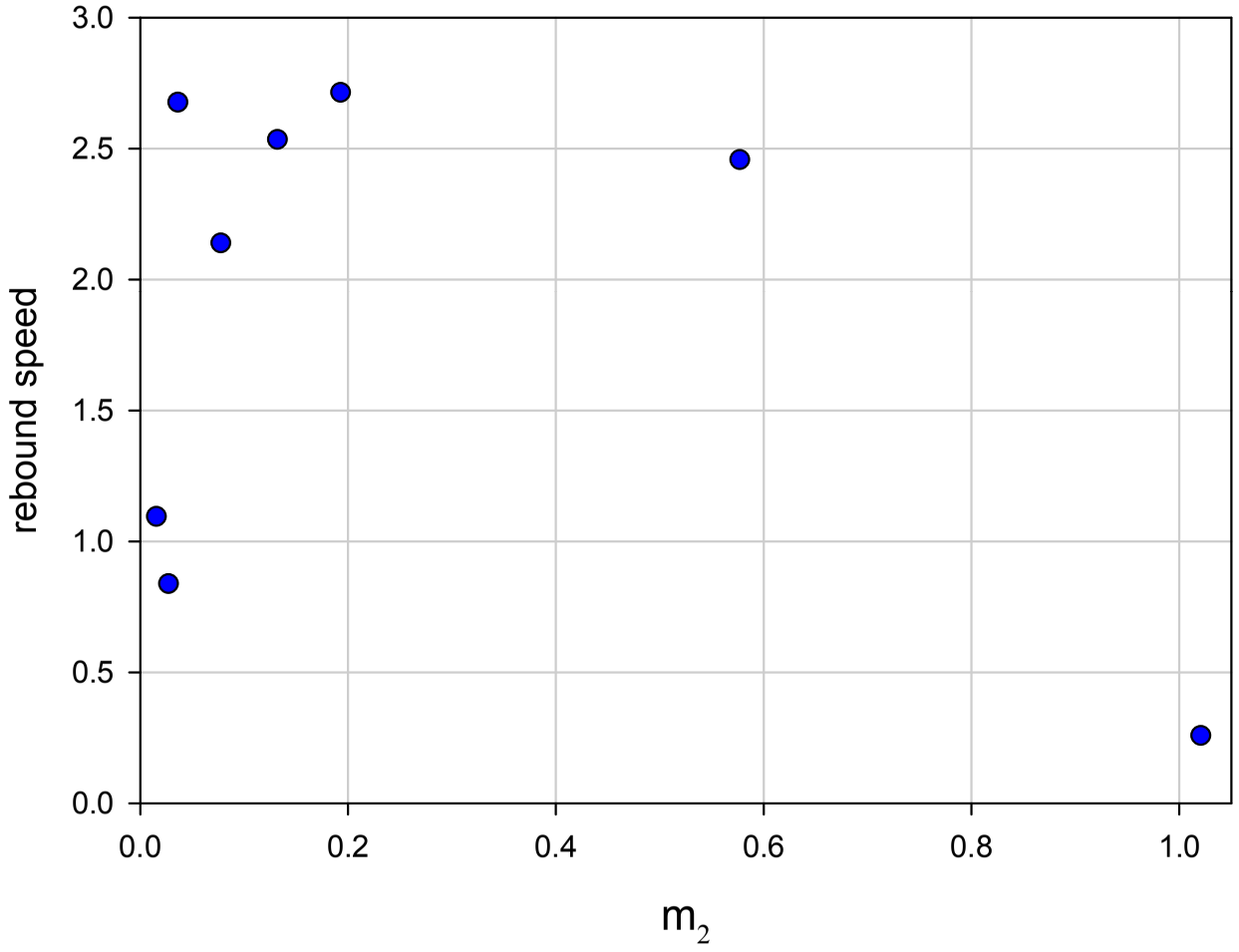
Results cont. (3 balls)



m_1=1,0

m_3=0,12

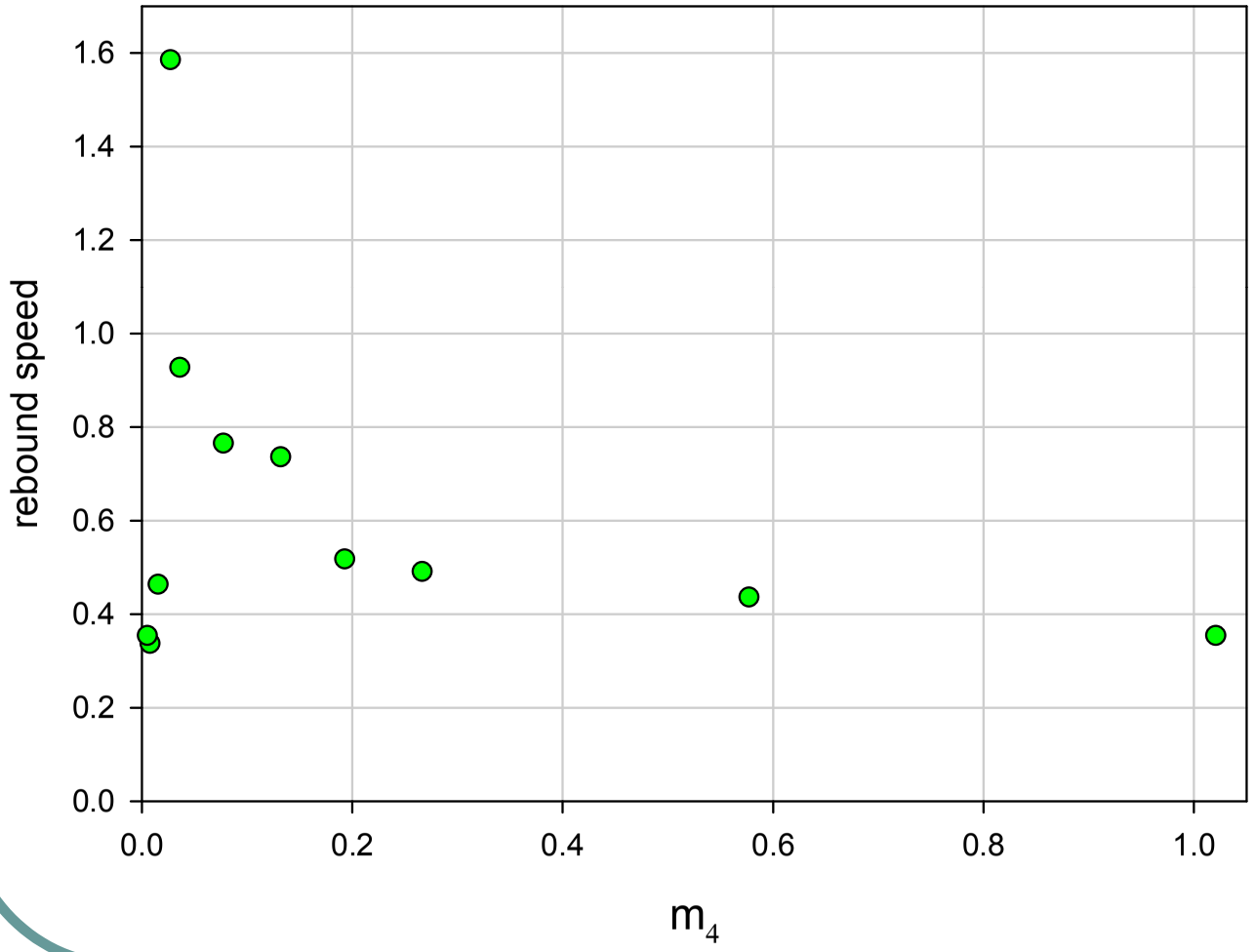
Results cont. (3 balls)



m_1=1,0

m_2=0,015

Results cont. (4 balls)

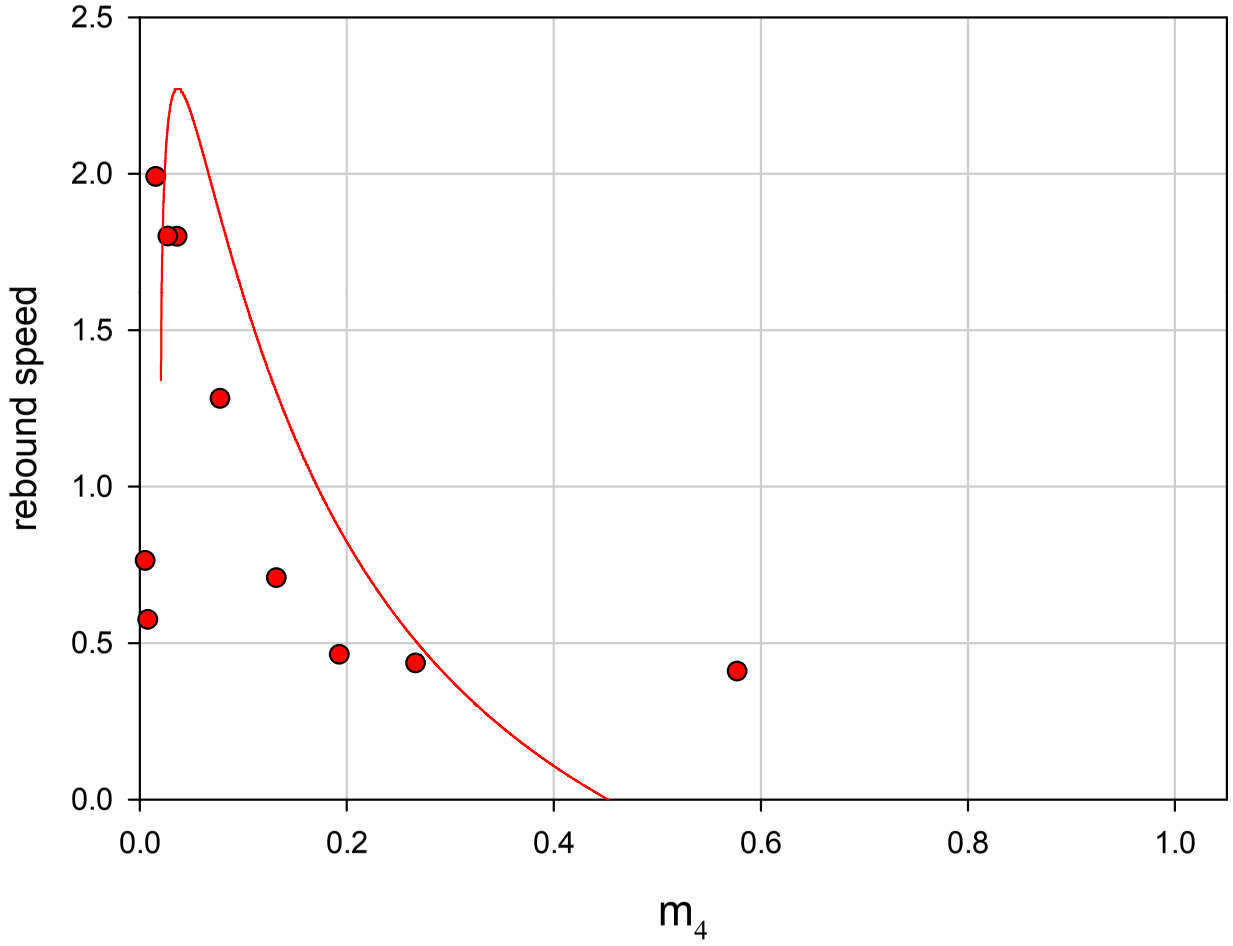


m_1=1,0

m_2=0,072

m_3=0,25

Results cont. (4 balls)

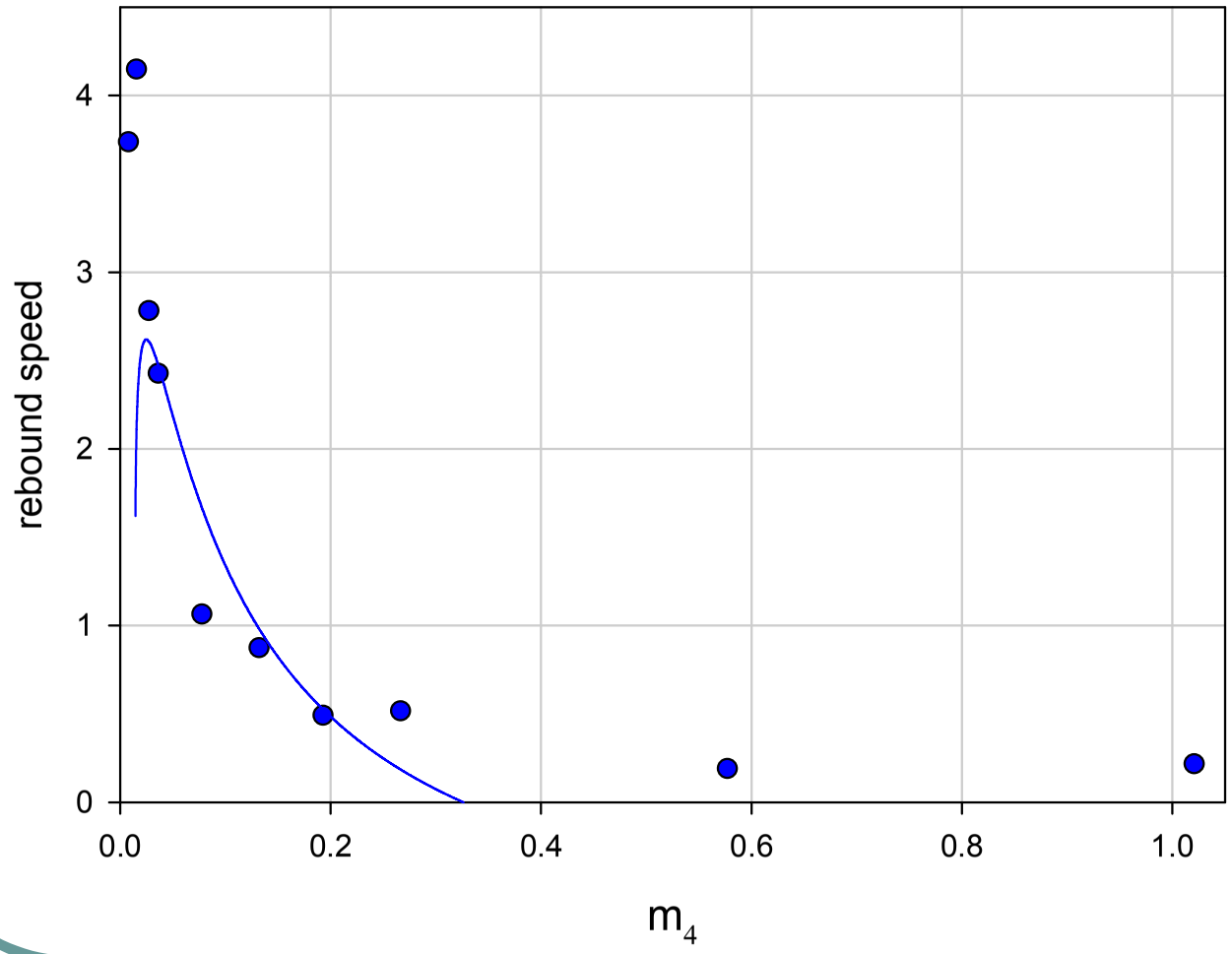


$m_1=1,0$

$m_2=0,18$

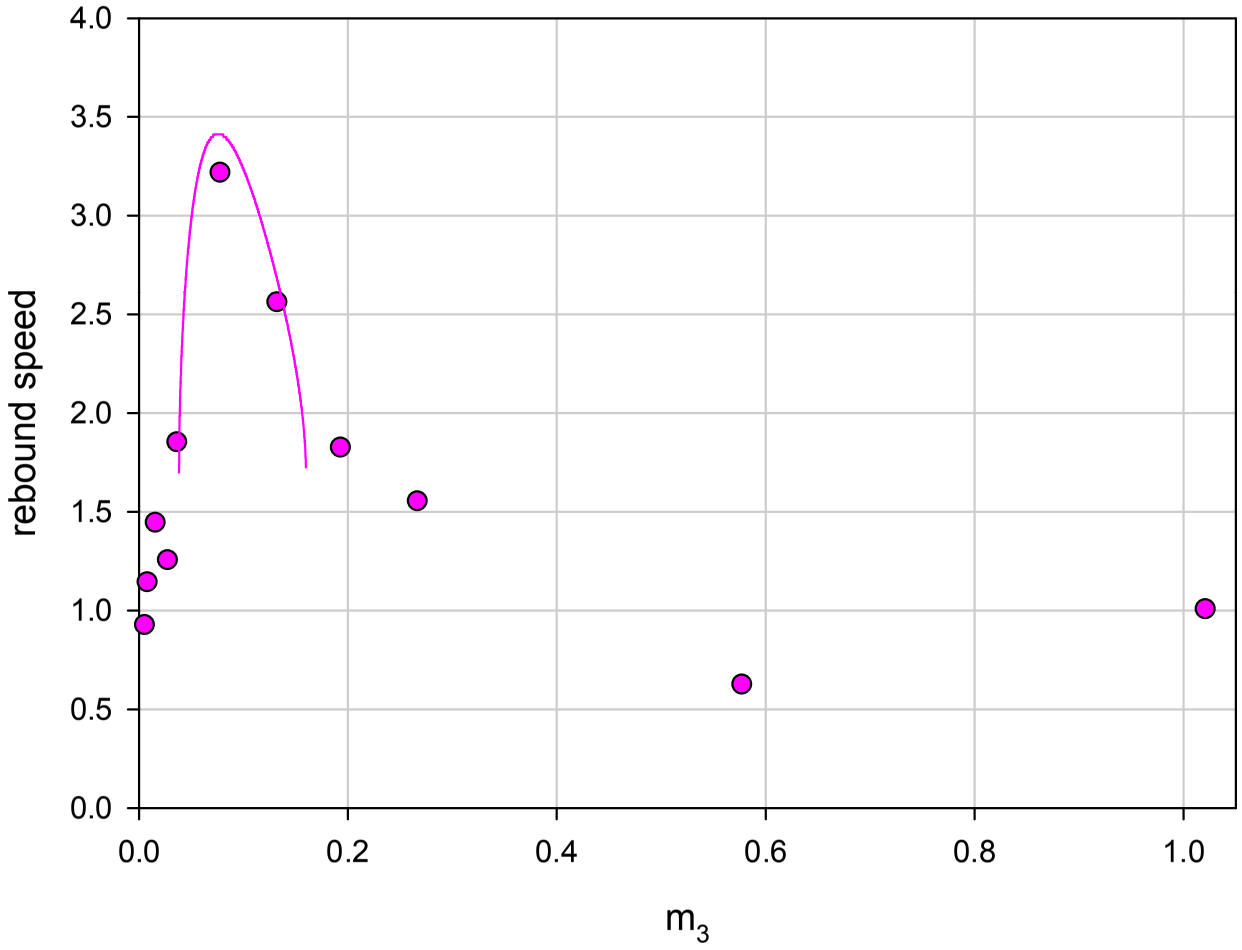
$m_3=0,12$

Results cont. (4 balls)



m_1=1,0
m_2=0,25
m_3=0,072

Results cont. (4 balls)

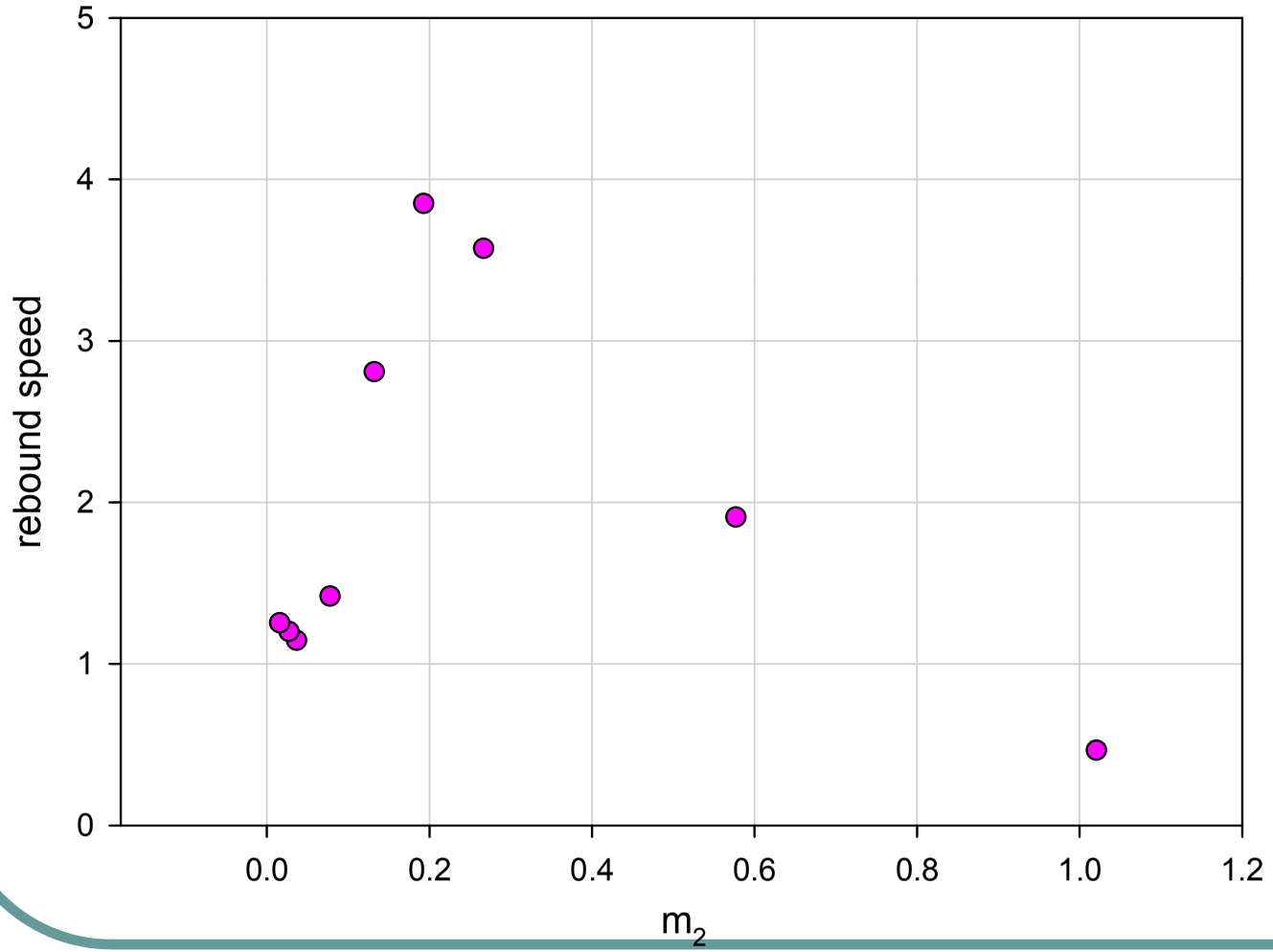


m_1=1,0

m_2=0,25

m_4=0,027

Results cont. (4 balls)



m_1=1,0
m_3=0,072
m_4=0,015

Interpretation of the Results

- Rebound speed smaller then the expected value for large masses and higher for small masses
- Explanation:
  - Larger mass → greater force → greater deformation → more energy lost due to elastic potential energy
  - → Restitution coefficient slightly depends on the mass
  - Considerable collision time → assumption of independent collisions doesn't hold for every combination

Optimization

- Position of the maxima on the x-axis slightly changes in both theory and experimental results

→Possible determination of the overall maxima

- Theoretical maxima calculated by the simulator
  - 1 – overall maximum
  - 2 – maximum obtainable by our masses

The Maximum

- 2 balls:

	m_2	u_2
Theoretical 1	0,19	1,52
Theoretical 2	0,19	1,52
Experimental	0,14	1,51

- 3 balls:

	m_2	m_3	u_3
Theoretical 1	0,19	0,054	2,13
Theoretical 2	0,27	0,072	2,08
Experimental	0,19	0,015	2,67

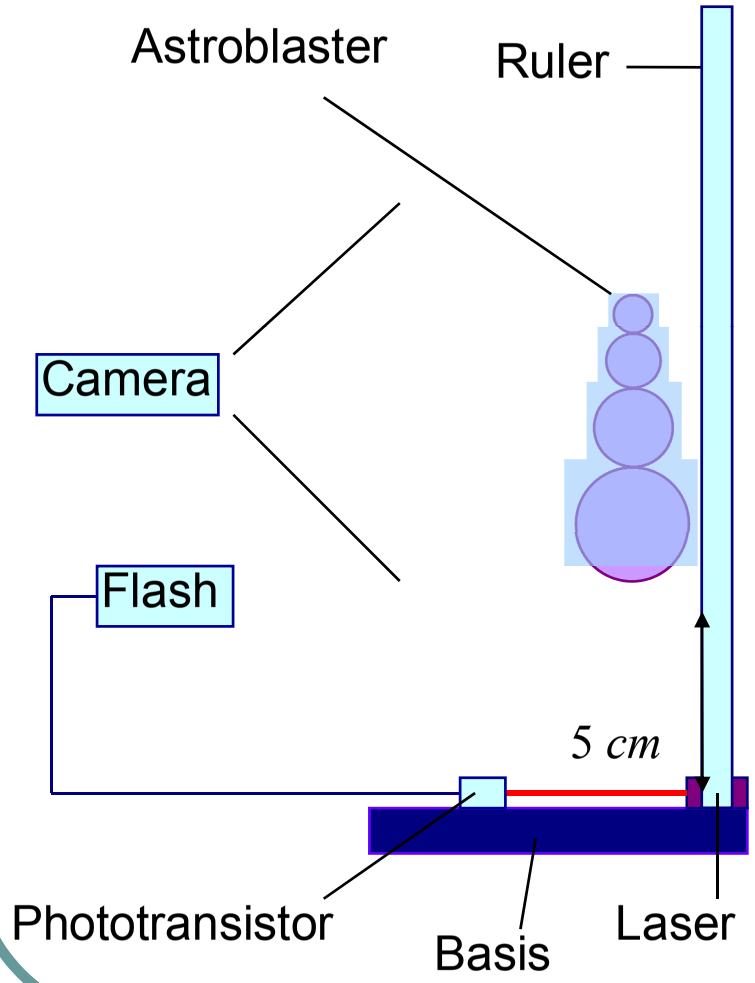
The Maximum

- 4 balls:

	m_2	m_3	m_4	u_4
Theoretical 1	0,19	0,054	0,019	2,64
Theoretical 2	0,27	0,072	0,027	2,60
Experimental	0,19	0,072	0,015	3,85

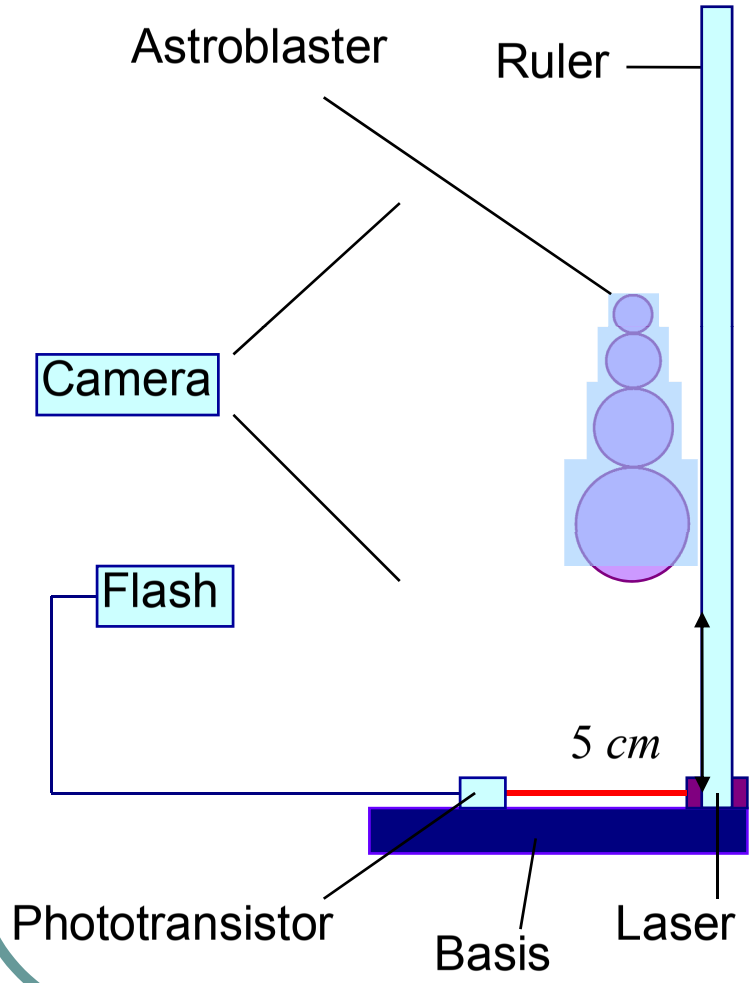
- There is a fixed ideal ratio of two consecutive masses
- Deflection of the results from the theory explained previously

Apparatus 2



- Astroblaster
  - 4 steel balls of radii 50 mm, 25 mm, 12.5 mm and 6,25 mm
  - Cylindric polystyrene aligner with maching inner radii
- Astroblaster dropped on a steel basis from the height $h=5\ cm$

Apparatus (2)



- The interruption of the laser ray triggers the flash
- The rebound photographed with long exposition, litten by the flash 60 times per second
- Ruller for calibration

Results (2)

- $\lambda = 0{,}88$ measured using one of the steel balls

	m_2	m_3	m_4	u_4
Theoretical 1	0,10	0,016	$3{,}2\times10^{-3}$	5,86
Theoretical 2	0,13	0,016	$2{,}0\times10^{-3}$	5,47
Experimental	0,13	0,016	$2{,}0\times10^{-3}$	5,24

- Experimental results in better correspondence with the theory because the aproximations are more applicable

h=1,40 m !



Conclusion

- The phenomenon is a consequence of momentum and energy conservation
- Rebound speed repeatedly increases in succesive collisions
- Dependence of the rebound speed on each mass was measured
- Theoretical and experimental maxima were found
- An optimized astroblaster was made
- Reached height 28 times greater then the initial