Problem 6.

Seebeck effect

Problem

Two long metal strips are bent into the form of an arc and are joined at both ends. One end is then heated. What are the conditions under which a magnetic needle placed between the strips shows maximum deviation?

The basics of the solution

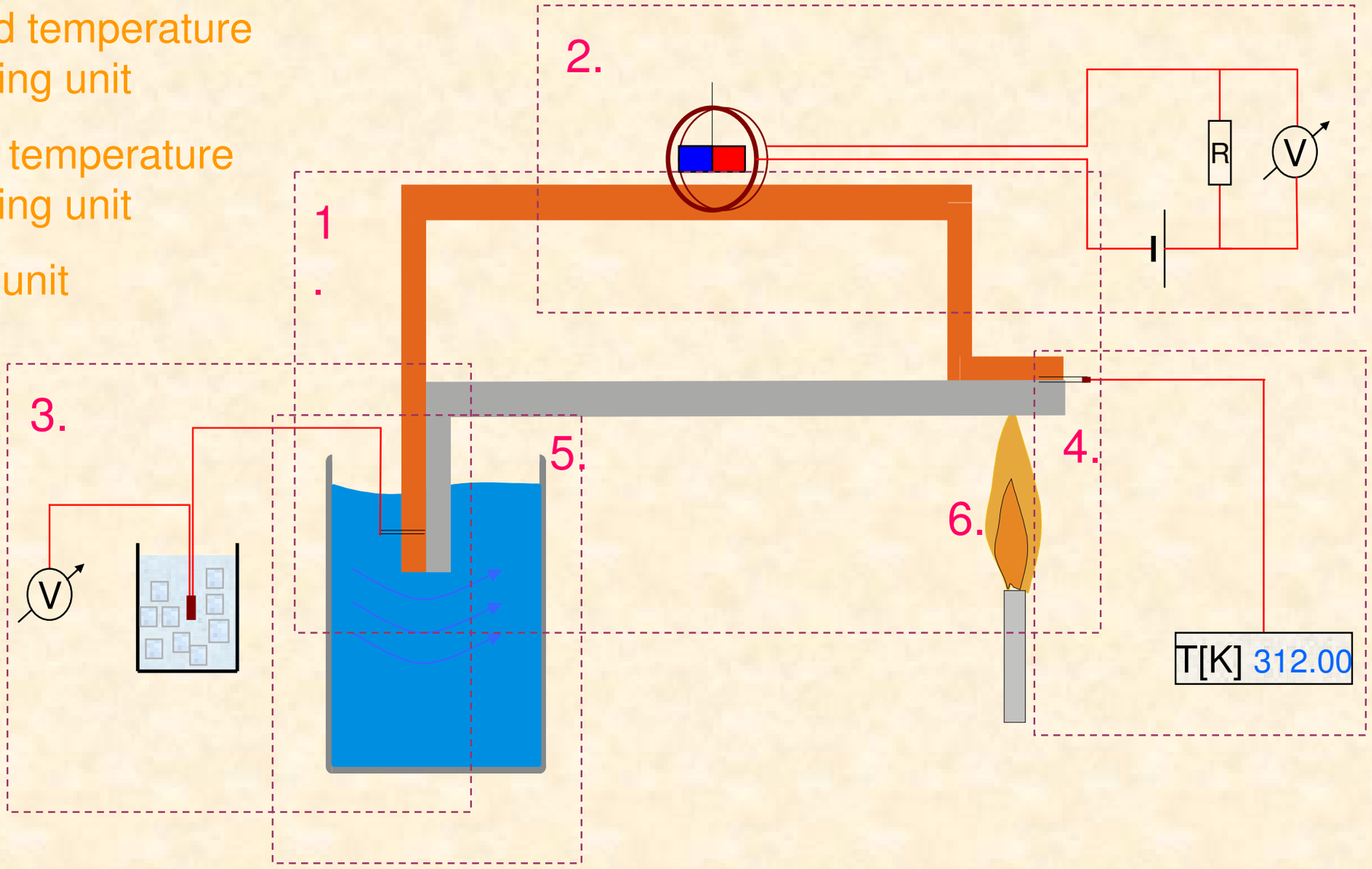
- For the thermocouple we used (Cu - Al) maximum deviation occurs for maximum possible temperature difference between the hot and cold ends
- We discuss the possibility that for couples other then Cu – Al the deviation may show a maximum

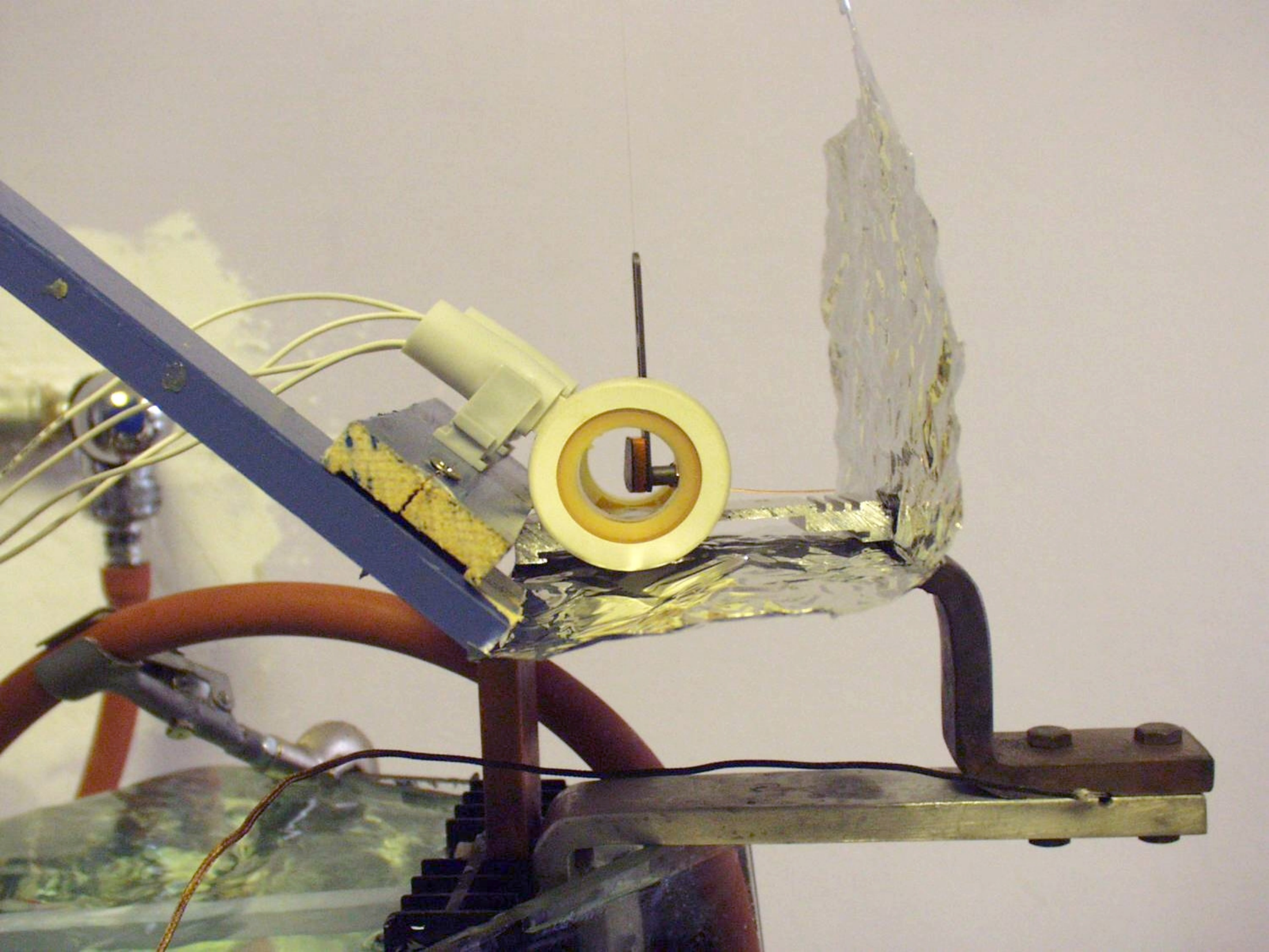
Experimental approach

- **Goals:**
  - Construct the thermocouple
  - Determine the dependency of magnetic field of the thermocouple on temperature difference of the ends
  - Find the field maximum (if present)

Apparatus

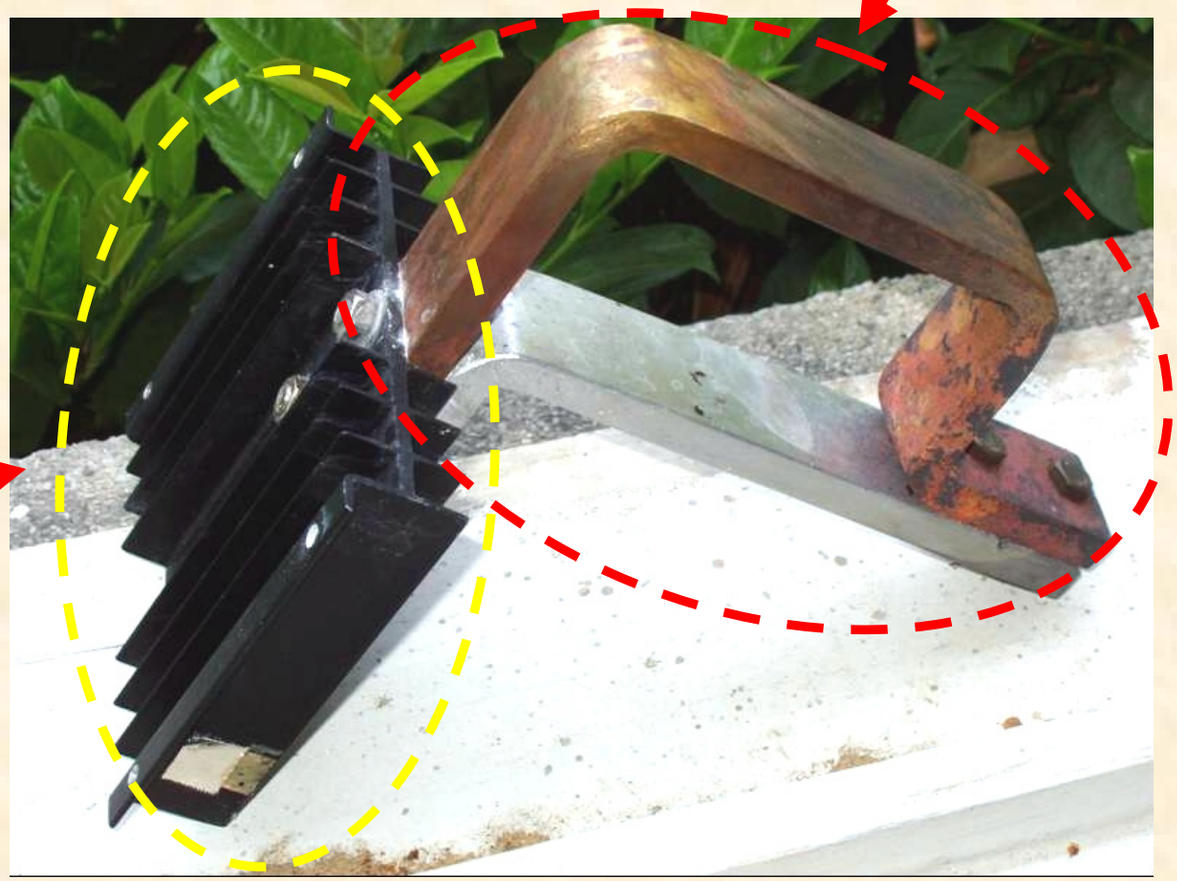
1. thermocouple
2. field measuring apparatus
3. cool end temperature measuring unit
4. hot end temperature measuring unit
5. cooling unit
6. heater



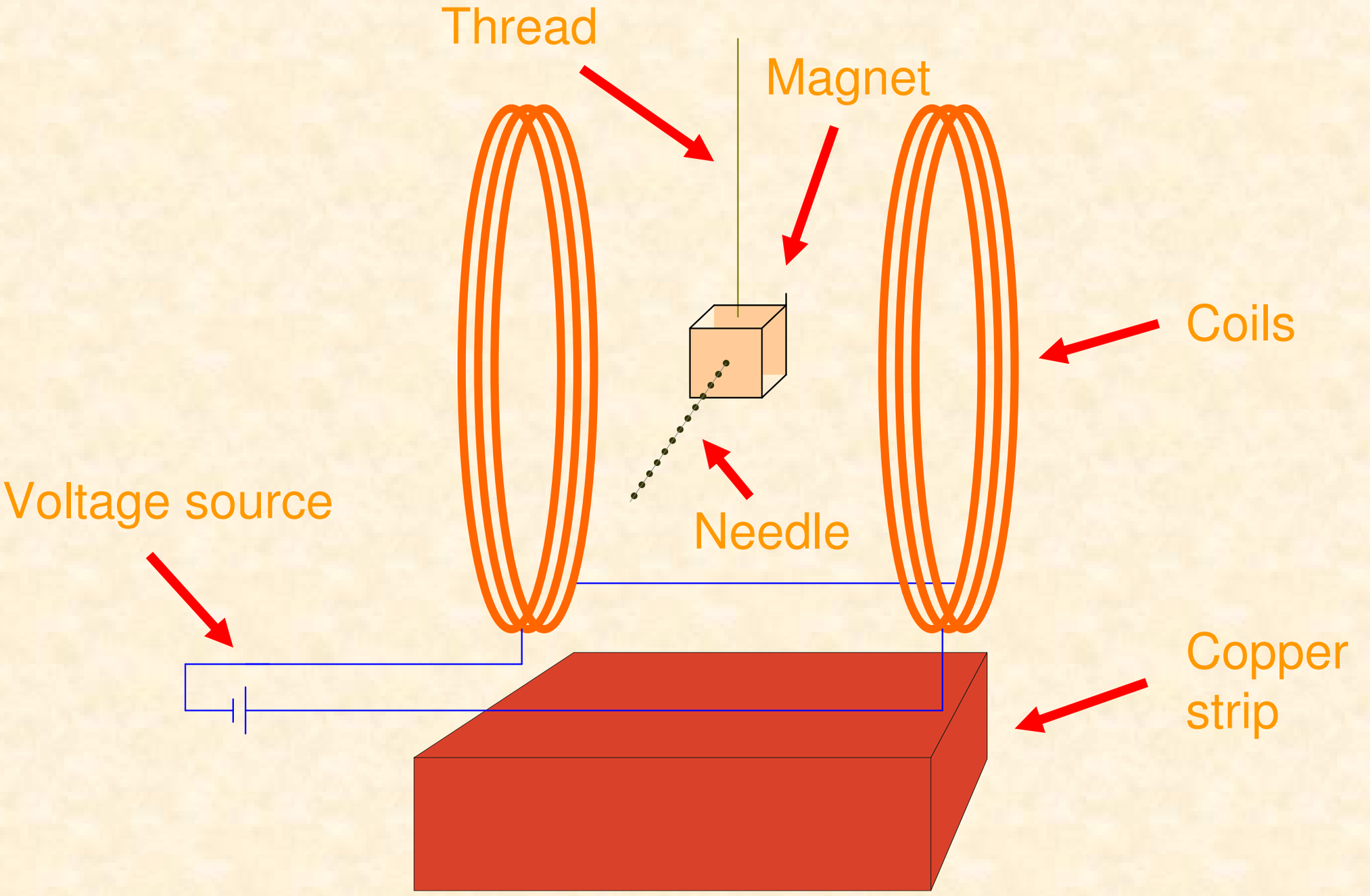


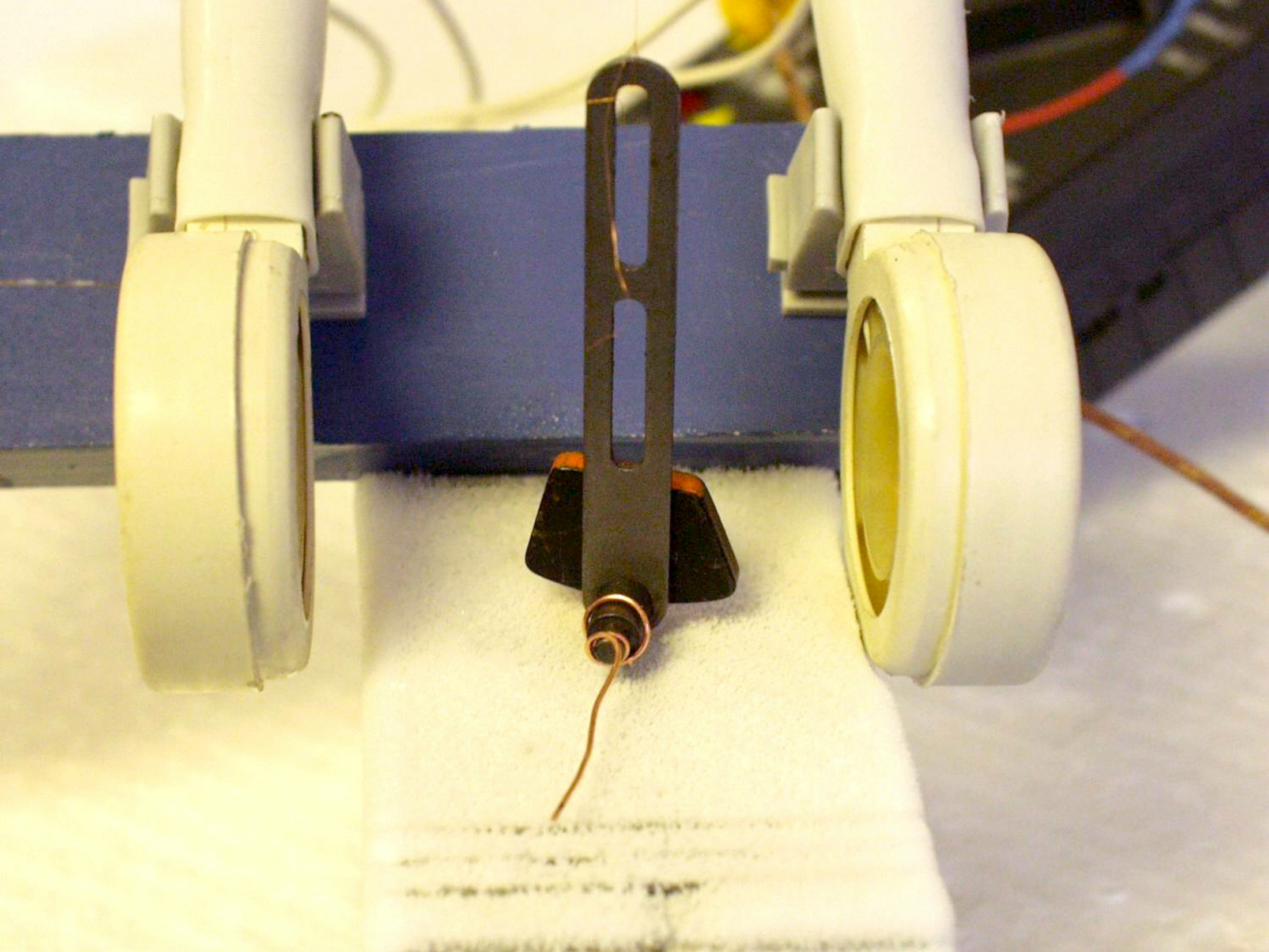
1. Thermocouple

- Material: copper – aluminium
- Width of metal strip: 3 cm
- Height of metal strip: 1 cm
- Net lenght: 17 cm

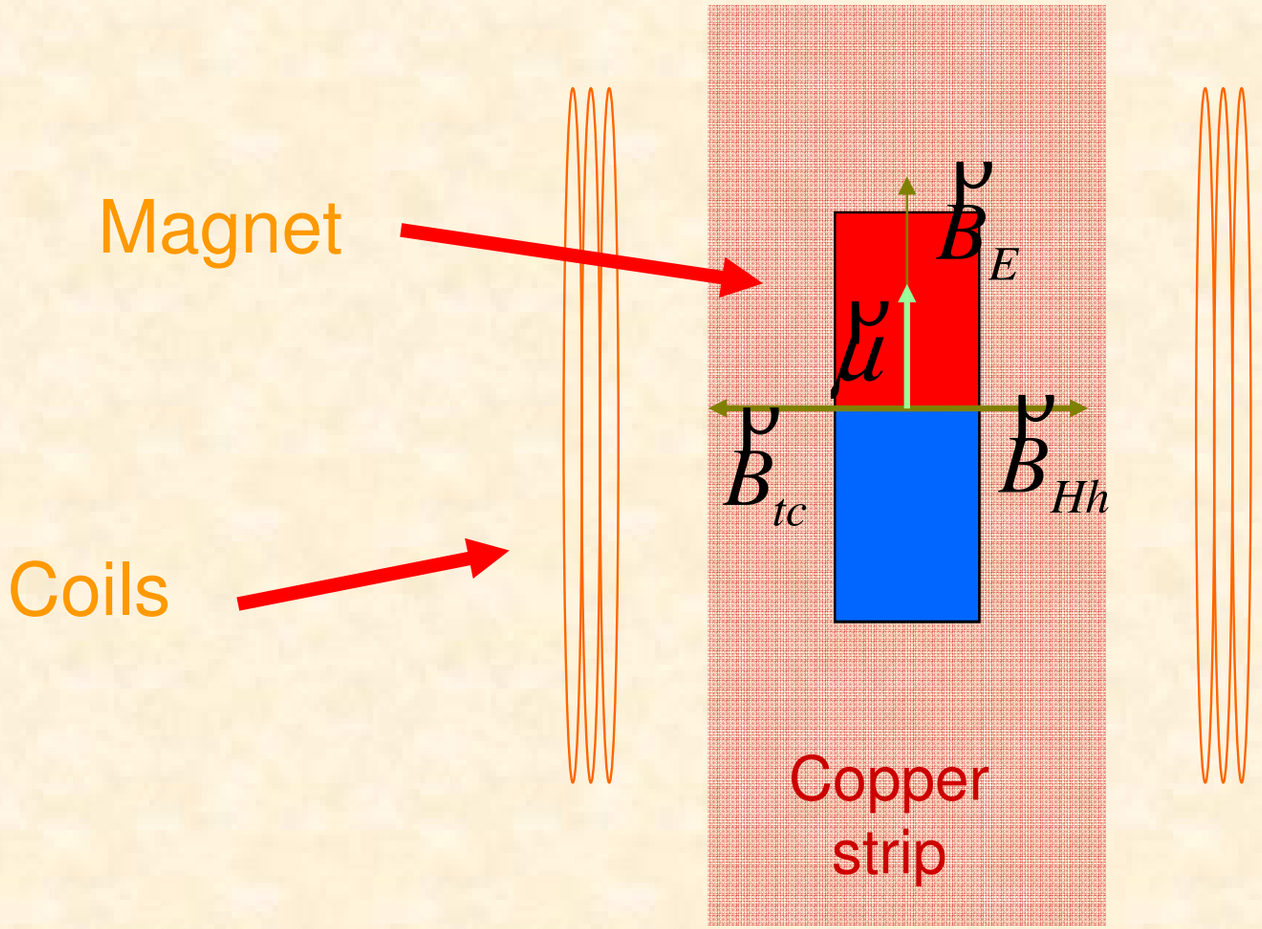


2. Field measuring apparatus





Principle of work



B_{tc} – thermocouple magnetic field

B_{Hh} – magnetic field of the coils

B_E – field of the Earth

μ – magnetic moment of the magnet

- When the field of the coils equals the thermocouple field the only remaining field acting on the magnet is the Earth field

- The magnet turns in the direction of the resultant field because of its magnetic moment:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

τ – torque

μ – magnetic moment of the magnet

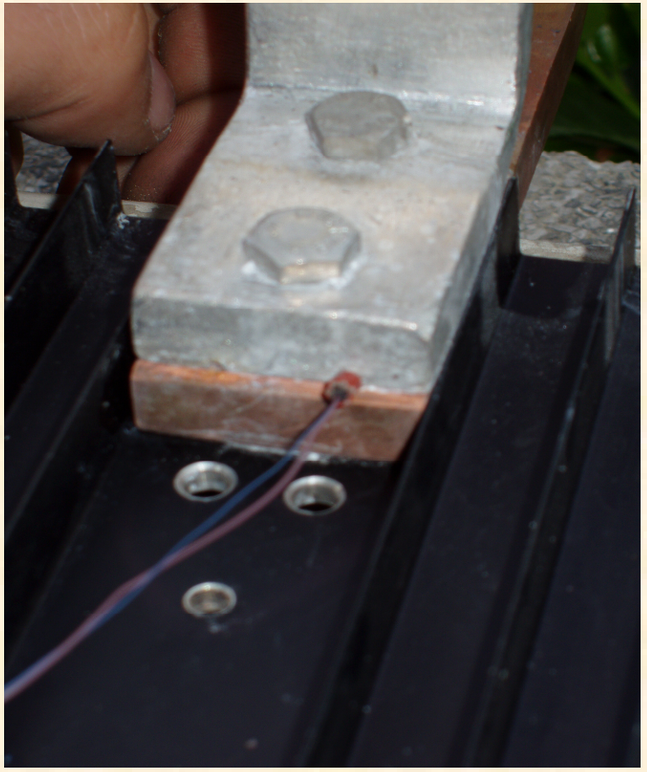
B – external magnetic field

- That torque causes rotation of the magnet until its magnetic moment is aligned with the field
- The equality of coils and thermocouple fields enables us to measure the thermocouple field easily

3. Temperature measuring units

Cool end

- A thermocouple was used



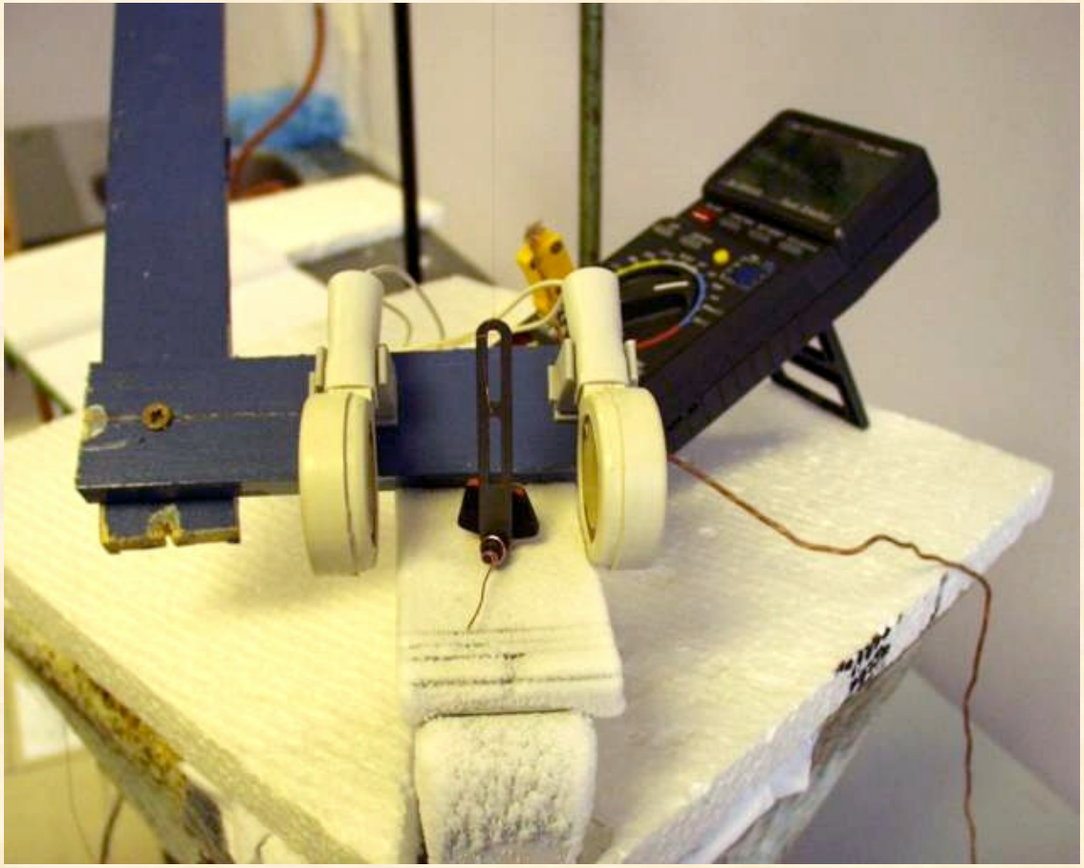
- The temperature was calculated from the obtained voltage

Hot end

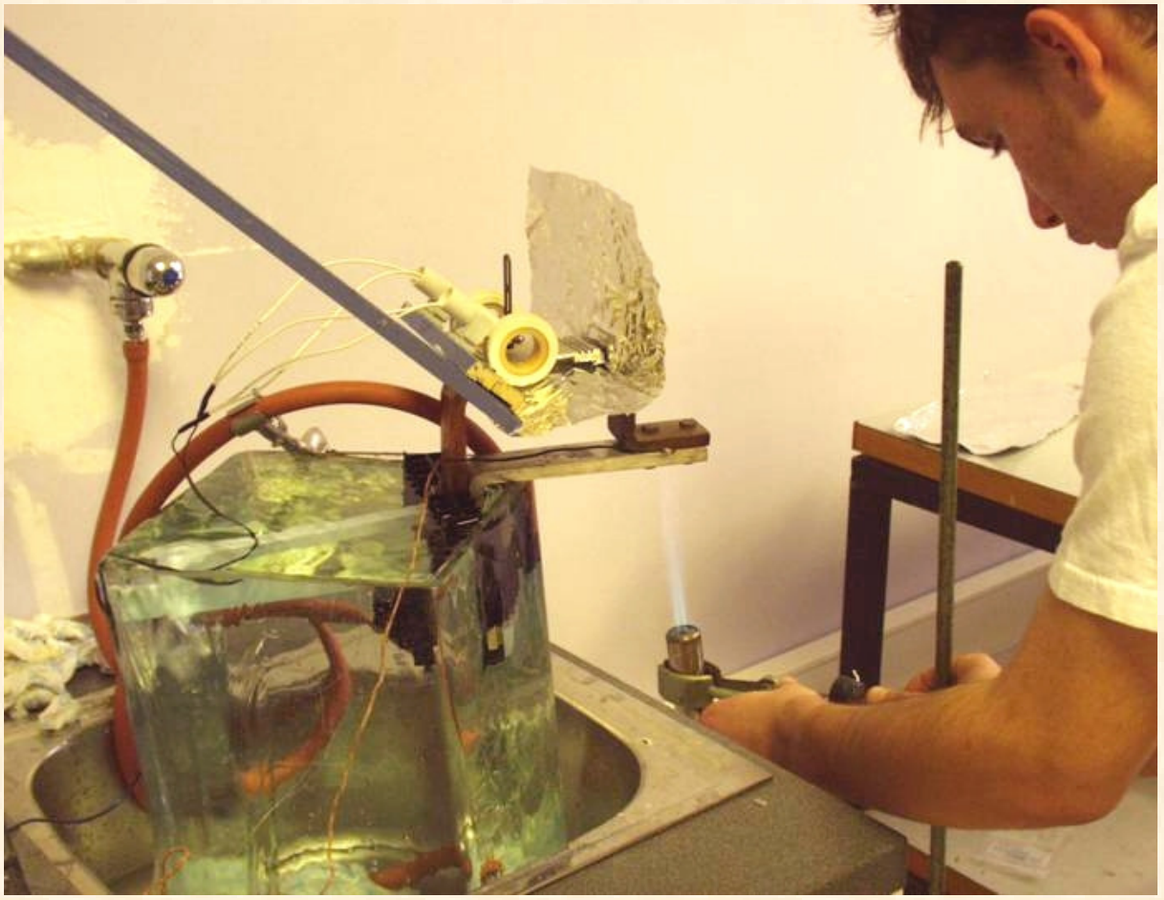
- Because of high temperatures it was appropriate to use Pt - Rh thermocouple

4. Cooler and heater

- The cooling was accomplished by
  - constant water flow over the cooling rib
  - Liquid nitrogen



- The heater was a Danniel burner



- The temperature range of the whole system was

Cool end	Hot end
-150 °C to 50°C	0°C to 600°C

Theoretical approach

Describing the conduction electrons

- Electron gas properties:
  - Great density
  - Momentum conservation (approximately)
  - Free electrons (lattice defects and interactions neglected)

For many metals the free electron model is not valid – introduction of correction parameters

- Mathematical description – Fermi - Dirac energy distribution:

$$f(u) = \frac{1}{1 + e^{\frac{u-\eta}{kT}}}$$

f(u) – probability of finding an electron with energy *u*

η – highest occupied energy level - ˝Fermi level˝

k – Boltzmann constant

T – absolute temperature

- The Fermi level is a characteristic of the metal and depends of electron concentration and temperature:

$$\eta(T) = \eta_0 \left[1 - \frac{\pi}{12} \left(\frac{kT}{\eta_0} \right)^2 + \ldots \right]$$

η_0 – Fermi level at 0 K

Contact potential

- At the join of two different metals a potential difference occurs
- This is due to different electron concentrations in the metals i.e. different Fermi levels
- Concentrations tend to equalize by electron diffusion
- The final potential difference is:

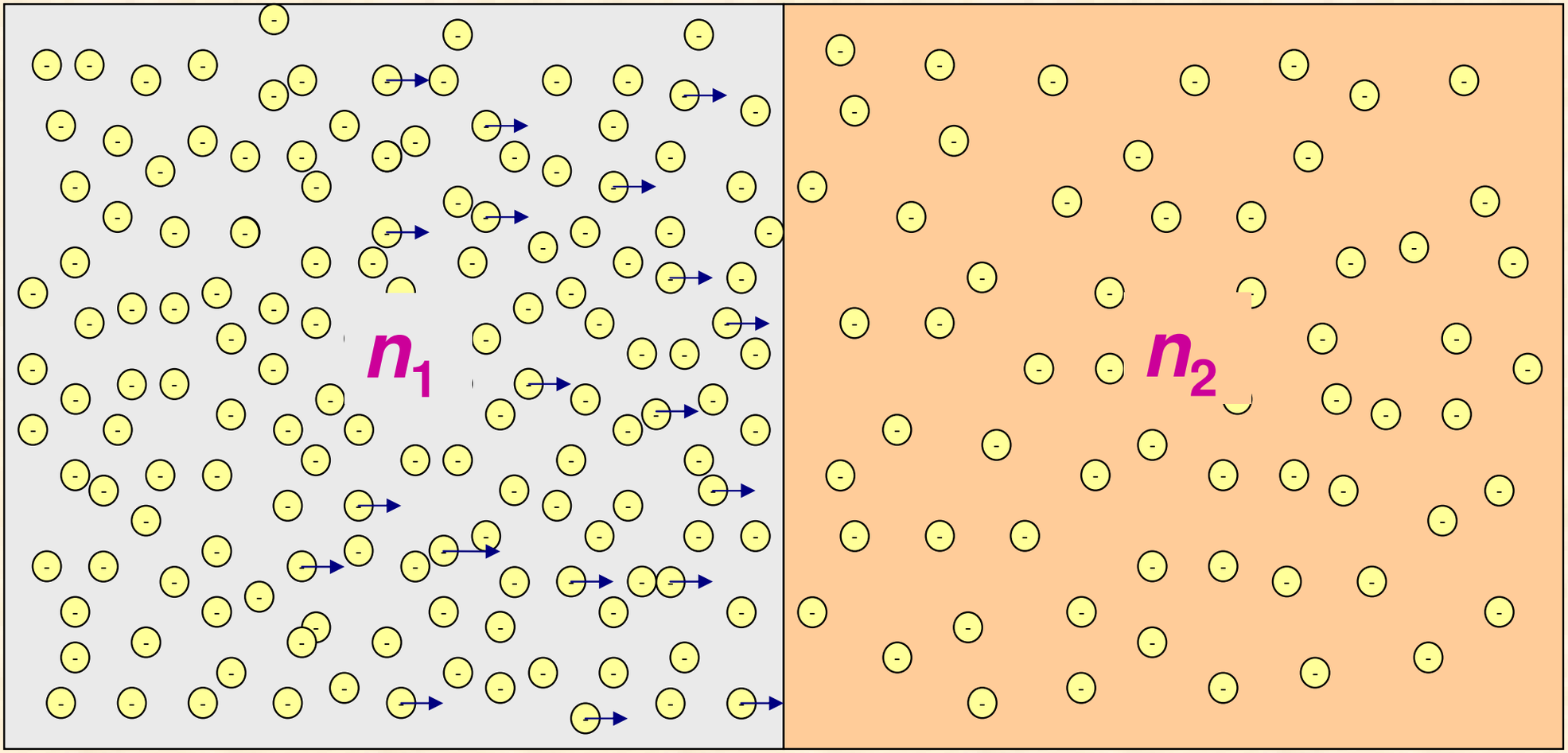
$$\Delta V = \frac{\eta_1 - \eta_2}{e}$$

ΔV – potential difference

η_i – Fermi level of the i – th metal

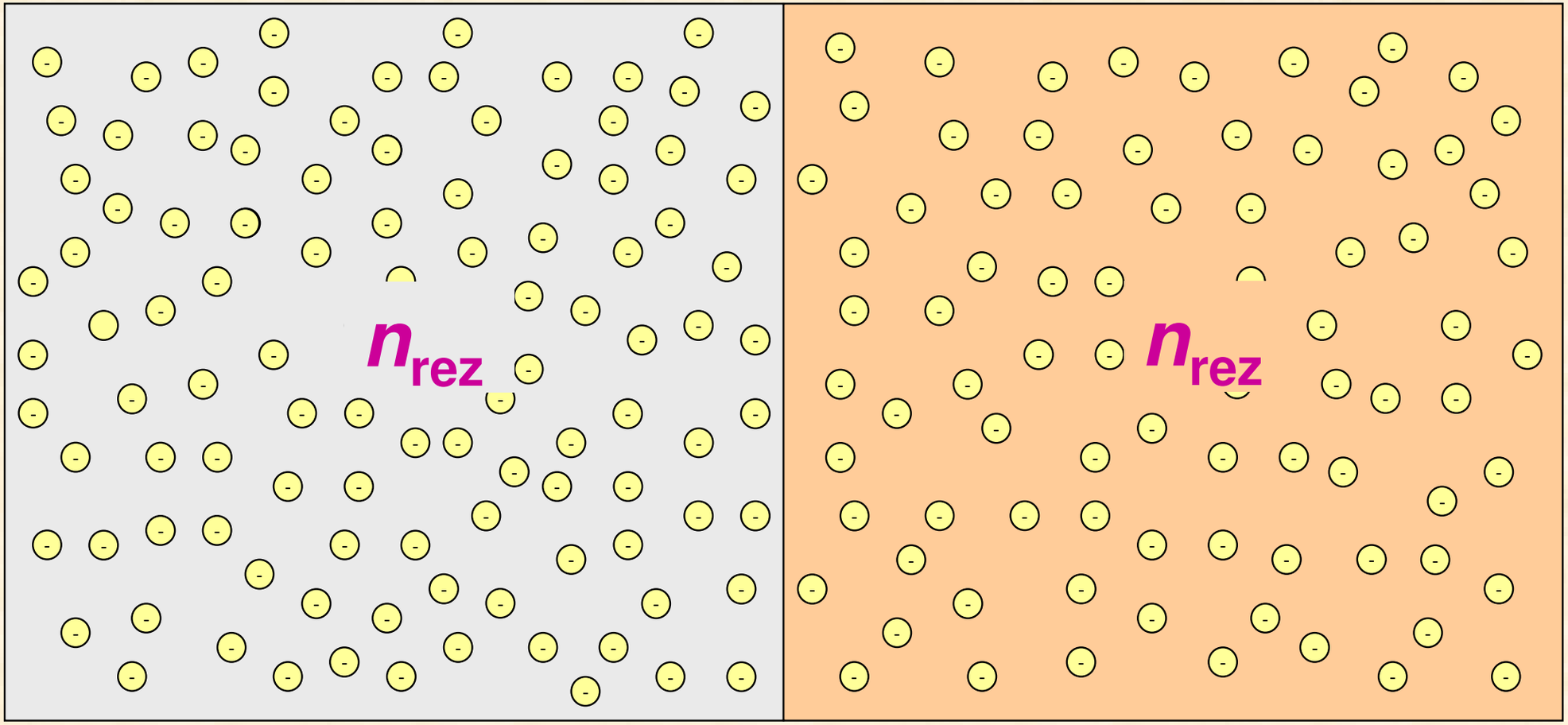
e – charge of the electron

Noneqilibrium



diffusion

Equilibrium

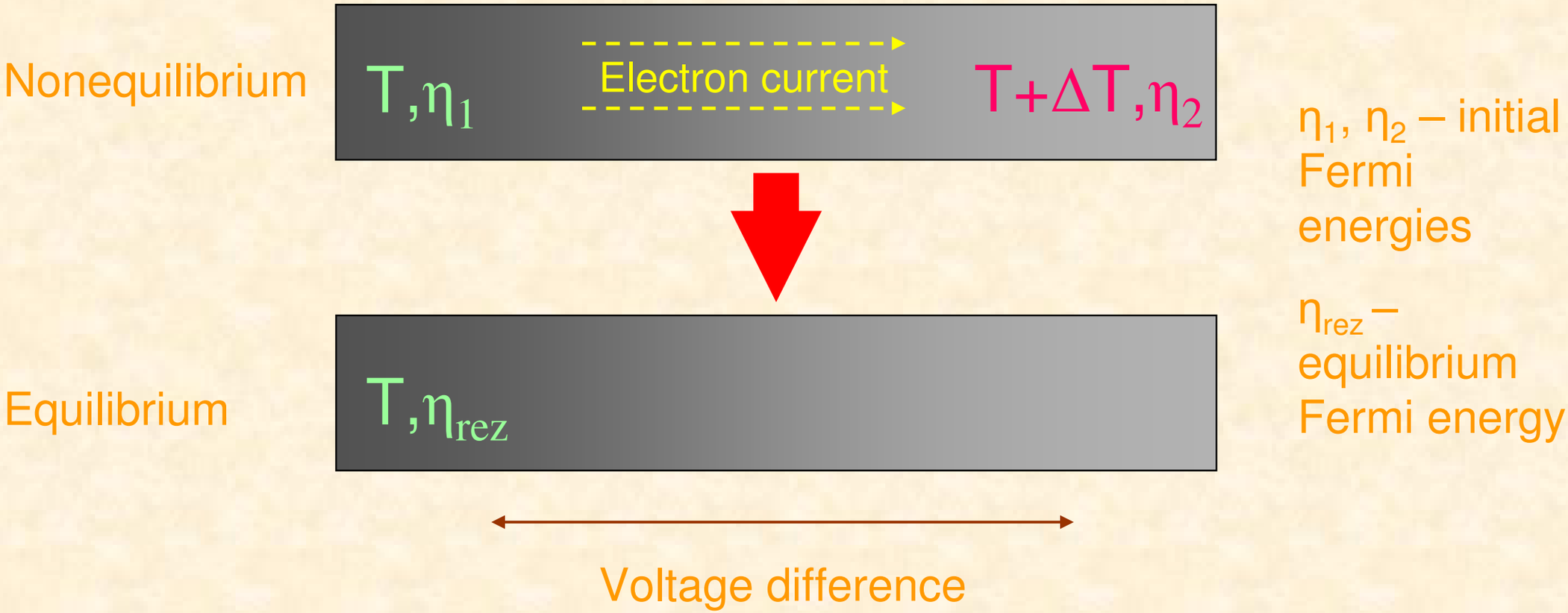


Shortage of electrons – net charge is positive

Surplus of electrons – net charge is negative

Thompson effect

- In a metal strip with ends at different temperatures a potential difference occurs (Thompson effect) because of different Fermi levels:



Seebeck effect

- The voltage in the thermocouple is:

$$dV = S_{AB}\,dT$$

dV – voltage difference

S_{AB} - ˝Seebeck coefficient˝ of the metal pair

$$S_{AB} = S_{AB}(T) \qquad S_{AB} = S_A - S_B$$

S_A, S_B – Seebeck coefficients of single metals (compared to a reference metal)

dT – temeprature difference of the joins

- The Seebeck coefficients can be determined considering the Thompson effect

$$S_A = \frac{\pi^2 k^2 T}{6\eta_0 e}$$

k – Boltzmann constant

T – temeprature of the metal

η_0 – Fermi level at 0 K

e – elementary charge

- For correcting the free electron model a correction constant of order 1 has to be introduced

$$S_A = \frac{\chi\pi^2 k^2 T}{6\eta_0 e}$$

a, b – constants

T_A, T_B – junction temperatures

$$\Rightarrow V_{AB} = a\left(T_B - T_A\right)^2 + b\left(T_B - T_A\right)$$

Seebeck coefficents

metal	Fermi - level at 0 K [eV]	measured S_A [μV/K]	theoretical S_A (free electrons) [μV/K]	hi
Na	3,10	-2,00	7,80	-0,26
Al	11,60	3,50	2,08	1,68
K	2,00	-9,00	12,09	-0,74
Cu	7,00	6,50	3,45	1,88
Ag	5,50	6,50	4,40	1,48
Au	5,50	6,50	4,40	1,48

Source for measured data:

www.materials.usask.ca

Experimental results

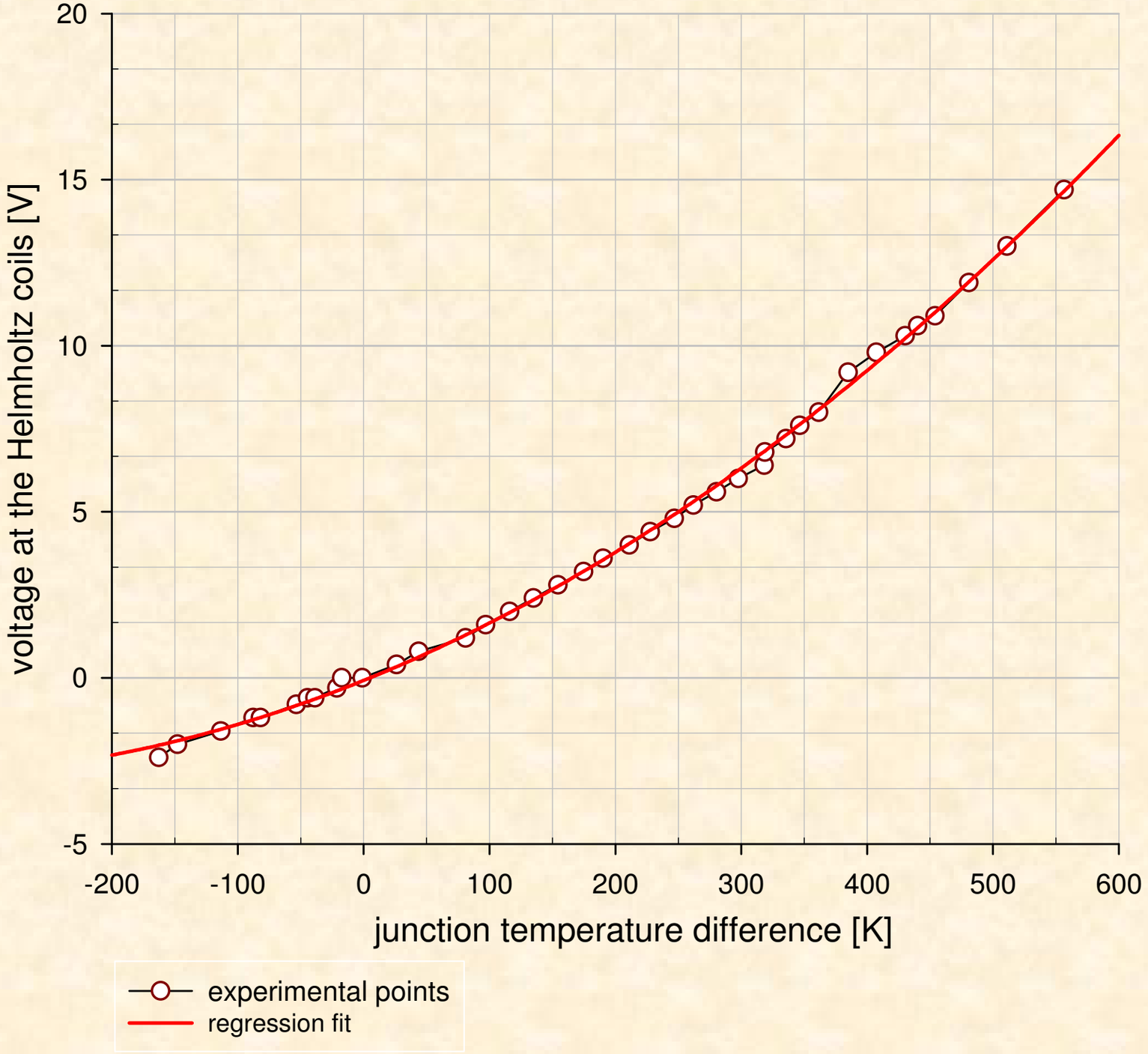
- The field we measured is approximately proportional to the potential difference:

$$\left|\vec{B}_{tc}\right| = \alpha\left(T_B - T_A\right)^2 + \beta\left(T_B - T_A\right)$$

α, β – constants

B_{tc} – thermocouple field

- The constants *A* and *B* have been obtained experimentally but a direct nimerical comparation to the theoretical voltage was impossible due to the unknown coefficient of proportionality:



Fit:

$$\Delta V_{Hh} = 0.0153\Delta T^2 - 0.0784\Delta T$$

ΔT – junction temperature difference

Field constants:

Conclusion

- Thermocouple voltage (and mgnetic field of the couple) will have a maximum if:
  - The constant *a* in the thermocouple equation is negative
  - The constant *b* in the thermocouple equation is positive
- The sign of these constants depends on the metals used in the couple
- For our Cu – Al couple the function $H(\Delta T)$ showed a minimum and no maximum

